YES/NO questions
1. For any finite language $L \subseteq \Sigma^*, \Sigma \neq \emptyset$ there is a finite automata $M$, such that $L = L(M)$.
   Justify: any finite language is regular

2. Given $L_1, L_2$ languages over $\Sigma$, then $((L_1 \cup (\Sigma^* - L_2)) \cap L_2)$ is regular.
   Justify: only when both are regular languages

3. For any deterministic automata $M$, $L(M) = \bigcup \{ R(1, j, n) : q_j \in K \}$, where $R(1, j, n)$ is the set of all strings in $\Sigma^*$ that may drive $M$ from state initial state to state $q_j$ without passing through any intermediate state numbered $n + 1$ or greater, where $n$ is the number of states of $M$.
   Justify: only when $q_j \in F$

4. If $L_1 \cap L_2$ is a regular language, so are $L_1$ and $L_2$.
   Justify: No, $L_1$ and $L_2$ may not be regular. Take $L_1 = \{a^n b^n : n \in \mathbb{N}\}$, $L_2 = \{a^n : n \in \text{Prime}\}$ $L_1 \cap L_2 = \emptyset$ is a regular language and $L_1, L_2$ are not regular.

5. $L = \{a^n a^n : n \geq 0\}$ is not regular.
   Justify: $L = a^n a^n = a^{2n} = (aa)^n$ and hence regular

6. If $L$ is regular, so is the language $L_1 = \{xy : x \in L, y \notin L\}$.
   Justify: Observe that $L_1 = L(\Sigma^* - L)$ and $L$ regular, hence $(\Sigma^* - L)$ is regular (closure under complement), so is $L_1$ by closure under concatenation.

7. Let $L$ be a regular language, and $L_1 \subseteq L$, then $L_1$ is regular.
   Justify: $L_1 = \{a^n b^n : n \geq 0\}$ is a non-regular subset of regular $L = a^* b^*$

8. Let $L$ be a language. The language $L^R = \{w^R : w \in L\}$ is regular.
   Justify: $L^R$ is accepted by finite automata $M^R$ constructed from $M$ such that $L(M) = L$

9. Let $L$ be a regular language $\Sigma$. Then the following condition holds.
   \[ \exists n \geq 1 \forall w \in L(|w| \geq n \Rightarrow \forall x, y, z \in \Sigma^*(w = xyz \Rightarrow (w \notin L) \leq n \forall i \geq 0(xy^i z \in L))) \]
   Justify: $\exists x, y, z \in \Sigma^*$.

10. Let $L$ be a regular language over $\Sigma \neq \emptyset$. Then the following holds.
    \[ \exists w \in \Sigma^* \exists x, y, z \in \Sigma^*(w = xyz \land y \neq e \land \forall n \geq 0(xy^n z \in L)) \]
Justify: only when $L$ is infinite.

PROBLEMS

QUESTION 1  (5pts)  Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata $M$ accepting

$L_1 = L = ((ba)^* \cup (cb)^*)ab$

You can just draw diagrams.

Solution

1. Diagrams for $M_1, M_2, M_3$ such that $L(M_1) = ab, L(M_2) = bc, L(M_3) = ba$

Solution

$M_1$ components:

$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$

$\Delta_{M_1} = \{(q_1, ba, q_2)\}$

$M_2$ components:

$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$

$\Delta_{M_2} = \{(q_2, cb, q_4)\}$

$M_3$ components:

$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$

$\Delta_{M_3} = \{(q_5, ab, q_6)\}$

2. Diagrams for $M_4, M_5$ such that $L(M_4) = L(M_1)^*, L(M_5) = L(M_2)^*$

Solution
M4 components:

\[ K = \{ q_1, q_2, q_7 \}, \Sigma = \{ a, b, c \}, s = q_7, F = \{ q_2, q_7 \}, \]
\[ \Delta_{M4} = \{ (q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1) \} \]

M5 components:

\[ K = \{ q_3, q_4, q_8 \}, \Sigma = \{ a, b, c \}, s = q_8, F = \{ q_4, q_8 \}, \]
\[ \Delta_{M4} = \{ (q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3) \} \]

3. Diagram for M6 such that \( L(M5) = L(M4) \cup L(M5) \)

Solution

M5 components:

\[ K = \{ q_1, q_2, q_3, q_4, q_7, q_8, q_9 \}, \Sigma = \{ a, b, c \}, s = q_9, F = \{ q_2, q_4, q_7, q_8 \}, \]
\[ \Delta_{M5} = \{ (q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3) \} \]

4. Diagram for \( M = M5M3 \), i.e. \( M \) is such that \( L(M) = L(M5) \cup L(M3) \).

M components:

\[ K = \{ q_1, q_2, q_3, q_4, q_7, q_8, q_9 \}, \Sigma = \{ a, b, c \}, s = q_9, F = \{ q_6 \}, \]
\[ \Delta_{M5} = \Delta_{M4} \cup \{ (q_7, e, q_5), (q_9, e, q_8), (q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3), \]
\[ (q_7, e, q_5), (q_9, e, q_8), (q_2, e, q_5), (q_4, e, q_3), (q_5, ab, q_6) \} \]

QUESTION 2  Evaluate \( r \), such that

\[ \mathcal{L}(r) = L(M) \]

using the Generalized Automata Construction, as described in example 2.3.2 page 80.

\[ M = (\{ q_1, q_2 \}, \{ a, b \}, s = q_1, \]
\[ \Delta = \{ (q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1) \}, F = \{ q_2 \} \]

Step 1: Construct a generalized \( GM \) that extends \( M \), i.e. such that \( L(M) = L(GM) \)

Solution

\[ GM = (\{ q_1, q_2, q_3, q_4 \}, \{ a, b \}, s = q_3, F = \{ q_4 \} \]
\[ \Delta = \{ (q_1, a, q_1), (q_1, a, q_2), (q_2, b, q_2), (q_2, a, q_1) \}, (q_3, e, q_1), (q_2, e, q_4) \} \]
**Step 2:** Construct $GM_1 \simeq GM \simeq M$ by elimination of $q_1$.

**Solution**

$$GM_1 = \{(q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2), (q_2, c, q_4)\}$$

**Step 3:** Construct $GM_2 \simeq GM_1 \simeq GM \simeq M$ by elimination of $q_2$.

**Solution**

$$GM_2 = \{(q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b(\text{ba}^*b \cup a)^*, q_4)\}$$

**Answer:** the language is

$$L(M) = a^*b(\text{ba}^*b \cup a)^*$$

**QUESTION 3** Show that the class of regular languages is not closed with respect to subset relation.

**Solution** Consider

$$L_1 = \{a^nb^n : n \in \mathbb{N}\}, \ L_2 = a^*b^*$$

$L_1 \subseteq L_2$ and $L_1$ is a non-regular subset of a regular $L_2$. 