CSE303 Q4 SOLUTIONS Spring 2019

PART 1 (10pts) YES/NO QUESTIONS - **(2pts) each**

1. $L \subseteq \Sigma^*$ is context-free if and only if $L = L(G)$
   Justify: It is a definition of context-free language PROVIDED G is a context -free grammar **n**

2. $L(G) = \{w \in V : S \Rightarrow^*_G w\}$
   Justify: $w \in \Sigma^*$ **n**

3. Any regular language is accepted by a pushdown automaton
   Justify: Any regular language is accepted by a finite automata and any finite automata is a pushdown automata operating on an empty stock. **y**

4. Context-free languages are closed under intersection
   Justify: Take $L_1 = a^n b^n c^m$, $L_2 = a^m b^n c^n$, both CF and we get that $L_1 \cap L_2 = a^n b^n c^n$ is not CF **n**

5. The union of a context-free language and regular language is a context-free language
   Justify: regular language is also a context free language and context free languages are closed under union **y**

6. Every subset of a regular language is a language.
   Justify: a subset of a set is a set. **y**

7. Any context -free language is accepted by some PD automata
   Justify: Main Theorem for Context Free Languages **y**

QUESTION 1 (8pts)

1. Construct a context-free grammar G such that

$$L(G) = \{w \in \{a,b\}^* : \ w = w^R\}.$$

Solution $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S\}, \quad \Sigma = \{a, b\},$$

$$R = \{S \to aSa \mid bSb \mid a \mid b \mid e\}.$$

2. Give an example of a derivation of a word *ababa* in G

Derivation example

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow ababa$$

3. Write a proof that the word $ababa \in L(G)$ fulfills the required property $w = w^R$

Proof

We know that $(xy)^R = y^R x^R$ and $(xyz)^R = z^R (xy)^R = z^R y^R x y^R$

We evaluate

$$(ababa)^R = ((ab)a(ba))^R = (ba)^R a^R (ab)^R = ababa$$

QUESTION 2 (7pts)

1. Construct a **pushdown** automaton M such that

$$L(M) = \{w \in \{a,b\}^* : \; w = w^R\}$$

JUST DRAW A DIAGRAM. Here ARE the Components

Solution $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$K = \{s, f\}, \;\; \Sigma = \{a, b\}, \;\; \Gamma = \{a, b\}, \;\; F = \{f\}, s = s,$$

$$\Delta = \{((s, a, e), (s, a)), \; ((s, b, e), (s, b)), \; ((s, e, e), (f, e)),$$

$$((s, a, e), (f, e)), \; ((s, b, e), (f, e)), \; ((f, a, a), (f, e)), \; ((f, b, b), (f, e))\}$$

2. Trace a transitions of M that lead to the acceptance of the string $ababa$.

Solution

s	$ababa$	e
s	$baba$	a
s	aba	ba
f	ba	ba
f	a	a
f	e	e

EXTRA

Proof - GENERAL CASE for Question 1

1. Construct a context-free grammar G such that

$$L(G) = \{w \in \{a,b\}^* : \; w = w^R\}.$$

Solution

$G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S\}, \;\; \Sigma = \{a, b\},$$

$$R = \{S \to aSa \mid bSb \mid a \mid b \mid e\}.$$

Here is a **general case- for you to study** (we did it in class!)

Grammar correctness justification

Observation We proved in class that for any $x, y \in \Sigma^*$, $(xy)^R = y^R x^R$. From this we have that

$$(xyz)^R = ((xy)z)^R = z^R(xy)^R = z^R y^R x^R$$

observe that the rules $S \to aSa \mid bSb \mid e$ generate the language $L_1 = \{ww^R : w \in \Sigma^*\}$. With additional rules $S \to a \mid b$ we get hence the language $L = L_1 \cup \{waw^R : w \in \Sigma^*\} \cup \{wbw^R : w \in \Sigma^*\}$. Now we are ready to prove that

$$L = L(G) = \{w \in \{a, b\}^* : w = w^R\}.$$

Proof Let $w \in L$, i.e. $w = xx^R$ or $w = xax^R$ or $w = xbx^R$. We show that in each case $w = w^R$ as follows.

c1: $w^R = (xx^R)^R = (x^R)^R x^R = xx^R = w$ (used property: $(x^R)^R = x$).

c2: $w^R = (xax^R)^R = (x^R)^R a^R x^R = xax^R = w$ (used Observation 1 and properties: $(x^R)^R = x$ and $a^R = a$).

c3: $w^R = (xbx^R)^R = (x^R)^R b^R x^R = xbx^R = w$ (used Observation and properties: $(x^R)^R = x$ and $b^R = b$).