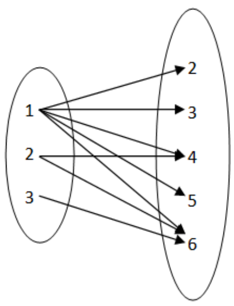
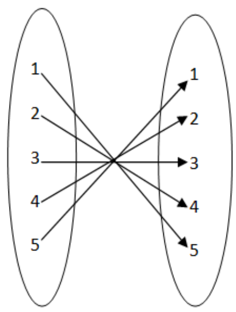
CSE213 Fall2007

Solution: Homework 3

(1) (i) $R_1 = \{(1,2),(1,3),(1,4),(1,5),(1,6),(2,4),(2,6),(3,6)\}$.
Domain(R_1) $= \{1,2,3\}$
Range(R_1) $= \{2,3,4,5,6\}$
(ii) $R_2 = \{(1,5),(2,4),(3,3),(5,1),(4,2)\}$.
Domain(R_2) $= \{1,2,3,4,5\}$
Range(R_2) $= \{1,2,3,4,5\}$



Graph: R1



Graph: R2

(2) (i) R_1 is reflexive, transitive and anti-symmetric.
(ii) R_2 is transitive and anti-symmetric.
(iii) R_3 is symmetric and transitive.
(iv) R_4 is reflexive, symmetric and transitive.

(3) $R = \{(1,2),(2,1),(1,1),(2,2)\}$.
It's symmetric because xRy and yRx both belongs to R. Its also transitive because whenever there are pairs like xRy and yRz, then xRz. But it's not reflexive because (3,3) is not in R.

(4) (i) $(R_1 \circ R_2) = \{(1,3),(1,4),(4,3),(4,4)\}$.
$(R_2 \circ R_1) = \{(2,2)\}$.
$(R_1 \circ R_2) \cap (R_2 \circ R_1) = \emptyset$
(ii) $(R_1 \cup R_2) = \{(1,2),(1,3),(2,3),(2,4),(4,2)\}$.
$(R_2 \cup R_1)^2 = (R_1 \cup R_2) \circ (R_1 \cup R_2)$.
$(R_2 \cup R_1)^2 = \{(1,3),(1,4),(2,2),(4,3),(4,4)\}$
(iii) $(R_1 \cup R_2) = \{(1,2),(1,3),(2,3),(2,4),(4,2)\}$.
$t(R_1 \cup R_2) = \{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(4,2),(4,3),(4,4)\}$.

(5) (i)False. Let, $R = \{(1,2),(2,3),(1,3)\}$ and $S = \{(3,4),(4,5),(3,5)\}$. Then $R \cup S = \{(1,2),(2,3),(1,3),(3,4),(4,5),(3,5)\}$. But $R \cup S$ is not transitive. Because it doesn't contain $\{(1,4),(1,5),(2,4),(2,5)\}$
(ii)True. Intersection of transitive relations will create another transitive relation.
(iii)True. If R is symmetric, then since converse preserves symmetry, R^c is also symmetric.

(iv)False. Let, $R = \{(1,2)\}$ and $S = \{(2,1)\}$. Then $R \cup S = \{(1,2),(2,1)\}$. But $R \cup S$ is not anti-symmetric.

(v)False. Because transitive property may not hold for r(s(t(R))). For example, if $R = \{(1,2),(1,3),(2,2)\}$ then $r(s(t(R))) = \{(1,1),(1,2),(1,3),(2,2),(2,1),(3,1),(3,3)\}$, which contains both (2,1) and (1,3), but it doesn't contain (2,3). So it's not transitive.

(vi)True. t(s(r(R))) is reflexive, symmetric and transitive. r(R) is always reflexive. Then s(r(R)) is symmetric and t(R) is always transitive. Hence its an equivalence relation.

(6) (i)R_1 is Equivalence Relation. Because it's reflexive, symmetric and transitive. $[1] = [2] = [3] = \{1,2,3\}, [4] = \{4\}, [5] = \{5\}$.

(ii)R_2 is not Equivalence Relation. Because it's reflexive, symmetric but it's not transitive. For example, $(2,4)$ is missing.