CSE303 Q2 SOLUTIONS

PART 1 YES/NO QUESTIONS

Circle the correct answer. Write SHORT justification.

1. The set F of final states of any non-deterministic finite automaton is always non-empty

   Justify: The definition says that F is a finite set, i.e. can be empty, hence for some M, $L(M) = \emptyset$ **n**

2. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a **transition relation** iff the following condition holds
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$

   Justify: Proper condition is: $(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$ **n**

3. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.

   Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$

   y

PART 2 Very Short Questions

Q1: M1 has components: $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1\}$

$\delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$

Solution

1. $M1$ is non-deterministic; δ is not a function with the domain $K \times \Sigma$. It can be completed to a function by adding some trap states. But the trap states information was not stated in the problem - so **M1** is NDFA

2. $L(M1) = aa^*$

Q2: M2 has $K = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $F = \{q_1, q_2\}$,

$\Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$

Solution

M2 is **not** DFA and is **not** NDFA because M2 **is NOT an automaton** It does not have the INITIAL state.

L(M2) does not exist - as languages are defined for automatas

PART 3 PROBLEMS

QUESTION 1

Components of an automaton M are:

$K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1, q_3\}$ and $\delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3)\}$

Complete the components to a full definition of a deterministic M by adding trap state(s)

Write a regular expression defining $L(M)$

Solution

YOU ONLY DRAW the diagram - and compare with the listed components below

You do NOT write components

We add a new state q_4 and extend δ to function δ_1 such that
$\delta_1 : (K \cup \{q_4\}) \times \Sigma \longrightarrow K \cup \{q_4\}$ as follows

$$\delta_1 = \delta \cup \{(q_0, b, q_4), (q_1, a, q_14), (q_3, a, q_4), (q_3, b, q_4)\}$$

$$L(M) = a \cup aba^*b$$

QUESTION 2

Let $M = (K, \ \Sigma, \ s, \ \Delta, \ F\ )$ for $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_1, q_2\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_0, b, q_2)\}$

Draw the diagram of an automaton M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution

ONLY DRAW the diagram - and compare with the listed BELOW components

DO NOT write components

We apply the **"stretching"** technique to M and the new M' is is as follows.

$$M' = (K \cup \{p_1, p_2, p_3\}, \ \ \Sigma, \ \ s = q_0, \ \ \Delta', \ \ F' = F\ )$$

$$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), \ (p_1, b, p_2), \ (p_2, c, q_0), \ (q_0, b, q_2), \ (q_0, a, p_3), \ (p_3, b, q_1)\}$$