PART 1: YES/NO QUESTIONS

Circle your answer and write a short justification (1pt) No justification- no points

1. \{a, \emptyset, 2\} \cap \emptyset = \emptyset
   Justify: \emptyset = \{\emptyset\} and \emptyset \notin \{a, \emptyset, 2\}  

2. The relation \(R = \{(n, m): n \in \mathbb{N} \text{ and } m = 1\}\) is a function
   Justify: \(R\) is a function defined by the formula is \(R(n) = 1\) for all \(n \in \mathbb{N}\)

3. If the \(A\) is uncountable, then \(|A| = \mathbb{C}\)
   Justify: The set \(2^\mathbb{R}\) of all subsets of real numbers \(R\) is uncountable, but by Cantor Theorem we have that \(|2^\mathbb{R}| > |\mathbb{R}| = \mathbb{C}\).

4. The set \(A = \{x \in \mathbb{N}: x \geq 0\}\) is finite
   Justify: \(A = \{x \in \mathbb{N}: x \geq 0\}\) = \(\mathbb{N}\) and \(\mathbb{N}\) is countably infinite

5. The set \(\{\emptyset, \{\emptyset\}, 1, 2\}\) is an alphabet 
   Justify: \(A\) is a finite set and any finite set is an alphabet by definition

6. Let \(\Sigma = \{\emptyset\}\). There are uncountably many languages over \(\Sigma\)
   Justify: \(\Sigma \neq \emptyset\) as \(\emptyset\) is element of \(\Sigma\). Hence there are infinitely countably many elements in \(\Sigma^*\) and uncountably many subsets of \(\Sigma^*\). In fact exactly as many as real numbers as \(|2^{\Sigma^*}| = |\mathbb{R}| = \mathbb{C}|\)

7. For any languages \(L_1, L_2, L\) over \(\Sigma \neq \emptyset\)
   \((L_1 \cap L_2) \cup L = (L_1 \cup L_2) \cap (L_2 \cup L)\)
   Justify: Languages are sets and this is the Law of Distributivity of union over intersection of sets

8. \(L^* = \{w_1...w_n: w_i \in L, i = 1, 2,..n, n \geq 1\}\)
   Justify: This is definition of \(L^+\); Kleene Star must have condition \(n \geq 0\).

9. Regular language is a regular expression.
   Justify: Regular language is defined by a regular expression.

10. For any language \(L\) over an alphabet \(\Sigma\), \(L^+ = L \cup L^*\).
    Justify: holds only only when \(e \in L\) as \(e \in L^*\)

PART 2

QUESTION 1

Given an alphabet \(\Sigma = \{a, b\}\) and a regular expression \(\alpha = a^*b \cup (a \cup b)^*\).

1. (3pts) Evaluate \(L = \mathcal{L}(\alpha)\).
Solution

1. We evaluate

\[ L = \mathcal{L}(a^*b \cup (a \cup b)^*) = \mathcal{L}(a^*)\mathcal{L}(b) \cup (\mathcal{L}(a) \cup \mathcal{L}(b))^* = \{a\}^*\{b\} \cup \{a, b\}^* \]

2. (4pts) Give a property describing the language \( L \) determined by \( \alpha \)

Solution

Observe that \( \{a\}^*\{b\} \subseteq \{a, b\}^* \) hence the language is

\[ L = \{w : w \in \{a, b\}^*\} = \Sigma^* \]

QUESTION 2

Let \( \Sigma = \{a, b\} \) and a language \( L \subseteq \Sigma^* \) be defined as follows:

\[ L = \{w \in \Sigma^* : w \text{ contains less then two } a's\} \]

Write a regular expression \( \alpha \), such that \( \mathcal{L}(\alpha) = L \). Use shorthand notation. Explain shortly your answer.

Solution

\[ \alpha = b^* \cup b^*ab^* \]

Explanation

"w contains less then two a's" means "contains zero or one a"

\( b^* \) contains no of a \ (case n=0)

\( b^*ab^* \) contains one occurrence of a \ (case n=1)