CSE303 Q3 SOLUTIONS

YES/NO questions Circle the correct answer. Write SHORT justification.

1. For any language $L \subseteq \Sigma^*, \Sigma \neq \emptyset$ there is a deterministic automata M, such that $L = L(M)$.
   Justify: only when L is regular **n**
2. Any finite language is regular.
   Justify: any finite language is a finite union of one element regular languages **y**
3. For any deterministic automata M, $L(M) = \bigcup\{R(1, j, n) : \ q_j \in F\}$, where where M has n states with $s = q_1$ and $R(1, j, n)$ is the set of all strings in Σ^* that may drive M from state initial state to state q_j without passing through any intermediate state numbered $n + 1$ or greater, where n is the number of states of M.
   Justify: basic fact and definition **y**
4. Σ in any Generalized Finite Automaton includes some regular expressions.
   Justify: definition of GFA **y**
5. For any finite automata M, there is a regular expression r, such that $L(M) = r$.
   Justify: main theorem **y**
6. Pumping Lemma says that we can always prove that a language is not regular.
   Justify: PL gives a certain characterization of infinite regular languages **n**
7. Let L be a regular language, and $L_1 \subseteq L$, then L_1 is regular.
   Justify: $L_1 = \{a^n b^n : n \geq 0\}$ is a non-regular subset of regular $L = a^* b^*$ **n**
8. Let L be a language. The language $L^R = \{w^R : \ w \in L\}$ is regular.
   Justify: L^R is accepted by finite automata M^R constructed from M such that $L(M) = L$ **y**
9. The class of regular languages is closed with respect to subset relation
   Justify:
   Consider
   $$L_1 = \{a^n b^n : n \in N\}, \quad L_2 = a^* b^*$$
   $L_1 \subseteq L_2$ and L_1 is a non-regular subset of a regular L_2 **n**
10. L (over Σ) is regular, so is the language $L_1 = \{xy : \ x \in L, y \notin L\}$
    Justify:
    $L_1 = L(\Sigma^* - L)$ and L regular, hence $(\Sigma^* - L)$ is regular (closure under complement), so is L_1 by closure under concatenation **y**

PROBLEMS

QUESTION 1 Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata M accepting

$$L_1 = \mathcal{L} = ((ab)^* \cup (bc)^*)ba$$

Solution

M1 components:

$$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$$

$$\Delta_{M1} = \{(q_1, ab, q_2)\}$$

M2 components:

$$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$$

$$\Delta_{M2} = \{(q_2, bc, q_4)\}$$

M3 components:

$$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$$

$$\Delta_{M3} = \{(q_5, ba, q_6)\}$$

2. $M4, M5$ such that $L(M4) = L(M1)^*, L(M5) = L(M2)^*$

Solution

M4 components:

$$K = \{q_1, q_2, q_7\}, \Sigma = \{a, b, c\}, s = q_7, F = \{q_2, q_7\},$$

$$\Delta_{M4} = \{(q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1)\}$$

M5 components:

$$K = \{q_3, q_4, q_8\}, \Sigma = \{a, b, c\}, s = q_8, F = \{q_4, q_8\},$$

$$\Delta_{M4} = \{(q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\}$$

3. $M6$ such that $L(M5) = L(M4) \cup L(M5)$

Solution

M5 components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_2, q_4, q_7, q_8\},$$

$$\Delta_{M5} = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\}$$

4. $M = M5M3$, i.e M is such that $L(M) = L(M5)L(M3)$.

M components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_6\},$$

$$\Delta_{M5} = \Delta_{M4} \cup \{(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\}$$
$$= \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3),$$
$$(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\}$$

Question 2 Evaluate r, such that

$$\mathcal{L}(r) = L(M)$$

using the Generalized Automata Construction for

$$M = (\{q_1, q_2\},\ \{a, b\},\ s = q_1,$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\},\ F = \{q_2\})$$

Step 1: Construct a generalized GM that extends M, i.e. such that $L(M) = L(GM)$

Solution

$$GM = (\{q_1, q_2, q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, (q_3, e, q_1), (q_2, e, q_4))$$

Step 2: Construct $GM1 \simeq GM \simeq M$ by elimination of q_1.

Solution

$$GM1 = (\{q_2, q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2)\}, (q_2, e, q_4))$$

Step 3: Construct $GM2 \simeq GM1 \simeq GM \simeq M$ by elimination of q_2.

Solution

$$GM2 = (\{q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b(ba^*b \cup a)^*, q_4))$$

Answer : the language is

$$L(M) = a^*b(ba^*b \cup a)^*$$

QUESTION 3

Show that the language
$$L = \{xyx^R : \ x, y \in \Sigma^*\}$$

is regular for any Σ.

Solution Take $x = e \in \Sigma^*$. The language

$$L_1 = \{eye^R : \ e, y \in \Sigma^*\} \subseteq L$$

and $L_1 = \Sigma^*$. We get $\Sigma^* \subseteq L \subseteq \Sigma^*$ and hence $L = \Sigma^*$ is regular.