CSE303 Q4 SOLUTIONS

PART 1: YES/NO questions

Circle the correct answer to all questions. Write SHORT justification.

1. Given a context-free grammar G , $L(G) = \{w \in V : S \Rightarrow^*{}_G w\}$
   Justify: $w \in \Sigma^*$ **n**

2. A language is context-free if and only if it is accepted by a context-free grammar.
   Justify: Generated, not accepted **n**

3. Any regular language is a context-free language
   Justify: 1. Any Finite Automata is a PDF automata
   2.Regular languages are generated by regular grammars, that are also CF.
   y

4. $L = \{w \in \{a,b\}^* : w = w^R\}$ is context-free
   Justify: G with the rules: $S \rightarrow aSa|bSb|a|b||e$ **y**

5. The stack alphabet of a pushdown automaton is always non- empty
   Justify: finite set can be empty **n**

6. $\Delta \subseteq (K \times \Sigma^* \times \Gamma^*) \times (K \times \Gamma^*$ is a transition relation of a pushdown automaton
   Justify: Δ must be finite **n**

7. $L(M) = \{w \in \Sigma^* : (s, w, e) \models^*{}_M (f, e, e)\}$
   Justify: $f \in F$ **n**

8. Any regular language is accepted by a pushdown automaton
   Justify: Any finite automata is a pushdown automata operating on an empty stock. **y**

9. Context-free languages are not closed under union
   Justify: we construct a CF grammar that is union of CF grammars **n**

10. Context-free languages are closed under intersection
    Justify: Take $L_1 = \{a^n b^n c^m : n, m \geq)\}$, $L_2 = \{a^m b^n c^n : n, m \geq)\}$ both are CF and we get that $L_1 \cap L_2 = \{a^n b^n c^n : n \geq 0\}$ is not CF **n**

PROBLEMS

QUESTION 1

Given a **Regular grammar** $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\}, \quad \Sigma = \{a, b\},$$

$$R = \{S \to aS \mid A \mid e, \quad A \to abA \mid a \mid b\}.$$

1. Draw a **diagram** of a finite automaton M, such that $L(G) = L(M)$.

Solution We construct a non-deterministic finite automata

$$M = (K, \Sigma, \Delta, s, F)$$

as follows:

$$K = (V - \Sigma) \cup \{f\}, \ \Sigma = \Sigma, s = S, \quad F = \{f\},$$

$$\Delta = \{(S, a, S), (S, e, A), (S, e, f), (A, ab, A), (A, a, f), (A, b, f)\}$$

2. Trace a transitions of M that lead to the acceptance of the string $aaaababa$, and compare with a derivation of the same string in G.

Solution

The accepting computation is:

$$(S, aaaababa) \vdash_M (S, aaababa) \vdash_M (S, aababa) \vdash_M (S, ababa) \vdash_M (A, ababa)$$
$$\vdash_M (A, aba) \vdash_M (A, a) \vdash_M (f, e)$$

G derivation is:

$$S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaS \Rightarrow aaaA \Rightarrow aaaabA \Rightarrow aaaababA \Rightarrow aaaababa$$

QUESTION 2

Use closure under union for CF languages to show that $L = \{a^n b^n : \ n \neq m\}$ is CF language

Solution 1 We know that $L_1 = \{a^m b^n : m > n\}$ and $L_2 = \{a^m b^n : m < n\}$ are context-free languages (we constructed proper grammars for both of them). $L = L_1 \cup L_2$, hence L is context free as the class of context free languages is closed under union.

Solution 2 Observe that $L_1 = \{a^m b^n : m > n\} = \{a\}^+\{a^n b^n : n \in N\}$ We proved that $\{a^n b^n : n \in N\}$, is context free, $\{a\}^+$ is regular and hence CF and the class of context free languages is closed under concatenation, hence L_1 is also context free.

Similarly, $L_2 = \{a^m b^n : m < n\} = \{a^n b^n : n \in N\}\{b\}^+$, so L_2 is context free. $L = L_1 \cup L_2$, hence L is context free as the class of context free languages is closed under union.

Also in Lecture 11

QUESTION 3

Construct a PD automaton M such that

$$L(M) = \{a^n b^{2n} : \quad n \geq 0\}$$

Draw the **diagram** of M

Trace a computation of M accept ion the word aabbbb

Show that $aab \notin L(M)$

Solution in Lecture 11