CSE303 Q1 Solutions

PART 1: YES/NO QUESTIONS

Circle your answer and **write** a short justification **(1pt)** **No justification- no points**

1. $\{a, \{\emptyset\}, 2\} \cap 2^{\emptyset} = \emptyset$
   Justify: $2^{\emptyset} = \{\emptyset\}$ and $\emptyset \notin \{a, \{\emptyset\}, 2\}$

   y

2. The relation $R = \{(n, m) : \ n \in N \text{ and } m = 1\}$ is a function
   Justify: R is a function defined by the formula is $R(n) = 1$ for all $n \in N$

   y

3. If the A is uncountable, then $|A| = \mathcal{C}$
   Justify: The set 2^R of all subsets of real numbers R is uncountable, but by Cantor Theorem we have that $|2^R| > |R| = \mathcal{C}$.

   n

4. The set $A = \{x \in N : \ x \geq 0\}$ is finite
   Justify: $A = \{x \in N : \ x \geq 0\} = N$ and N is countably infinite

   n

5. The set $\{\emptyset, \{\emptyset\}, 1, 2\}$ is an alphabet
   Justify: A is a finite set and any finite set is an alphabet by definition

   y

6. Let $\Sigma = \{\emptyset\}$. There are uncountably many languages over Σ
   Justify: $\Sigma \neq \emptyset$ as $\emptyset$ is element of Σ. Hence there are infinitely countably many elements in $\Sigma*$ and uncountably many subsets of Σ^*. In fact exactly as many as real numbers as $|2^{\Sigma^*}| = |R| = \mathcal{C}$.

   y

7. For any languages $L_1, \ L_2, \ L$ over $\Sigma \neq \emptyset$
   $(L_1 \cap L_2) \cup L = (L_1 \cup L) \cap (L_2 \cup L)$
   Justify: Languages are sets and this is the Law of Distributivity of union over intersection of sets

   y

8. $L^* = \{w_1 ... w_n : w_i \in L, \ i = 1, 2, ..n, \ n \geq 1\}$
   Justify: This is definition of L^+; Kleene Star must have condition
   $n \geq 0$.

   n

9. Regular language is a regular expression.
   Justify: Regular language is *defined* by a regular expression.

   n

10. For any language L over an alphabet Σ, $L^+ = L \cup L^*$.
    Justify: holds only only when $e \in L$ as $e \in L^*$

    n

PART 2

QUESTION 1

Given an alphabet $\Sigma = \{a, b\}$ and a regular expression $\alpha = a^\star b \cup (a \cup b)^\star$.

1. (3pts) Evaluate $L = \mathcal{L}(\alpha)$.

Solution

1. We evaluate

$$L = \mathcal{L}(a^\star b \cup (a \cup b)^\star) = \mathcal{L}(a^\star)\mathcal{L}(b) \cup (\mathcal{L}(a) \cup \mathcal{L}(b))^\star = \{a\}^*\{b\} \cup \{a, b\}^*$$

2. (4pts) Give a property describing the language L determined by α

Solution

Observe that $\{a\}^*\{b\} \subseteq \{a, b\}^*$ hence the language is

$$L = \{w : w \in \{a, b\}^\star\} = \Sigma^\star$$

QUESTION 2

Let $\Sigma = \{a, b\}$ and a language $L \subseteq \Sigma^\star$ be defined as follows:

$$L = \{w \in \Sigma^\star : \text{ w contains less then two a's}\}$$

Write a regular expression α, such that $\mathcal{L}(\alpha) = L$. Use shorthand notation. **Explain** shortly your answer.

Solution

$$\alpha = b^\star \cup b^\star a b^\star$$

Explanation

"w contains less then two a's" means "contains zero or one a"

$b^\star$ contains no of a (case n=0)

$b^\star a b^\star$ contains one occurrence of a (case n=1)