CSE 213 F07 : Quiz 2-Solutions

November 10, 2007

1. **a)** Recall the definition of composition of two functions (see book pg. 195)
   R *o* S = {(1,2),(1,4)}
   To obtain pair (1,2), we should have composed two pairs of the form $(1, a) \in R$ and $(a, 2) \in S$; where **a** is any value in our universe. Similarly, to obtain pair (1,4) we should have had two pairs of the form $(1, b) \in R$ and $(b, 4)\ inS$; where **b** is any value in our universe. Therefore, any answer of the form: R̄={(1,a),(1,b)} ; S={(a,2),(b,4)} is valid.

   b) Yes, because it does not violate the transitivity property.

2. **a)** Answer= { 1,3 }
   The diagonal set with respect to R is $D = \{2, 8, 32, 128, ...\}$ because 2+2, 8+8, 32+32, ... are perfect squares. Its complement will be $\{1, 3, 4, 5), ...\}$. Therefore, the two smallest elements are 1 and 3.

   b) Yes. For any pair, (x, y) such that $x + y$ is a perfect square and since addition has the commutative property we know that $y+x$ is a perfect square also and pair (y, x) can be in set R. Therefore, R is symmetric.

   c) Yes

   d) Recall that for a relation to be equivalent, it must be transitive, symmetric and reflexive. Therefore, we have to show that this is the case.

   Transitive
   $t(R)$ is transitive by definition or transitive closure.

   Symmetric
   $t(R)$ is symmetric since R is symmetric and transitivity closure does not affect symmetry property.

   Reflexive
   $t(R)$ is reflexive. $(1, 3), (3, 1) \in R$, when we obtain $t(R)$, $(1, 1), (3, 3)$ are added to the set. The same will apply for similar pairs, so pair of the form (n, n) will be added when performing the transitive closure.
   Precisely, if x is a perfect square, then
   $(1, x - 1) \in R$ and $(x - 1, 1) \in R$

$(2, x-2) \in R$ and $(x-2, 2) \in R$
$\ldots$
when we get $t(R)$, we add $(1,1),(2,2),(3,3),\ldots,(x-1,x-1)$ for every perfect square$x$, all previous members will be adde to $t(R)$. There are infinitely many perfect squares so for all members of $t(R)$ there will be $(a,a) \in t(R)$. Therefore $t(R)$ is reflexive.