PART 1  YES/NO QUESTIONS - (2pts) each

Circle the correct answer. Write SHORT justification.

1. The set $F$ of final states of any non-deterministic finite automaton is always non-empty
   **Justify:** The definition says that $F$ is a finite set, i.e. can be empty, hence for some $M$, $L(M) = \emptyset$  

2. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a transition relation iff the following condition holds
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q'$
   **Justify:** Proper condition is: $(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$

3. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture.
   **Justify:** $\Sigma \cup \{e\} \subseteq \Sigma^*$

PART 2 Very Short Questions - (3pts) each

Q1: $M_1$ has components: $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1\}$

$\delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$

**Solution**

1. $M_1$ is non-deterministic; $\delta$ is not a function with the domain $K \times \Sigma$. It can be completed to a function by adding some trap states. But the trap states information was not stated in the problem - so $M_1$ is NDFA

2. $L(M_1) = aa^*$

Q2: $M_2$ has $K = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $F = \{q_1, q_2\}$,

$\Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$

**Solution**

$M_2$ is not DFA and is not NDFA because $M_2$ is NOT an automaton. It does not have the INITIAL state.

$L(M_2)$ does not exist - as languages are defined for automatas

PART 3 PROBLEMS

**QUESTION 1** (6pts)

Components of an automaton $M$ are:

$K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1, q_3\}$ and $\delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3)\}$

**Complete** the components to a full definition of a deterministic $M$ by adding trap state(s)

**Write** a regular expression defining $L(M)$

**Solution**

YOU ONLY DRAW the diagram - and compare with the listed components below

You do NOT write components
We add a new state $q_4$ and extend $\delta$ to function $\delta_1$ such that

$\delta_1 : (K \cup \{q_4\}) \times \Sigma \rightarrow K \cup \{q_4\}$ as follows

$\delta_1 = \delta \cup \{(q_0, b, q_4), (q_1, a, q_1), (q_3, a, q_4), (q_3, b, q_4)\}$

$L(M) = a \cup aba^*b$

QUESTION 2 (7pts)

Let $M = (K, \Sigma, s, \Delta, F)$ for $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_1, q_2\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_0, b, q_2)\}$

Draw the diagram of an automaton $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

Solution

YOU ONLY DRAW the diagram - and compare with the listed BELOW components

You do NOT write components

We apply the "stretching" technique to $M$ and the new $M'$ is as follows.

$M' = (K \cup \{p_1, p_2, p_3\}, \Sigma, s = q_0, \Delta', F' = F')$

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, b, q_2), (q_0, a, p_3), (p_3, b, q_1)\}$