(1) Consider the set $A = \{1, 2, 3, 4, 5, 6\}$ and relations $R_1$ and $R_2$ on $A$. List relations as sets of ordered pairs, find their domain and range (similar to function) and draw their graphs. Domain of a relation on $A$ is a subset of $A$ defined by $\text{Domain}(R) = \{x \mid (x, y) \in R\}$. Range($R$) is a set $\{y \mid (x, y) \in R\}$.

(i) $R_1 = \{(x, y) \mid (y \text{ is a multiple of } x) \text{ and } (x \neq y)\}$

(ii) $R_2 = \{(x, y) \mid x + y = 6\}$

(2) For the following relations on $X$, determine if each relation is reflexive, symmetric, antisymmetric, and/or transitive. A relation could have more than one such property mentioned above. In that case state all properties. Explain your answers.

Let $P = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$ (set of all positive integers).

$L_1 = \{(x, y) \mid x \geq y, \text{ where } x, y \in P\}$

$L_2 = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

$L_3 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

$L_4 = \{(x, y) \mid \text{ the positive difference } |x - y| \text{ is evenly divisible by } 4\}$

(3) Let $X = \{1, 2, 3\}$. List a relation $R$ on $X$ (with at least 4 pairs) which is symmetric and transitive but not reflexive. Explain your answer.

(4) Let $X = \{1, 2, 3, 4\}$. Let $R_1 = \{(1, 2), (1, 3), (4, 2)\}$ and $R_2 = \{(2, 3), (2, 4)\}$

(i) List all elements/pairs of $(R_1 \circ R_2) \cap (R_2 \circ R_1)$. Show all steps.

(ii) List all pairs for $(R_1 \cup R_2)^2$.

(iii) Obtain transitive closure of $(R_1 \cup R_2)$.

(5) If each of the following statements is true, explain why. If false, give a counter example. Assume that $R$ and $S$ are some relations on a nonempty set $A$.

(i) If $R$ is transitive and $S$ is transitive then $R \cup S$ is transitive.

(ii) If $R$ is transitive and $S$ is transitive then $R \cap S$ is transitive.

(iii) If $R$ is symmetric then $R^c$ (converse of $R$) is symmetric.

(iv) If $R$ and $S$ both are antisymmetric then so is $R \cup S$.

Let $r$, $s$, and $t$ denote the closure operators as discussed in class.

(v) The relation $r(s(t(R)))$ is always an equivalence relation for any $R$.

(vi) The relation $t(s(r(R)))$ is always an equivalence relation for any $R$.

(6) For $X = \{1, 2, 3, 4, 5\}$, determine if each of the following relations is an equivalence relation. If yes, explain your answer and give equivalence classes. If no, explain why.

(i) $R_1 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4), (5, 5)\}$

(ii) $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4), (5, 5)\}$