

## CSE303 Q2 SOLUTIONS

### PART 1: YES/NO QUESTIONS (10 points)

Circle the correct answer. Write **SHORT justification**.

**Answers without justification will not receive credit**

1. The set  $K$  of states of a deterministic finite automaton can be empty.  
**Justify:**  $s \in K$  **n**
2. A configuration of a deterministic finite automaton  $M = (K, \Sigma, \delta, s, F)$  is any element of  $K \times \Sigma^* \times K$ .  
**Justify:** element of  $K \times \Sigma^*$  **n**
3. Given  $M = (K, \Sigma, \delta, s, F)$ ,  
 $L(M) = \{w \in \Sigma^* : \exists q \in K(s, w) \vdash^*_M(q, e)\}$ .  
**Justify:** only when  $q \in F$  **n**
4. For any  $M = (K, \Sigma, \delta, s, F)$ ,  $L(M) \neq \emptyset$   
**Justify:** for any  $M$ , such that  $F = \emptyset$  or  $\Sigma = \emptyset$  we have that  $L(M) = \emptyset$ . **n**
5. There is a deterministic  $M = (K, \Sigma, \delta, s, F)$ , such that  $e \in L(M)$   
**Justify:** any  $M$  such that  $s \in F$  as we we proved a **Theorem:**  
 $e \in L(M)$  iff  $s \in F$  **y**
6. Given a non-deterministic automaton  $M = (K, \Sigma, \Delta, s, F)$  as defined in the book, a binary relation  $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$  is a one step computation iff the following condition holds  
 $(q, aw) \vdash_M(q', w)$  iff  $(q, a, q') \in \Delta$  and  $a \in \Sigma \cup \{e\}$ .  
**Justify:** it is the book definition **y**
7. If  $M = (K, \Sigma, \Delta, s, F)$  is a non-deterministic as defined in the book, then  $M$  is also non-deterministic, as defined in the lecture.  
**Justify:**  $\Sigma \cup \{e\} \subseteq \Sigma^*$ . **y**
8. If  $M = (K, \Sigma, \delta, s, F)$  is a deterministic, then  $M$  is also non-deterministic.  
**Justify:** any function is a relation and  $\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K$  **y**
9. We define, for any (deterministic or non-deterministic  $M = (K, \Sigma, \Delta, s, F)$ ) a computation of the length  $n$  from  $(q, w)$  to  $(q', w')$  as a sequence  
 $(q_1, w_1), (q_2, w_2), \dots, (q_n, w_n), n \geq 1$   
of configurations, such that  $q_1 = q, q_n = q', w_1 = w, w_n = w'$  and  
 $(q_i, w_i) \vdash^*_M(q_{i+1}, w_{i+1})$  for  $i = 1, 2, \dots, n-1$ .

Statement: For any  $M$ , a computation of the length 1 exists.

**Justify:** TRIVIAL path (case  $n=1$ ) of length one always exists **y**

10. A **trap state** of a DFA automaton  $M$  is any of its states that traps computations of  $M$

**Justify:** Definition: A **trap state** of a DFA automaton  $M$  is any of its states that does not influence the language  $L(M)$  of  $M$  **n**

## TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

**BOOK DEFINITION:**  $M = (K, \Sigma, \Delta, s, F)$  is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that  $\Delta$  is always finite because  $K, \Sigma$  are finite sets.

**LECTURE DEFINITION:**  $M = (K, \Sigma, \Delta, s, F)$  is non-deterministic when  $\Delta$  is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that  $\Delta$  is finite because  $\Sigma^*$  is infinite.

**SOLVING PROBLEMS** you can use any of these definitions.

**PART 2: Few Very Short Questions** For the automata  $M$  below

1. State and explain whether  $M$  represents a deterministic or a non-deterministic automaton.
2. Write down a regular expression representing  $L(M)$ .

**Q1: M1** has components:  $K = \{q\}$ ,  $s = q$ ,  $\Sigma = \emptyset$ ,  $\delta = \emptyset$ ,  $F = \emptyset$ .

**Solution**

1.  $M1$  is deterministic.
2.  $L(M1) = \emptyset$ .

**Q2: M2** has components:  $K = \{q_0, q_1, q_2\}$ ,  $s = q_0$ ,  $\Sigma = \{a, b\}$ ,  $\delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$ ,  $F = \{q_1\}$ .

1.  $M2$  is non-deterministic;  $\delta$  is not a function with the domain  $K \times \Sigma$ . It can be completed to a function by adding some trap states. But the trap states information was not it was not stated in the problem.
2.  $L(M2) = aa^*$ .

**Q3: M3**  $K = \{q_0, q_1, q_2\}$ ,  $\Sigma = \{a, b\}$ ,  $\Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$ ,  $F = \{q_1, q_2\}$ .

**Solution**  $M_3$  is NOT an automaton. It does not have the INITIAL state!

### PART 3: PROBLEMS

**QUESTION 1** Construct a deterministic finite automaton  $M$  such that

$$L(M) = \{w \in \{a, b\}^* : \text{neither } bb \text{ nor } aa \text{ is a substring of } w\}.$$

Draw a state diagram and specify all components  $K, \Sigma, \delta, s, F$  of  $M$ . Justify your construction.

**Solution**

**Components** of  $M = (K, \Sigma, \delta, s, F)$  are:

$\Sigma = \{a, b\}$ ,  $K = \{q_0, q_1, q_2, q_3\}$ ,  $s = q_0$ ,  $q_3$  is a trap state,  $F = \{q_0, q_1, q_2\}$ .

We define  $\delta$  on non-trap states as follows.

$$\delta(q_0, a) = q_1, \delta(q_0, b) = q_2,$$

$$\delta(q_1, b) = q_2,$$

$$\delta(q_2, a) = q_1.$$

$M$  **accepts** strings  $a, aba, ababa, \dots$  or  $b, bab, babab, \dots$  etc and never  $aa, bb$ , etc...

**QUESTION 4** Let  $M$  be defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for  $K = \{q_0, q_1, q_2, q_3\}$ ,  $s = q_0$

$\Sigma = \{a, b, c\}$ ,  $F = \{q_3\}$  and

$$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}.$$

1. Find the regular expression describing the  $L(M)$ . Simplify it as much as you can. Explain your steps.

**Solution**  $L(M) = (abc)^*abbb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa = (abc)^*(abbb \cup baa).$

We used the property:

$$LL_1 \cup LL_2 = L(L_1 \cup L_2).$$

2. Write down (you can draw the diagram) an nondeterministic automata  $M'$  such that  $M' \equiv M$  and  $M'$  is defined by the BOOK definition.

**Solution** We apply the "stretching" technique to  $M$  and the new  $M'$  is as follows.

$$M' = (K \cup \{p_1, p_2, \dots, p_5\}, \Sigma, s = q_0, \Delta', F' = F)$$

for  $K = \{q_0, q_1, q_2\}$ ,  $s = q_0$   
 $\Sigma = \{a, b\}$ ,  $F = \{q_3\}$  and  
 $\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1),$   
 $(q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}.$