CSE 213 F07 : Quiz 2-Solutions

November 10, 2007

1. a) Recall the definition of composition of two functions (see book pg. 195)
   \[ R \circ S = \{(1,2),(1,4)\} \]
   To obtain pair (1,2), we should have composed two pairs of the form
   \( (1,a) \in R \) and \( (a,2) \in S \) ; where a is any value in our universe.
   Similarly, to obtain pair (1,4) we should have had two pairs of the
   form \( (1,b) \in R \) and \( (b,4) \in S \) ; where b is any value in our universe.
   Therefore, any answer of the form: \( \overline{R} = \{(1,a),(1,b)\} ; \overline{S} = \{(a,2),(b,4)\} \) is valid.

   b) Yes, because it does not violate the transitivity property.

2. a) Answer= \{ 1,3 \}
   
   The diagonal set with respect to R is \( D = \{2, 8, 32, 128, \ldots\} \) because
   \( 2+2, 8+8, 32+32, \ldots \) are perfect squares. Its complement will be
   \( \{1, 3, 4, 5, \ldots\} \) . Therefore, the two smallest elements are 1 and 3.

   b) Yes. For any pair, \( (x, y) \) such that \( x + y \) is a perfect square and since
   addition has the commutative property we know that \( y+x \) is a perfect
   square also and pair \( (y, x) \) can be in set R. Therefore, R is symmetric.

   c) Yes

   d) Recall that for a relation to be equivalent, it must be transitive, sym-
   metric and reflexive. Therefore, we have to show that this is the case.

   **Transitive**
   
   \( t(R) \) is transitive by definition or transitive closure.

   **Symmetric**
   
   \( t(R) \) is symmetric since R is symmetric and transitivity closure
   does not affect symmetry property.

   **Reflexive**
   
   \( t(R) \) is reflexive. \( (1, 3), (3, 1) \in R \), when we obtain \( t(R) \) , \( (1, 1), (3, 3) \)
   are added to the set. The same will apply for similar pairs, so
   pair of the form \( (n, n) \) will be added when performing the tran-
   sitive closure.
   Precisely, if x is a perfect square, then
   \( (1, x - 1) \in R \) and \( (x - 1, 1) \in R \)
(2, x - 2) ∈ R and (x - 2, 2) ∈ R

... when we get t(R), we add (1, 1), (2, 2), (3, 3), ... , (x - 1, x - 1) for every perfect square x, all previous members will be added to t(R). There are infinitely many perfect squares so for all members of t(R) there will be (a, a) ∈ t(R). Therefore t(R) is reflexive.