1. All questions have REGULAR points assigned to them- as in a regular FINAL test.

2. TAKE YOUR TEST at home - for NOT LONGER then 2.5 hours- time allocated for your FINAL

3. GRADE your test according to these points and write down the final GRADE POINTS you would get on the real test. Write the (with the TOTAL sum of POINTS on front page of the test.

4. WRITE correct solutions for problems you didn’t solve or got it wrong and bring it to class together with graded TEST on THURSDAY, May 5 to class

5. You will get up to 25 extra points for the corrections - or for the test- in a case there was no need for corrections. The GRADE you assign to yourself is there only for you to KNOW how you would have done; it will not influence the EXTRA CREDIT POINTS!

There are 50 yes/no questions and 6 Problems and 1 extra credit Problem

Write CAREFUL solutions to all of them and don’t forget justifications for yes/no questions

PART 1: Yes/No Questions (2pts each question)

Circle the correct answer. Write ONE-SENTENCE justification

No justification- no credit

1. Set $A$ is uncountable   iff   $R \subseteq A$ ($R$ is the set of real numbers)

   Justify:

   y  n

2. All infinite sets have the same cardinality

   Justify:

   y  n

3. $R^* = R \cup \{(a,b) : there \ is \ a \ path \ from \ a \ to \ b\}$

   Justify:

   y  n
4. \((ab \cup a^*b)^*\) is a regular language

Justify:

5. Let \(\Sigma = \phi\), there is \(L \neq \phi\) over \(\Sigma\)

Justify:

6. There are uncountably many languages over \(\Sigma = \{a\}\)

Justify:

7. Let \(R\) be a set of regular expressions and \(L\) be a function that maps \(R\) into set of all subsets of \(\Sigma^*\). Then the following is true.
\(L \subseteq \Sigma^*\) is a regular language iff \(L = L^r\), for some \(r \in R\)

Justify:

8. \(L^* = \{w \in \Sigma^* : \exists_q \in F(s,w) \vdash_M^* (q,e)\}\)

Justify:

9. \((a^*b \cup \phi^*)\) is a regular expression

Justify:

10. \(\{a\}^*\{b\} \cup \{ab\}\) is a regular language

Justify:

11. Let \(L\) be a language defined by \((a^*b \cup ab)\), i.e (shorthand) \(L = a^*b \cup ab\). Then \(L \subseteq \{a,b\}^*\)

Justify:

12. \(\Sigma = \{a\}\), there are \(c\) (continuum) languages over \(\Sigma\)

Justify:

13. \(L^* = L^+ \setminus \{e\}\)

Justify:

14. \(L^* = \{w_1 \ldots w_n, w_i \in L, i = 1, \ldots, n\}\)

Justify:
15. For any languages $L_1, L_2, L_3 \subseteq \Sigma^*$
   $L_1 \cup (L_2 \cap L_3) = (L_1 \cup L_2) \cap (L_1 \cup L_3)$

   Justify:

16. For any languages $L_1, L_2 \subseteq \Sigma^*$,
   if $L_1 \subseteq L_2$, then $(L_1 \cup L_2)^* = L_2^*$

   Justify:

17. $((\phi^* \cap a) \cup (\phi \cup b^*)) \cap \phi^*$ represents a language $L = \phi$

   Justify:

18. $L(M) = \{ w \in \Sigma^* : (q, w) \vdash^*_{M} (s, e) \}$

   Justify:

19. For every i deterministic automaton $M_i$, $L(M) \neq \phi$

   Justify:

20. If $M$ is a nondeterministic FA, then $L(M) \neq \emptyset$

   Justify:

21. $L(M_1) = L(M_2)$ iff $M_1$ and $M_2$ are finite automata

   Justify:

22. A language is regular iff $L = L(M)$ and $M$ is a deterministic automaton

   Justify:

23. If $M$ is a deterministic automaton, then there is a nondeterministic $N$, such that $L(N) = L(M)$

   Justify:

24. Any finite language is CF

   Justify:

25. Intersection of any two regular languages is a CF language

   Justify:
26. Union of a regular and a CF language is a CF language
   Justify:

27. \( L_1 \) is regular, \( L_2 \) is CF and \( L_1, L_2 \subseteq \Sigma^* \), then \( L_1 \cap L_2 \subseteq \Sigma^* \) is a CF language
   Justify:

28. If \( L \) is regular, then there is a PDA \( M \) such that \( L = L(M) \)
   Justify:

29. If \( L \) is regular, then there is a CF grammar \( G \), such that \( L = L(G) \)
   Justify:

30. \( L = \{ a^n b^n c^n : n \geq 0 \} \) is CF
   Justify:

31. \( L = \{ a^n b^n : n \geq 0 \} \) is CF
   Justify:

32. Let \( \Sigma = \{ a \} \), then for any \( w \in \Sigma^* , w^R w \in \Sigma^* \)
   Justify:

33. \( A \rightarrow Ax, A \in V, x \in \Sigma^* \) is a rule of a regular grammar
   Justify:

34. Regular grammar has only rules \( A \rightarrow xA, A \rightarrow x, x \in \Sigma^* , A \in V - \Sigma \)
   Justify:

35. Let \( G = (\{ S, (, )\}, \{(, )\}, R, S) \) for \( R = \{ S \rightarrow SS | (S) \} \)
   \( L(G) \) is regular
   Justify:

36. \( L = \{ w \in \{ a, b \}^* : w = w^R \} \) is regular
   Justify:
37. We can always show that \( L \) is regular using Pumping Lemma

**Justify:**

38. \(((p, e, \beta), (q, \gamma)) \in \Delta\) means: read nothing, move from \( p \) to \( q \)

**Justify:**

39. \( L = \{a^n b^m e^n : n, m \in \mathbb{N}\} \) is CF

**Justify:**

40. If \( L \) is not regular, then there is a CF grammar \( G \), such that \( L = L(G) \)

**Justify:**

41. There is countably many non CF languages over \( \Sigma \neq \phi \)

**Justify:**

42. Every subset of a regular language is a language

**Justify:**

43. Any regular language is accepted by some PD automata

**Justify:**

44. \( \Delta \subseteq (K \times \Sigma^* \times \Gamma^*) \times (K \times \Gamma^* \times (K \times \Sigma^* \times \Gamma^*) \) is a transition relation of a pushdown automaton

**Justify:**

45. Let \( M \) be a pushdown automaton

\[ L(M) = \{w \in \Sigma^* : (s, w, e) \models^*_M (f, e, e)\} \]

**Justify:**

46. There is uncountably many regular languages

**Justify:**

47. Every subset of a regular language is a regular language.

**Justify:**

48. A CF language is a regular language.

**Justify:** \( L = \{a^n b^n : n \geq 0\} \) is CF and not regular

**Justify:**
49. A regular language is a CF language.

Justify: y n

50. Every subset of a regular language is a regular language.

Justify: y n

PART 2: PROBLEMS (100pts + 15 extra pts)

QUESTION 1 (10pts)

Let $\Sigma$ be any alphabet, $L_1, L_2$ two languages over $\Sigma$ such that $e \in L_1$ and $e \in L_2$. Show that

$$(L_1 \Sigma^* L_2)^* = \Sigma^*$$
QUESTION 2 (15pts)

Let $L$ be a language defined as follows

$$L = \{w \in \{a, b\}^* : \text{every } a \text{ is either immediately proceeded or followed by } b\}$$

Part 1

Describe a regular expression $r$ such that $L(r) = L$

Part 2  Construct a finite state automata $M$, such that $L(M) = L$.

1. **Draw** a diagram of $M$

2. **List** some elements of $L(M)$
QUESTION 3 (15pts)

Given a Regular grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\}, \quad \Sigma = \{a, b\},$$

$$R = \{S \rightarrow aS | A | \epsilon, \quad A \rightarrow abA | a | b\}.$$

Part 1

Use the construction in the proof of L-GTheorem:

Language $L$ is regular if and only if there exists a regular grammar $G$ such that $L = L(G)$

to construct a finite automaton $M$, such that $L(G) = L(M)$.

Diagram of $M$

Part 2

Trace a transitions of $M$ that lead to the acceptance of the string $aaaababa$, and compare

with a derivation of the same string in $G$

The accepting computation of $aaaababa$ is:

$G$ derivation of $aaaababa$ is:
QUESTION 4 (15pts)
Define a grammar $G$ such that $L(G) = \{a^k b^j : k < j\}$

Justify your construction

QUESTION 5 (15pts)
Prove that the Class of context-free languages is NOT closed under intersection
QUESTION 6 (15pts)

Construct a pushdown automaton $M$ such that

$$L(M) = \{w \in \{a, b\}^* : w = w^R\}$$

Draw a DIAGRAM of $M$

$M$ components are

Trace a transitions of $M$ that lead to the acceptance of the string $ababa$. 
QUESTION 7 (15pts)

Construct a PD automaton \( M \), such that

\[
L(M) = \{ b^n a^{2n} : n \geq 0 \}.
\]

Draw a DIAGRAM

**Explain** the construction. Write motivation.

**Trace formal** transitions of \( M \) that leads to the acceptance of the string \( bbaaa \), i.e complete:

\((s, bbaaa, e) \vdash_M\)
QUESTION 9 (EXTRA CREDIT - 15pts)

Use closure under union for CF languages to show that

\[ L = \{a^n b^m : n \neq m \} \]

is a CF language.