CSE 303 FINAL SOLUTIONS GROUP 1

PART 1: Yes/No Questions Circle the correct answer. Write ONE-SENTENCE justification.

1. $(ab \cup a^*b)^*$ defines a regular language
   Justify: this is a regular expression and hence define a regular language

   y

2. There are two languages over $\Sigma = \emptyset$

   Justify: $\Sigma^* = \emptyset^* = \{e\}$ and $\emptyset \subseteq \{e\}$, $\{e\} \subseteq \{e\}$, so we have two languages $L_1 = \emptyset$ and $L_2 = \{e\}$ over $\Sigma = \emptyset$

   y

3. $L^* = \{w \in \Sigma^* : \exists_{q \in F}(s, w) \vdash_M^* (q, e)\}$.
   Justify: this is definition of $L(M)$, not L^*

   n

4. $((\phi^* \cap a) \cup (\phi \cup b^*)) \cap \phi^*$ represents a language $L = \{e\}$
   Justify: $((\{e\} \cap \{a\}) \cup \{b\}^*) \cap \{e\} = \{b\}^* \cap \{e\} = \{e\}$

   y

5. If M is a FA, then $L(M) \neq \phi$.
   Justify: take M with $\Sigma = \phi$

   n

6. A language is regular iff $L = L(M)$ and M is a finite automaton
   Justify: Main Theorem

   y

7. If L is regular, there is a PDA M such that $L = L(M)$.
   Justify: FA is a PDA operating on an empty stock

   y

8. Every subset of a regular language is a language.
   Justify: subset of a set is a set

   y

9. Let L be a regular language. The language $L^R = \{w^R : w \in L\}$ is regular.

   Justify: L^R is accepted by finite automata M^R constructed from M such that $L(M) = L$ and such automata EXISTS as L is regular

   y

10. $L = \{a^n a^n : n \geq 0\}$ is not regular.

    Justify: $L = (aa)^*$ and hence regular

    n

11. $((p, e, \beta), (q, \gamma)) \in \Delta$ means: read nothing, move from p to q
    Justify: and replace β by γ on the top of the stack

    n

12. Any finite language is CF
    Justify: any finite language is regular and $RL \subset CFL$

    y

13. Intersection of any two regular languages is CF language.
Justify: Regular languages are closed under intersection and $RL \subset CFL$
y

14. Union of a regular and a CF language is a CF language.
Justify: $RL \subseteq CFL$ and FCL are closed under union
y

15. If L is regular, there is a CF grammar G, such that $L = L(G)$.
Justify: we proved that $RL \subseteq CFL$
y

16. If L is regular, then there is a CF grammar G, such that $L = L(G)$

Justify: We proved: $RL \subset CFL$
y

17. $L = \{a^n b^n c^n : n \geq 0\}$ is CF.
Justify: is not CF, as proved by Pumping Lemma for CF languages
n

18. $L = \{a^n b^n : n \geq 0\}$ is CF.
Justify: $L = L(G)$ for G with $R = \{S \to aSb|e\}$
y

19. Let $\Sigma = \{a\}$, then for any $w \in \Sigma^*, w^R = w$
Justify: $a^R = a$ and $w^R = w$ for $w \in \{a\}^*$
y

20. Let $G = (\{S, (,)\}, \{(,)\}, R, S)$ for $R = \{S \to SS \mid (S)\}$. $L(G)$ is regular.
Justify: $L(G) = \emptyset$ and hence regular
y

21. Any regular language is accepted by some PD automata.
Justify: Any regular language is accepted by a finite automata, and a finite automaton is a PD automaton (that never operates on the stock). **y**

22. $L = \{a^n b^m c^n : n, m \in N\}$ is CF.
Justify: $S \to aSc|B,\ B \to bB|e$ **y**

23. If L is regular, then there is a CF grammar G, such that $L = L(G)$.
Justify: $RL \subseteq CF$
y

24. Class of context-free languages is closed under intersection.
Justify: $L_1 = \{a^n b^n c^m, n, m \geq 0\}$ is CF, $L_1 = \{a^m b^n c^n, n, m \geq 0\}$ is CF, but $L_1 \cap L_2 = \{a^n b^n c^n, n \geq 0\}$ is not CF
n

25. A CF language is a regular language.
Justify: $L = \{a^n b^n : n \geq 0\}$ is CF and not regular
n

PART 2: PROBLEMS

QUESTION 1

Let $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0, q_1, q_2\},\ s = q_0,\quad \Sigma = \{a, b, c\},\ F = \{q_1, q_2\}$ and $\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_0, b, q_2)\}$

Draw the diagram an automaton M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution

We apply the **"stretching"** technique to M and the new M' is is as follows.

$M' = (K \cup \{p_1, p_2, p_3\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1),\ (p_1, b, p_2),\ (p_2, c, q_0),\ (q_0, b, q_2),\ (q_0, a, p_3),\ (p_3, b, q_1)\}$

QUESTION 2

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton M, such that

$$L(M) = (ab)^*(ba)^*.$$

Draw a state diagram and specify all components K, Σ, Δ, s, F of M. Justify your construction by listing some strings accepted by the state diagram.

Solution 1 We use the lecture definition.

Components of M are: $\Sigma = \{a, b\},\ \ K = \{q_0, q_1\},\ s = q_0,\ F = \{q_0, q_1\}$.
We define Δ as follows.
$\Delta = \{(q_0, ab, q_0), (q_0, e, q_1), (q_1, ba, q_1)\}$.

Strings accepted : $ab, abab, abba, ababba, ababbaba, ...$

Solution 2 We use the book definition.

Components of M are: $\Sigma = \{a, b\},\ \ K = \{q_0, q_1, q_2, q_3\},\ s = q_0,\ F = \{q_2\}$.
We define Δ as follows.
$\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}$.

Strings accepted : $ab, abab, abba, ababba, ababbaba, ...$

QUESTION 3

Given a grammar G with the following rules

$$R = \{S \rightarrow\ SS \mid (S) \mid e\}$$

1. Trace a derivation in G generating a word ()().

Solution in Lecture 11

2. Construct a PD automaton M, such that $L(M) = L(G)$.

HINT: Use construction described in the proof of the **Lemma** : "Each context free language is accepted by some PD automaton"

DRAW A DIAGRAM and LIST components.

Solution in Lecture 11

3. Trace formally a computation of M that leads to the acceptance of the string ()(), i.e. complete the following statement.

The accepting computation is:

$$(s, ()(), e) \vdash_M (f, ()(), SS)) \vdash_M (f, ()(), (S)S) \vdash_M (f,)(), S)S)$$
$$\vdash_M (f,)(),)S)) \vdash_M (f, (), S) \vdash_M (f, (), (S)) \vdash_M (f,), S))$$
$$\vdash_M (f,),)) \vdash_M (f, e, e)$$

We proved that

$$()() \in L(M)$$

QUESTION 4 Construct a PDA M, such that

$$L(M) = \{b^n a^{3n} : n \geq 0\}.$$

Solution

$M = (K, \Sigma, \Gamma, \Delta, s, F)$ for

$$K = \{s, f\}, \Sigma = \{a, b\}, \Gamma = \{a\}, s, F = \{f\},$$
$$\Delta = \{((s, b, e), (s, aaa)), ((s, e, e), (f, e)), ((f, a, a), (f, e))\}$$

Explain the construction. Write motivation.

Solution M operates as follows: Δ pushes aaa on the top of the stock while M is reading b, switches to f (final state) non-deterministically; and pops a while reading a (all in final state). M puts on the stock three a's for each b, and then remove all a's from the stock comparing them with a's in the word while in the final state.

Trace a transitions of M that leads to the acceptance of the string $bbaaaa$.

Solution The accepting computation is:

$$(s, bbaaaaaa, e) \vdash_M (s, baaaaaa, aaa) \vdash_M (s, aaaaaa, aaaaaa) \vdash_M (f, aaaaaa, aaaaaa)$$
$$\vdash_M (f, aaaaa, aaaaa) \vdash_M (f, aaaa, aaaa) \vdash_M (f, aaa, aaa) \; ... \; \vdash_M (f, e, e)$$

QUESTION 5

Prove that the Class of context-free languages is NOT closed under intersection

Proof

Assume that the context-free languages are **are closed** under **intersection**

Observe that both languages

$$L_1 = \{a^n b^n c^m : \quad m, n \geq 0\} \quad \text{and} \quad L_2 = \{a^m b^n c^n : \quad m, n \geq 0\}$$

are **context-free**

So the language

$$L_1 \cap L_2$$

must be **context-free**, but

$$L_1 \cap L_2 = \{a^n b^n c^n : \quad n \geq 0\}$$

and we have proved that $L = \{a^n b^n c^n : \quad n \geq 0\}$ is **not** context-free. **Contradiction**