Introduction to Functional Programming

OCaml Programming Basics

Data Structures in OCaml

Writing Efficient Programs in OCaml

Functional Programming

- Programs are viewed as functions transforming input to output
- Complex transformations are achieved by composing simpler functions (i.e. applying functions to results of other functions)

**Purely Functional Languages:** Values given to “variables” do not change when a program is evaluated

- “Variables” are names for values, not names for storage locations.
- Functions have *referential transparency*:
  - Value of a function depends solely on the values of its arguments
  - Functions do not have *side effects.*
  - Order of evaluation of arguments does not affect the value of a function’s output.
Features of Functional Programming Languages

- Support for complex (recursive) data types
  ... with automatic memory management (e.g. garbage collection)
- Functions themselves may be treated as values
  - Higher-order functions: Functions that functions as arguments.
  - Functions as first-class values: no arbitrary restrictions that distinguish functions from other data types (e.g. int)

History of Functional Programming

- LISP ('60)
- Scheme ('80s): a dialect of LISP; more uniform treatment of functions
- ML ('80s): Strong typing and type inference
  - Standard ML (SML, SML/NJ: '90s)
  - Categorical Abstract Machine Language (CAML, CAML Light, O’CAML: late '90s)
- Haskell, Gofer, HUGS, ... (late '90s): “Lazy” functional programming
ML

Developed initially as a “meta language” for a theorem proving system (Logic of Computable Functions)

- The two main dialects, SML and CAML, have many features in common:
  - data type definition, type inference, interactive top-level, …
- SML and CAML have different syntax for expressing the same things. For example:
  - In SML: variables are defined using `val` and functions using `fun`
  - In CAML: both variables and functions defined using `equations`
- Both have multiple implementations (Moscow SML, SML/NJ; CAML, OCAML) with slightly different usage directives and module systems.

OCAML

- CAML with “object-oriented” features.
- Compiler and run-time system that makes OCAML programs run with performance comparable imperative programs!
- A complete development environment including libraries building UIs, networking (sockets), etc.
- We will mainly use the non-oo part of OCAML
  - Standard ML (SML) has more familiar syntax.
  - CAML has better library and runtime support and has been used in more “real” systems.
The OCAML System

- OCAML interactive toplevel
  - Invocation:
    - UNIX: Run `ocaml` from command line
    - Windows: Run `ocaml.exe` from Command window or launch `ocamlwin.exe` from windows explorer.
  - OCAML prompts with “#”
  - User can enter new function/value definitions, evaluate expressions, or issue OCAML directives at the prompt.
  - Control-D to exit OCAML

- OCAML compiler:
  - `ocamlc` to compile OCAML programs to object bytecode.
  - `ocamlopt` to compile OCAML programs to native code.

Learning OCAML

- We will use OCAML interactive toplevel throughout for examples.
- What we type in can be entered into a file (i.e. made into a “program”) and executed.
- Read
  1. Tutorials at https://ocaml.org
  2. OCAML textbook: https://realworldocaml.org/
OCaml Expressions

- Syntax: \( (expression) ;; \)
- Two semicolons indicate the end of expression

**Example:**

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML’s Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 \times 3 ;; )</td>
<td>(- : \text{ int} = 6)</td>
</tr>
</tbody>
</table>

OCAML’s response:
- ‘\(-\)’ : The last value entered
- ‘\(\cdot\)’ : is of type
- ‘\text{int}’ : integer
- ‘\(=\)’ : and the value is
- ‘\(6\)’ : 6

**Expressions (contd.)**

- More examples:

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML’s Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 + 3 \times 4 ;; )</td>
<td>(- : \text{ int} = 14)</td>
</tr>
<tr>
<td>(-2 + 3 \times 4 ;; )</td>
<td>(- : \text{ int} = 10)</td>
</tr>
<tr>
<td>((-2 + 3) \times 4 ;; )</td>
<td>(- : \text{ int} = 4)</td>
</tr>
<tr>
<td>(4.4 ** 2.0 ;; )</td>
<td>(- : \text{ float} = 19.36)</td>
</tr>
<tr>
<td>(2 + 2.2 ;; )</td>
<td>... This expression has type float but is used here with type int</td>
</tr>
<tr>
<td>(2.7 + 2.2 ;; )</td>
<td>... This expression has type float but is used here with type int</td>
</tr>
<tr>
<td>(2.7 +. 2.2 ;; )</td>
<td>(- : \text{ float} = 4.9)</td>
</tr>
</tbody>
</table>
Operators

<table>
<thead>
<tr>
<th>Operators</th>
<th>Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Integer arithmetic</td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td></td>
</tr>
<tr>
<td>/</td>
<td></td>
</tr>
<tr>
<td>mod</td>
<td></td>
</tr>
<tr>
<td>+.</td>
<td>Floating point arithmetic</td>
</tr>
<tr>
<td>-.</td>
<td></td>
</tr>
<tr>
<td>*.</td>
<td></td>
</tr>
<tr>
<td>./</td>
<td></td>
</tr>
<tr>
<td>**</td>
<td></td>
</tr>
<tr>
<td>&amp; &amp; ,</td>
<td>, not</td>
</tr>
<tr>
<td></td>
<td>Boolean operations</td>
</tr>
</tbody>
</table>

Value definitions

- Syntax: `let ⟨name⟩ = ⟨expression⟩ ;;`
- Examples:

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML’s Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>let x = 1;;</td>
<td>val x : int = 1</td>
</tr>
<tr>
<td>let y = x + 1;;</td>
<td>val y : int = 2</td>
</tr>
<tr>
<td>let x = x + 1;;</td>
<td>val x : int = 3</td>
</tr>
<tr>
<td>let z = &quot;OCAML rocks!&quot;;;</td>
<td>val z : string = &quot;OCAML rocks!&quot;</td>
</tr>
<tr>
<td>let w = &quot;21&quot;;;</td>
<td>val w : string = &quot;21&quot;</td>
</tr>
<tr>
<td>let v = int_of_string(w);</td>
<td>val v : int = 21</td>
</tr>
</tbody>
</table>
Functions

- Syntax: `let ⟨name⟩ {⟨argument⟩} = ⟨expression⟩ ;;`
- Examples:

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML’s Response</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>let f x = 1;;</code></td>
<td><code>val f : 'a -&gt; int = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>let g x = x;;</code></td>
<td><code>val g : 'a -&gt; 'a = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>let inc x = x + 1;;</code></td>
<td><code>val inc : int -&gt; int = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>let sum(x,y) = x+y;;</code></td>
<td><code>val sum : int * int -&gt; int = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>let add x y = x+y;;</code></td>
<td><code>val add : int -&gt; int -&gt; int = &lt;fun&gt;</code></td>
</tr>
</tbody>
</table>

Note the use of *parametric polymorphism* in functions `f` and `g`

More Examples of Functions

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML’s Response</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>let max(x, y) =</code></td>
<td><code>val max : 'a * 'a -&gt; 'a = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>    if x &lt; y</code></td>
<td></td>
</tr>
<tr>
<td><code>    then y</code></td>
<td></td>
</tr>
<tr>
<td><code>    else x;;</code></td>
<td></td>
</tr>
<tr>
<td><code>let mul(x, y) =</code></td>
<td><code>Unbound value mul</code></td>
</tr>
<tr>
<td><code>    if x = 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    then 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    else y+mul(x-1,y);;</code></td>
<td></td>
</tr>
<tr>
<td><code>let rec mul(x, y) =</code></td>
<td><code>val mul : int * int -&gt; int = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>    if x = 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    then 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    else y+mul(x-1,y);;</code></td>
<td></td>
</tr>
<tr>
<td><code>let rec mul(x, y) =</code></td>
<td><code>val mul : int * int -&gt; int = &lt;fun&gt;</code></td>
</tr>
<tr>
<td><code>    if x = 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    then 0</code></td>
<td></td>
</tr>
<tr>
<td><code>    else let i = mul(x-1,y)</code></td>
<td></td>
</tr>
<tr>
<td><code>    in y+i;;</code></td>
<td></td>
</tr>
</tbody>
</table>
An Aside on Polymorphism

- Java supports two kinds of polymorphism:
  - *Subtype* polymorphism: a method defined for objects of class \( A \) can be applied to objects of \( A \)'s subclasses.
  - *Ad-hoc* polymorphism: a method name can be overloaded, with same name representing many different methods.

- *Templates* in C++ support an additional kind of polymorphism:
  ```cpp
template <typename T>
  int f(T x) { return 1; }

  template <typename T>
  T g(T x) { return x; }
```

Data structures

Examples of built-in data structures (lists and tuples):

<table>
<thead>
<tr>
<th>User Input</th>
<th>OCAML's Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1];</td>
<td>- : int list = [1]</td>
</tr>
<tr>
<td>[4.1; 2.7; 3.1];</td>
<td>- : float list = [4.1; 2.7; 3.1]</td>
</tr>
<tr>
<td>[4.1; 2];</td>
<td>... This expression has type int but is used here with type float</td>
</tr>
<tr>
<td>[[1;2]; [4;8;16]];</td>
<td>- : int list list = [[1;2], [4;8;16]]</td>
</tr>
<tr>
<td>1::2::[]</td>
<td>- : int list = [1; 2]</td>
</tr>
<tr>
<td>1::(2::[])</td>
<td>- : int list = [1; 2]</td>
</tr>
<tr>
<td>(1,2);</td>
<td>- : int * int = (1, 2)</td>
</tr>
<tr>
<td>()</td>
<td>- : unit = ()</td>
</tr>
</tbody>
</table>
| let (x,y) = (3,7);            | val x : int = 3  
                              | val y : int = 7 |
Pattern Matching

- Used to “deconstruct” data structures.
- Example:
  
  ```ocaml
  let rec sumlist l =
    match l with
    [] -> 0
    | x::xs -> x + sumlist(xs);
  ```

- When evaluating `sumlist [2; 5]`
  - The argument `[2; 5]` matches the pattern `x::xs`,
  - ... setting `x` to 2 and `xs` to `[5]`
  - ... then evaluates `2 + sumlist([5])`

---

Match

`match` is analogous to a “switch” statement

- Each case describes
  - a pattern (lhs of ‘->’) and
  - an expression to be evaluated if that pattern is matched (rhs of ‘->’)
  - patterns can be constants, or terms made up of constants and variables

- The different cases are separated by ‘|’
- A matching pattern is found by searching in order (first case to last case)
- The first matching case is selected; others are discarded

```ocaml
let emptyList l =
  match l with
    [] -> true
    | _ -> false;;
```
Pattern Matching (contd.)

- Pattern syntax:
  - Patterns may contain “wildcards” (i.e. ‘.’); each occurrence of a wildcard is treated as a new anonymous variable.
  - Patterns are linear: any variable in a pattern can occur at most once.
- Pattern matching is used very often in OCAML programs.
- OCAML gives a shortcut for defining pattern matching in functions with one argument.
  Example:

```ocaml
let rec sumlist l = match l with
  | [] -> 0
  | x::xs -> x + sumlist(xs);
```

- OCAML gives a shortcut for defining pattern matching in functions with one argument.
  Example:

```ocaml
let rec sumlist = function
  | [] -> 0
  | x::xs -> x + sumlist(xs);
```

Functions on Lists

- Add one list to the end of another:
  ```ocaml
  let rec append v1 v2 = match v1 with
    | [] -> v2
    | x::xs -> x::(append xs v2);
  ```

  - Note that this function has type append: ‘a list -> ‘a list -> ‘a list
  and hence can be used to concatenate arbitrary lists, as long as the list elements are of the same type.
  - This function is implemented by builtin operator @
- Many list-processing functions are available in module Lists.
  Examples:
  - Lists.hd: get the first element of the given list
  - Lists.rev: reverse the given list
Enumerated Types

- A finite set of values
- Two values can be compared for equality
- There is no order among values
- Example:

  ```
  type primaryColor = RED | GREEN | BLUE;;
  type status = Freshman | Sophomore | Junior | Senior;;
  ```

- Syntax: `type ⟨name⟩ = ⟨name⟩ { | ⟨name⟩ } ;;`
- A note about names:
  - Names of constants must begin with an `uppercase` letter.
  - Names of types, functions and variables must begin with a `lowercase` letter.
  - Names of constants are global within a module and not local to its type.

Records

- Used to define structures with named fields.
- Example:

  ```
  type student = {name:string; gpa:float; year:status;};
  ```

- Syntax: `type ⟨name⟩ = { ⟨name⟩ : ⟨name⟩ ; } ;;`
- Usage:
  - Creating records:
    ```
    let joe = {name="Joe"; gpa=2.67; year=Sophomore};;
    ```
  - Accessing fields:
    ```
    let x = joe.gpa;; (* using "." operator *)
    let {gpa=x} = joe;; (* using pattern matching *)
    ```
  - Field names are global within a module and not local to its type.
### Union types

- Used to define (possibly recursive) structured data with tags.

  - Example:
    ```
    type iTree = Node of int * iTree * iTree | Empty;;
    ```

- The empty tree is denoted by `Empty`

- The tree with one node, with integer 2, is denoted by `Node(2,Empty,Empty)`

- **Tree**

  ```
  Tree
  1
  ↙ ↘
  2 3
  ↙ ↘ ↘
  4 5 6
  ```

  **Denoted by**

  ```
  Node(1, Node(2, Node(4, Empty, Empty), Node(5, Empty, Empty)), Node(3, Empty, Node(6, Empty, Empty))))
  ```

### Union Types (contd.)

- Generalizes enumerated types

- Constants that tag the different structures in an union (e.g. `Node` and `Empty`) are called *data constructors*.

- Usage example: counting the number of elements in a tree:

  ```
  let rec nelems tree =
  match tree with
  Node(i, lst, rst) ->
  (* 'i' is the value of the node; 'lst' is the left sub tree; and 'rst' is the right sub tree *)
  1 + nelems lst + nelems rst
  | Empty -> 0;;
  ```
Polymorphic Data Structures

- Structures whose components may be of arbitrary types.
- Example:

  ```ml
  type 'a tree = Node of 'a * 'a tree * 'a tree | Empty;;
  'a in the above example is a type variable ... analogous to the
  typename parameters of a C++ template
  ``

- Parameteric polymorphism enforces that all elements of the tree are
  of the same type.
- Usage example: traversing a tree in preorder:

  ```ml
  let rec preorder tree = 
    match tree with 
    Node(i, lst, rst) -> i::(preorder lst)@(preorder rst) 
  | Empty -> [];;
  ``

Exceptions

- **Total function**: function is defined for every argument value.
  Examples: +, length, etc.
- **Partial function**: function is defined only for a subset of argument
  values.
  - Examples: /, Lists.hd, etc.
  - Another example:

    ```ml
    (* find the last element in a list *)
    let rec last = function
    x::[] -> x 
    | _::xs -> last xs;;
    ``

- Exceptions can be used to signal invalid arguments.
- Failed pattern matching (due to incomplete matches) is signalled with
  (predefined) Match_failure exception.
- Exceptions also signal unexpected conditions (e.g. I/O errors)
Exceptions (contd.)

- Users can define their own exceptions.
- Exceptions can be thrown using `raise`.

```ocaml
(* Exception to signal no elements in a list *)
extension NoElements;

let rec last = function
  | [] -> raise NoElements
  | x::[] -> x
  | _::xs -> last xs;;
```

- Exceptions can be handled using `try ... with`.

```ocaml
extension DumbCall;
let test l y =
  try (last l) / y
  with
    NoElements -> 0
    | Division_by_zero -> raise DumbCall;;
```

Functions of functions

- Add 1 to every element in list:
  ```ocaml
  let rec add_one = function
    | [] -> []
    | x::xs -> (x+1)::(add_one xs);;
  ```

- Multiply every element in list by 2:
  ```ocaml
  let rec double = function
    | [] -> []
    | x::xs -> (x*2)::(double xs);;
  ```

- Perform function `f` on every element in list:
  ```ocaml
  let rec map f = function
    | [] -> []
    | x::xs -> (f x)::(map f xs);;
  ```

- Now we can write `add_one` and `double` as:
  ```ocaml
  let add_one = map ((+) 1);; let double = map (( * ) 2);;
  ```
More examples of higher-order functions

- Sum all elements in a list
  
<table>
<thead>
<tr>
<th>OCAML</th>
<th>C</th>
</tr>
</thead>
</table>
  | let rec sumlist = function  
  |   [] -> 0  
  |   | x::xs -> x + sumlist xs;;  
  | let sumlist = foldr (+) 0;; |

- Multiply all elements in a list
  
<table>
<thead>
<tr>
<th>OCAML</th>
<th>C</th>
</tr>
</thead>
</table>
  | let rec prodlist = function  
  |   [] -> 1  
  |   | x::xs -> x * prodlist xs;;  
  | let prodlist = foldr ( * ) 1;; |

Accumulate over a list:

- Using foldr:

<table>
<thead>
<tr>
<th>OCAML</th>
<th>C</th>
</tr>
</thead>
</table>
  | let rec foldr f b = function  
  |   (* f is the function to apply at element;  
  |   b is the base case value *)  
  |   [] -> b  
  |   | x::xs -> f x (foldr f b xs); |

  | let sumlist = foldr (+) 0;; |

Using foldr:

<table>
<thead>
<tr>
<th>OCAML</th>
<th>C</th>
</tr>
</thead>
</table>
| let sumlist = foldr (+) 0;; | int sumlist(List l) {
| | if (l == NULL) {
| |   return 0;
| | } else {
| |   return (l->element) +
| |   sumlist(l->next);
| | }

Writing Efficient OCAML Programs

- Using recursion to sum all elements in a list:

<table>
<thead>
<tr>
<th>OCAML</th>
<th>C</th>
</tr>
</thead>
</table>
  | let rec sumlist = function  
  |   [] -> 0  
  |   | x::xs -> x + sumlist xs;;  
  | let sumlist = foldr (+) 0;; |

- Iteratively summing all elements in a list (C):

  int acc = 0;
  for(l=list; l!=NULL; l = l->next)
      acc += l->element;
Iteration vs. Recursion

- Recursive summation takes stack space proportional to the length of the list

\[
\text{sumlist([1;2])} \Rightarrow \text{sumlist([1;2])} \Rightarrow \text{sumlist([1;2])} \\
2 \Rightarrow \text{sumlist([1;2])} \Rightarrow \text{sumlist([1;2])} \\
3 \Leftarrow \text{sumlist([1;2])} \Leftarrow \text{sumlist([1;2])}
\]

- Iterative summation takes constant stack space.

Tail Recursion

- let rec last = function
  [] -> raise NoElements
  | x::[] -> x
  | _::xs -> last xs;;

- Evaluation of last [1;2;3];;

\[
\text{last([1;2;3])} \Rightarrow \text{last([1;2;3])} \Rightarrow \text{last([1;2;3])} \\
3 \Leftarrow \text{last([1;2;3])} \Leftarrow \text{last([1;2;3])}
\]
Tail Recursion (contd.)

```ocaml
let rec last = function
    [] -> raise NoElements
  | x::[] -> x
  | _::xs -> last xs;;
```

- Note that when the 3rd pattern matches, the result of `last` is whatever is the result of `last xs`. Such calls are known as *tail recursive calls*.

- Tail recursive calls can be evaluated without extra stack:

  \[
  \begin{array}{c}
  \text{last([1;2;3])} \\
  \Rightarrow \text{last([2;3])} \\
  \Rightarrow \text{last([3])} \\
  \downarrow \\
  3
  \end{array}
  \]

Efficient programs using tail recursion

Example: efficient recursive function for summing all elements:

<table>
<thead>
<tr>
<th><strong>C</strong></th>
<th><strong>OCAML</strong></th>
</tr>
</thead>
</table>
| int acc_sumlist(int acc, List l) { 
  if (l == NULL) 
    return acc;
  else 
    return acc_sumlist( 
      acc + (l->element),
      l->next);
} int sumlist(List l) { 
  return acc_sumlist(0, l);} | let rec acc_sumlist acc = function 
  [] -> acc 
  | x::xs -> acc_sumlist 
    (acc+x) 
  xs;; 
  let sumlist l = 
  acc_sumlist 0 l;; |

\[
\begin{array}{c}
\text{acc_sumlist(0,[1;2])} \\
\Rightarrow \text{acc_sumlist(1,[2])} \\
\Rightarrow \text{acc_sumlist(3,[])} \\
\downarrow \\
3
\end{array}
\]