CSE303 Q3 SOLUTIONS

YES/NO questions Circle the correct answer. Write SHORT justification.

1. For any language $L \subseteq \Sigma^*, \Sigma \neq \emptyset$ there is a deterministic automata $M$, such that $L = L(M)$.
   **Justify:** only when $L$ is regular  

2. Any finite language is regular.
   **Justify:** any finite language is a finite union of one element regular languages

3. For any deterministic automata $M$, $L(M) = \bigcup \{R(1, j, n) : q_j \in F\}$, where where $M$ has $n$ states with $s = q_1$ and $R(1, j, n)$ is the set of all strings in $\Sigma^*$ that may drive $M$ from state initial state to state $q_j$ without passing through any intermediate state numbered $n + 1$ or greater, where $n$ is the number of states of $M$.
   **Justify:** basic fact and definition

4. $\Sigma$ in any Generalized Finite Automaton includes some regular expressions.
   **Justify:** definition of GFA

5. For any finite automata $M$, there is a regular expression $r$, such that $L(M) = r$.
   **Justify:** main theorem

6. Pumping Lemma says that we can always prove that a language is not regular.
   **Justify:** PL gives a certain characterization of infinite regular languages

7. Let $L$ be a regular language, and $L_1 \subseteq L$, then $L_1$ is regular.
   **Justify:** $L_1 = \{a^n b^n : n \geq 0\}$ is a non-regular subset of regular $L = a^* b^*$

8. Let $L$ be a language. The language $L^R = \{w^R : w \in L\}$ is regular.
   **Justify:** $L^R$ is accepted by finite automata $M^R$ constructed from $M$ such that $L(M) = L$

9. The class of regular languages is closed with respect to subset relation
   **Justify:**
   Consider
   
   $L_1 = \{a^n b^n : n \in N\}, \quad L_2 = a^* b^*$
   
   $L_1 \subseteq L_2$ and $L_1$ is a non-regular subset of a regular $L_2$

10. $L$ (over $\Sigma$) is regular, so is the language $L_1 = \{xy : x \in L, y \notin L\}$
    **Justify:**
    $L_1 = L(\Sigma^* - L)$ and $L$ regular, hence $(\Sigma^* - L)$ is regular (closure under complement), so is $L_1$ by closure under concatenation
PROBLEMS

QUESTION 1 Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a finite automata $M$ accepting

$L_1 = L = ((ab)^* \cup (bc)^*)ba$

Solution

M1 components:

$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$

$\Delta_{M1} = \{(q_1, ab, q_2)\}$

M2 components:

$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$

$\Delta_{M2} = \{(q_2, bc, q_4)\}$

M3 components:

$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$

$\Delta_{M3} = \{(q_5, ba, q_6)\}$

2. $M4, M5$ such that $L(M4) = L(M1)^*, L(M5) = L(M2)^*$

Solution

M4 components:

$K = \{q_1, q_2, q_7\}, \Sigma = \{a, b, c\}, s = q_7, F = \{q_2, q_7\},$

$\Delta_{M4} = \{(q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1)\}$

M5 components:

$K = \{q_3, q_4, q_8\}, \Sigma = \{a, b, c\}, s = q_8, F = \{q_4, q_8\},$

$\Delta_{M4} = \{(q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\}$

3. $M6$ such that $L(M5) = L(M4) \cup L(M5)$
Solution

**M5 components:**

\[ K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_2, q_4, q_7, q_8\}, \]

\[ \Delta_{M5} = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\} \]

4. \( M = M5M3 \), i.e. \( M \) is such that \( L(M) = L(M5)L(M3) \).

**M components:**

\[ K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_6\}, \]

\[ \Delta_{M5} = \Delta_{M4} \cup \{(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\} \]

\[ = \{(q_3, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3), (q_7, e, q_3), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\} \]

**Question 2** Evaluate \( r \), such that \( L(r) = L(M) \)

using the Generalized Automata Construction for

\[ M = \{(q_1, q_2), \{a, b\}, s = q_1, \]

\[ \Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, F = \{q_2\} \]

**Step 1:** Construct a generalized \( GM \) that extends \( M \), i.e. such that \( L(M) = L(GM) \)

**Solution**

\[ GM = \{(q_1, q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \]

\[ \Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, (q_3, e, q_1), (q_2, e, q_4)\} \]

**Step 2:** Construct \( GM1 \cong GM \cong M \) by elimination of \( q_1 \).

**Solution**

\[ GM1 = \{(q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \]

\[ \Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2)\}, (q_2, e, q_4)\} \]

**Step 3:** Construct \( GM2 \cong GM1 \cong GM \cong M \) by elimination of \( q_2 \).

**Solution**

\[ GM2 = \{(q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \]

\[ \Delta = \{(q_3, a^*b(ba^*b \cup a)^*, q_4) \]
Answer: the language is

\[ L(M) = a^*b(ba^*b \cup a)^* \]

**QUESTION 3**

Show that the language

\[ L = \{xyx^R : x, y \in \Sigma^*\} \]

is regular for any \( \Sigma \).

**Solution** Take \( x = e \in \Sigma^* \). The language

\[ L_1 = \{e\hat{y}e^R : e, y \in \Sigma^*\} \subseteq L \]

and \( L_1 = \Sigma^* \). We get \( \Sigma^* \subseteq L \subseteq \Sigma^* \) and hence \( L = \Sigma^* \) is regular.