CSE303 Q3 SOLUTIONS

YES/NO questions

1. For any finite language $L \subseteq \Sigma^*, \Sigma \neq \emptyset$ there is a finite automata M, such that $L = L(M)$.
   Justify: any finite language is regular **y**
2. Given L_1, L_2 languages over Σ, then $((L_1 \cup (\Sigma^* - L_2)) \cap L_2)$ is regular.
   Justify: only when both are regular languages **n**
3. For any deterministic automata M, $L(M) = \bigcup\{R(1, j, n) : \ q_j \in K\}$, where $R(1, j, n)$ is the set of all strings in Σ^* that may drive M from state initial state to state q_j without passing through any intermediate state numbered $n+1$ or greater, where n is the number of states of M.
   Justify: only when $q_j \in F$ **n**
4. If $L_1 \cap L_2$ is a regular language, so are L_1 and L_2.

   Justify: No, L_1 and L_2 may not be regular . Take $L_1 = \{a^n b^n : n \in N\}$, $L_2 = \{a^n : n \in Prime\}$ $L_1 \cap L_2 = \emptyset$ is a regular language and L_1, L_2 are not regular. **n**
5. $L = \{a^n a^n : n \geq 0\}$ is not regular.
   Justify: $L = a^n a^n = a^{2n} = (aa)^*$ and hence regular **n**
6. If L is regular, so is the language $L_1 = \{xy : \ x \in L, y \notin L\}$.
   Justify: Observe that $L_1 = L(\Sigma^* - L)$ and L regular, hence $(\Sigma^* - L)$ is regular (closure under complement), so is L_1 by closure under concatenation. **y**
7. Let L be a regular language, and $L_1 \subseteq L$, then L_1 is regular.
   Justify: $L_1 = \{a^n b^n : n \geq 0\}$ is a non-regular subset of regular $L = a^* b^*$ **n**
8. Let L be a language. The language $L^R = \{w^R : \ w \in L\}$ is regular.
   Justify: L^R is accepted by finite automata M^R constructed from M such that $L(M) = L$ **y**
9. Let L be a **regular** language Σ. Then the following condition holds.

   $$\exists n \geq 1 \forall w \in L(|w| \geq n \Rightarrow \forall x, y, z \in \Sigma^*(w = xyz \cap y \neq e \cap |xy| \leq n \cap \forall i \geq 0(xy^i z \in L)))$$

   Justify: $\exists x, y, z \in \Sigma^*$. **n**
10. Let L be a regular language over $\Sigma \neq \emptyset$. Then the following holds

    $$\exists w \in \Sigma^* \exists x, y, z \in \Sigma^*(w = xyz \cap y \neq e \cap \forall n \geq 0(xy^n z \in L))$$

    Justify: only when L is infinite. **n**

PROBLEMS

QUESTION 1 (5pts) Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata M accepting

$$L_1 = \mathcal{L} = ((ba)^* \cup (cb)^*)ab$$

You can just draw a diagrams.

Solution

1. Diagrams for $M1, M2, M3$ such that $L(M1) = ab, L(M2) = bc, L(M3) = ba$

Solution

M1 components:

$$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$$
$$\Delta_{M1} = \{(q_1, ba, q_2)\}$$

M2 components:

$$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$$
$$\Delta_{M2} = \{(q_2, cb, q_4)\}$$

M3 components:

$$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$$
$$\Delta_{M3} = \{(q_5, ab, q_6)\}$$

2. Diagrams for $M4, M5$ such that $L(M4) = L(M1)^*, L(M5) = L(M2)^*$

Solution

M4 components:

$$K = \{q_1, q_2, q_7\}, \Sigma = \{a, b, c\}, s = q_7, F = \{q_2, q_7\},$$
$$\Delta_{M4} = \{(q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1)\}$$

M5 components:

$$K = \{q_3, q_4, q_8\}, \Sigma = \{a, b, c\}, s = q_8, F = \{q_4, q_8\},$$
$$\Delta_{M4} = \{(q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3)\}$$

3. Diagram for $M6$ such that $L(M5) = L(M4) \cup L(M5)$

Solution

M5 components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_2, q_4, q_7, q_8\},$$

$$\Delta_{M5} = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3)\}$$

4. Diagram for $M = M5M3$, i.e M is such that $L(M) = L(M5)L(M3)$.

M components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_6\},$$

$$\begin{aligned}\Delta_{M5} &= \Delta_{M4} \cup \{(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ab, q_6)\} \\ &= \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ba, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, cb, q_4), (q_4, e, q_3), \\ &\quad (q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ab, q_6)\}\end{aligned}$$

QUESTION 2 Evaluate r, such that

$$\mathcal{L}(r) = L(M)$$

using the Generalized Automata Construction, as described in example 2.3.2 page 80.

$$M = (\{q_1, q_2\},\ \{a, b\},\ s = q_1,$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\},\ F = \{q_2\})$$

Step 1: Construct a generalized GM that extends M, i.e. such that $L(M) = L(GM)$

Solution

$$GM = (\{q_1, q_2, q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_1, a, q_1), (q_1, a, q_2), (q_2, b, q_2), (q_2, a, q_1)\}, (q_3, e, q_1), (q_2, e, q_4))$$

Step 2: Construct $GM1 \simeq GM \simeq M$ by elimination of q_1.

Solution

$$GM1 = (\{q_2, q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2)\}, (q_2, e, q_4))$$

Step 3: Construct $GM2 \simeq GM1 \simeq GM \simeq M$ by elimination of q_2.

Solution

$$GM2 = (\{q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b(ba^*b \cup a)^*, q_4)\}$$

Answer : the language is

$$L(M) = a^*b(ba^*b \cup a)^*$$

QUESTION 3 Show that the class of regular languages is not closed with respect to subset relation.

Solution Consider

$$L_1 = \{a^n b^n : n \in N\}, \quad L_2 = a^*b^*$$

$L_1 \subseteq L_2$ and L_1 is a non-regular subset of a regular L_2.