CSE303 Q1 SOLUTIONS

Quiz has TWO PARTS

Part 1: yes/no questions **(15pts)**

Must write justifications- **no justification, no points!**

Part 2 (10pts) Solve only **one problem** of your choice

PART 1: Yes/No Questions

1. $\{\emptyset\} \subseteq \{a, b, c\}$
   Justify: $\emptyset \notin \{a, b, c\}$ **n**
2. $2^{\{1,2\}} \cap \{\{1, 2\}\} \neq \emptyset$
   Justify: $\{1, 2\} \subseteq \{1, 2\}$ i.e. $\{1, 2\} \in 2^{\{1,2\}}$ and $\{1, 2\} \in \{\{1, 2\}\}$ **y**
3. Some $R \subseteq A \times B$ are functions that map A into B
   Justify: Functions are special type of relations. **y**
4. Let $A \neq \emptyset$ such that there are exactly 25 partitions of A. It is possible to define 20 equivalence relations on A
   Justify: one can define up to 25 (as many as partitions) of equivalence classes **y**
5. There is a relation that is equivalence and *order* at the same time
   Justify: equality relation **y**
6. Let $A = \{n \in N : n^2 + 1 \leq 15\}$. It is possible to define 8 *alphabets* $\Sigma \subseteq A$
   Justify:A has 4 elements, so we have $2^4 > 8$ subsets **y**
7. Let $A = \{\{n\} \in 2^N : n^2 + 1 \leq 15\}$. A is infinite.
   Justify: $\{n \in N : n^2+1 \leq 15\} = \{0, 1, 2, 3\}$, hence $A = \{\{0\}, \{1\}, \{2\}, \{3\}\}$ is a finite set **n**
8. Let $\Sigma = \{a\}$. There are countably many languages over Σ.
   Justify: There is any many languages as subsets of $\Sigma^\star$, i.e. uncountably many and exactly as many as real numbers. **n**
9. Let $\Sigma = \{n \in N : n \leq 1\}$
   There are uncountably many finite languages over Σ
   Justify: $\Sigma = \{0, 1\}$ and so $\Sigma^\star$ is countably infinite. The set of all finite subsets of any countably infinite set is countably infinite **n**
10. $L^* = \{w_1 ... w_n : w_i \in L, i = 1, 2, ..n, n \geq 1\}$
    Justify: $n \geq 0$. **n**
11. For any langauge L over an alphabet Σ, $L^+ = L \cup L^*$
    Justify: $e \in L^*$ and $e \notin L^+$ **n**

12. For any language L over an alphabet Σ, $L^+ = L \cup L^*$.
    Justify: Take L such that $e \notin L$. We get that $e \in L \cup L^*$ as $e \in L^*$ and $e \notin L^+$. **n**

13. For any languages $L_1,\ L_2,\ L$ over $\Sigma \neq \emptyset$

$$(L_1 \cup L_2) \circ L = L_1 \circ L \cup L21 \circ L$$

    Justify: proved in class **y**

14. Regular language is a regular expression.
    Justify: Regular Language is represented by the function $\mathcal{L}$

    n

15. Regular expression $\alpha = (a \cup b)^\star a$ defines a language $L = \{w \in \{a,b\}^\star : \quad w \text{ ends with } a\ \}$
    Justify: $\{a,b\}^\star\{a\} = \Sigma^\star\{a\} = \{w \in \{a,b\}^\star : \quad w \text{ ends with } a\ \}$. **y**

PART 2: PROBLEMS

Chose ONE problem and solve it

Circle the problem you want us to correct

QUESTION 1

1. Let $A = \{(\{n, n+1\}, n) \in 2^N \times N :\ 1 \leq n \leq 3\}$. List all elements of A.

2. Let now $A = \{(\{n\}, n) \in 2^N \times N : \quad 1 \leq n \leq n+1\}$. Prove that A is infinitely countable.

Solution

1. Let $A = \{(\{n, n+1\}, n) \in 2^N \times N :\ 1 \leq n \leq 3\}$. List all elements of A.

$$A = \{(\{n, n{+}1\}, n) \in 2^N \times N : n = 1, 2, 3\} = \{(\{1,2\},1), (\{2,3\},2), (\{3,4\},3)\}$$

2. Let now $A = \{(\{n\}, n) \in 2^N \times N : \quad 1 \leq n \leq n+1\}$. Prove that A is infinitely countable.

Solution

$$A = \{(\{n\}, n) \in 2^N \times N :\ \ 1 \leq n \leq n{+}1\} = \{(\{n\}, n) \in 2^N \times N :\ \ 1 \leq n\}$$

because $n \leq n+1$ for all $n \in N$.
The set $B = \{\{n\} : n \in N\}$ has the same cardinality as N by the function $f(n) = \{n\}$. $A = B \times N$ is a Cartesian product of two infinitely countable sets, and hence is also infinitely countable.

QUESTION 2 Let $\Sigma = \{a, b\}$. Let $L_1 \subseteq \Sigma^\star$ be defined as follows:

$$L_1 = \{w \in \Sigma^\star : \text{ number of } b \text{ in } w \text{ is divisible three}\}$$

Write a regular expression α, such that $\mathcal{L}(\alpha) = L_1$. You can use shorthand notation. Explain shortly your answer.

Solution:

$$\alpha = a^\star(a^*ba^\star ba^\star ba^\star)^\star = a^\star(ba^\star ba^\star ba^\star)^\star.$$

Explanation: the part $a^*ba^\star ba^\star ba^\star$ says that there must be 3 occurrences of b in L_1. The part $(a^*ba^\star ba^\star ba^\star)^\star$ says that we the number of b's is $3n$ for $n \geq 1$.

Observe that 0 is divisible by 3, so we need to add the case of 0 number of b's ($n = 0$), i.e. words $e, a, aa, aaa, , ...$ We do so by concatenating $(a^*ba^\star ba^\star ba^\star)^\star$ with a^*.