Data Structures and Algorithms for Big Databases

Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
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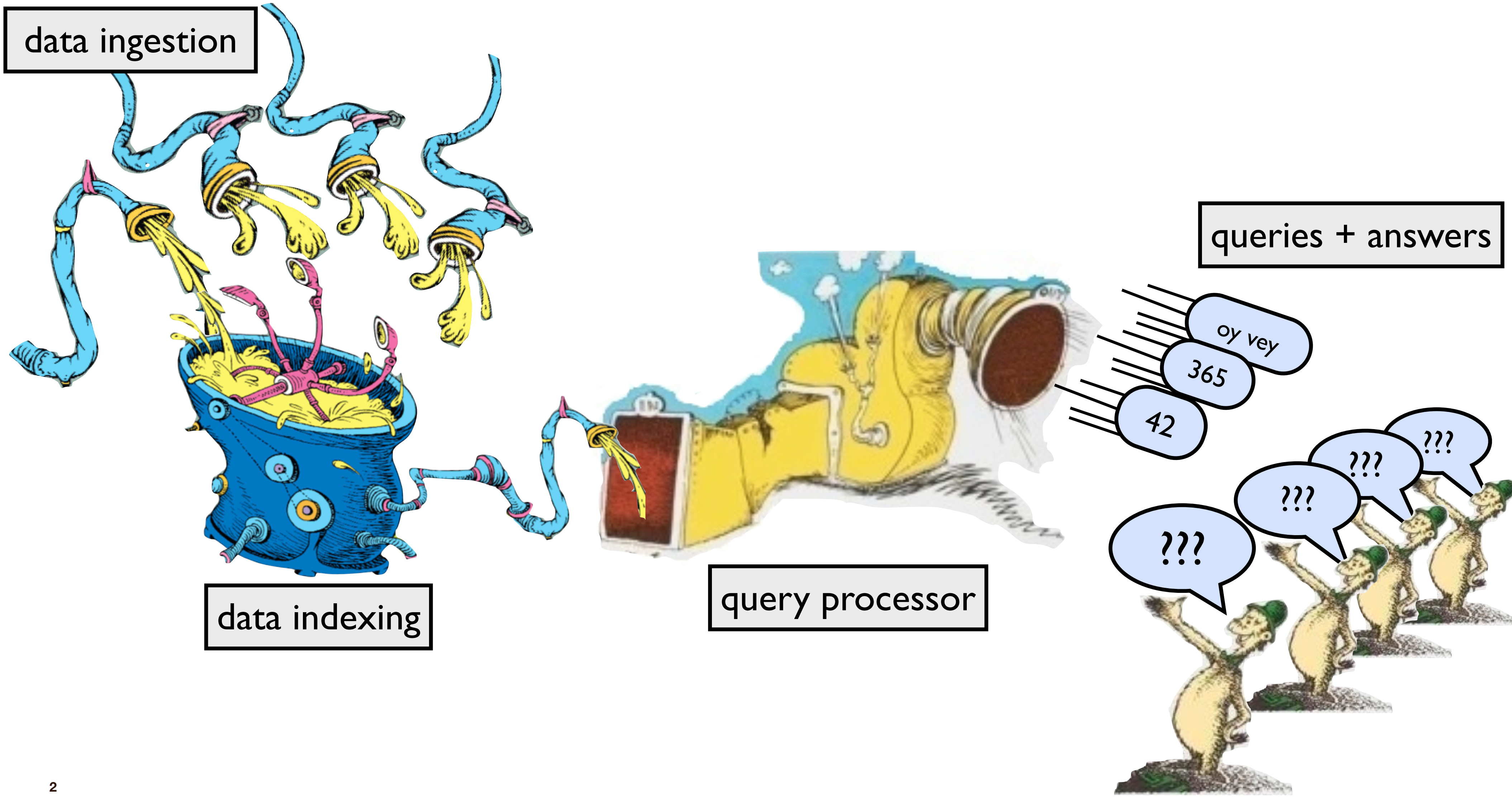
data ingestion
data indexing
query processor
queries + answers
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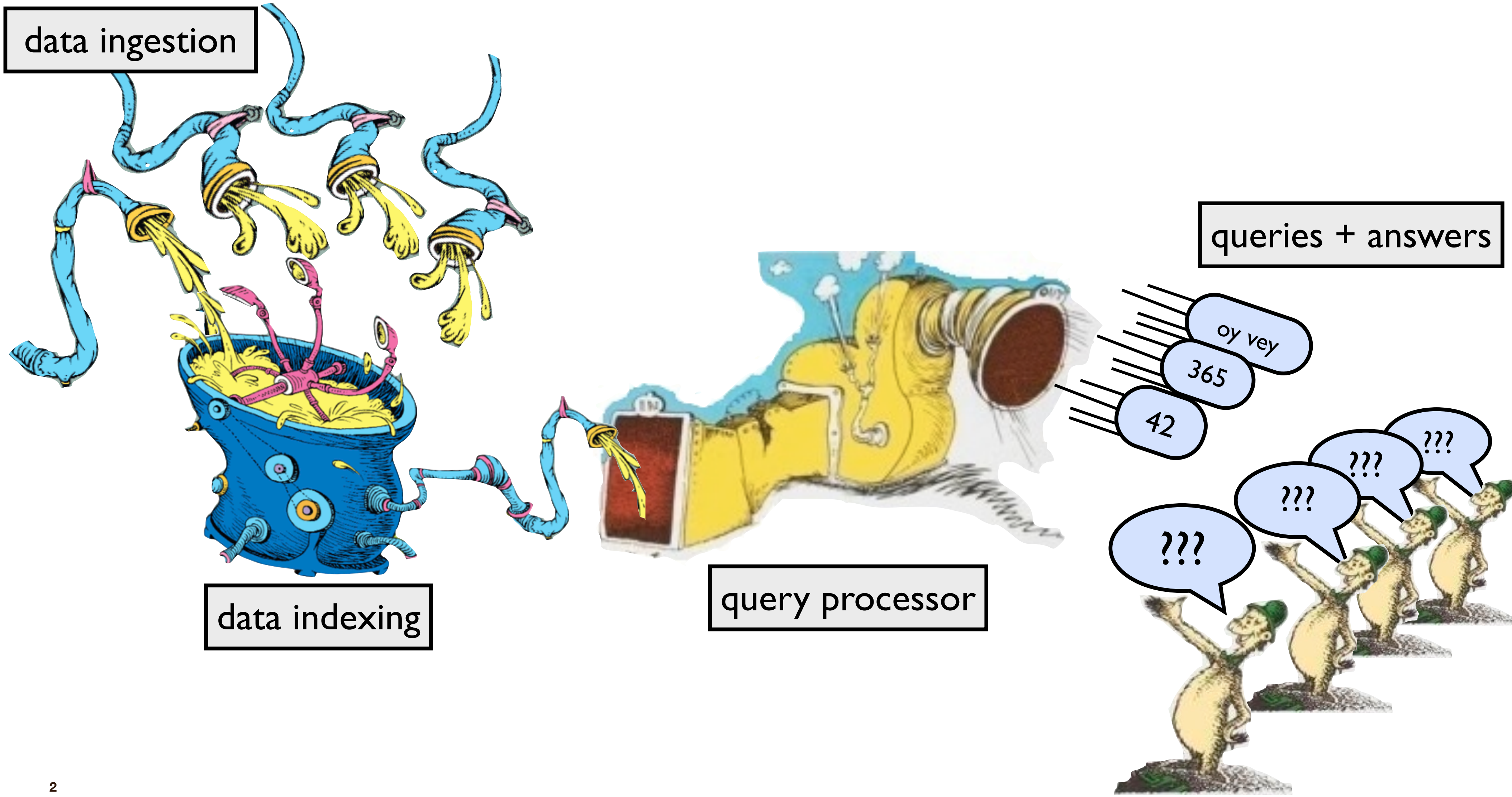
For on-disk data, one sees funny tradeoffs in the speeds of data ingestion, query speed, and freshness of data.



Funny tradeoff in ingestion, querying, freshness

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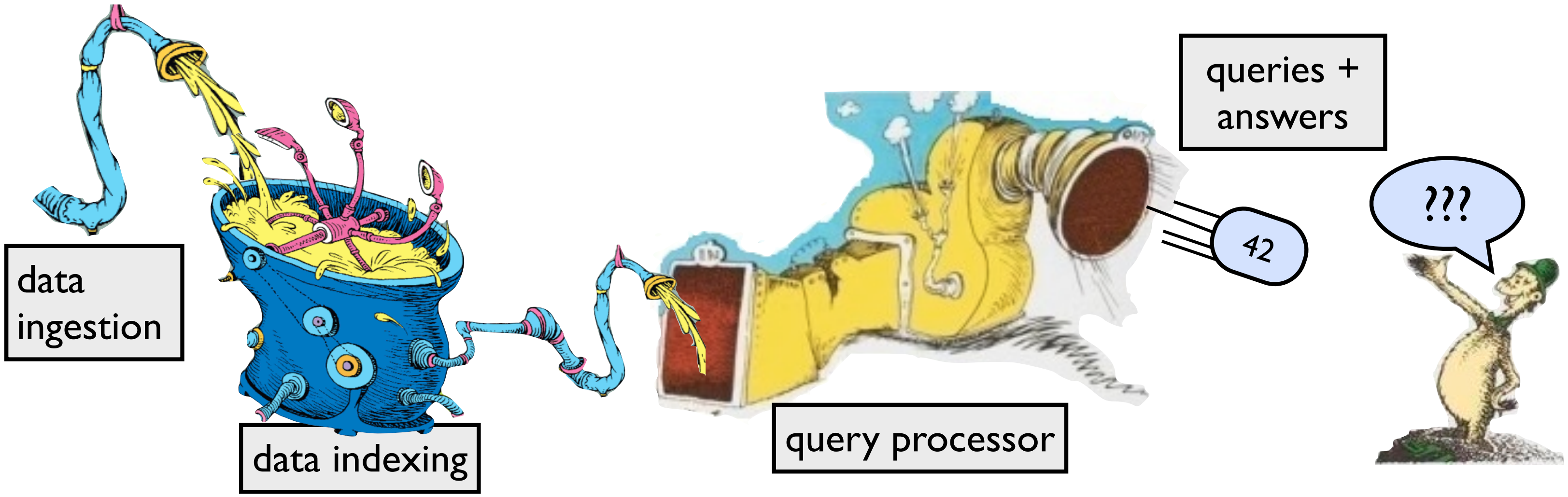

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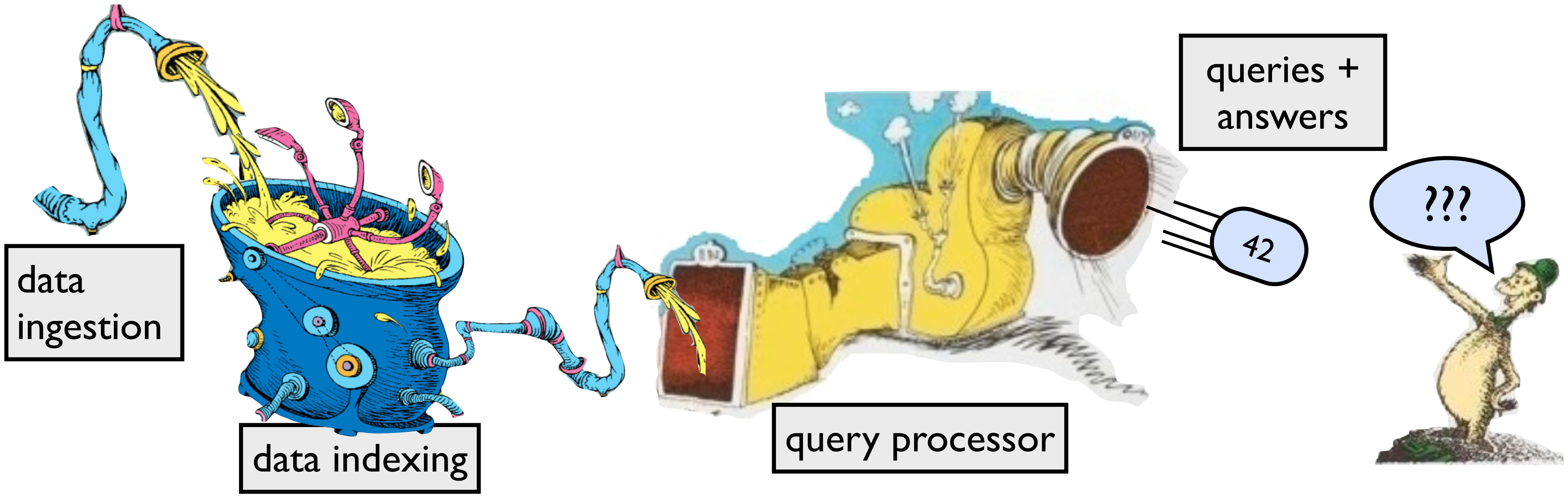

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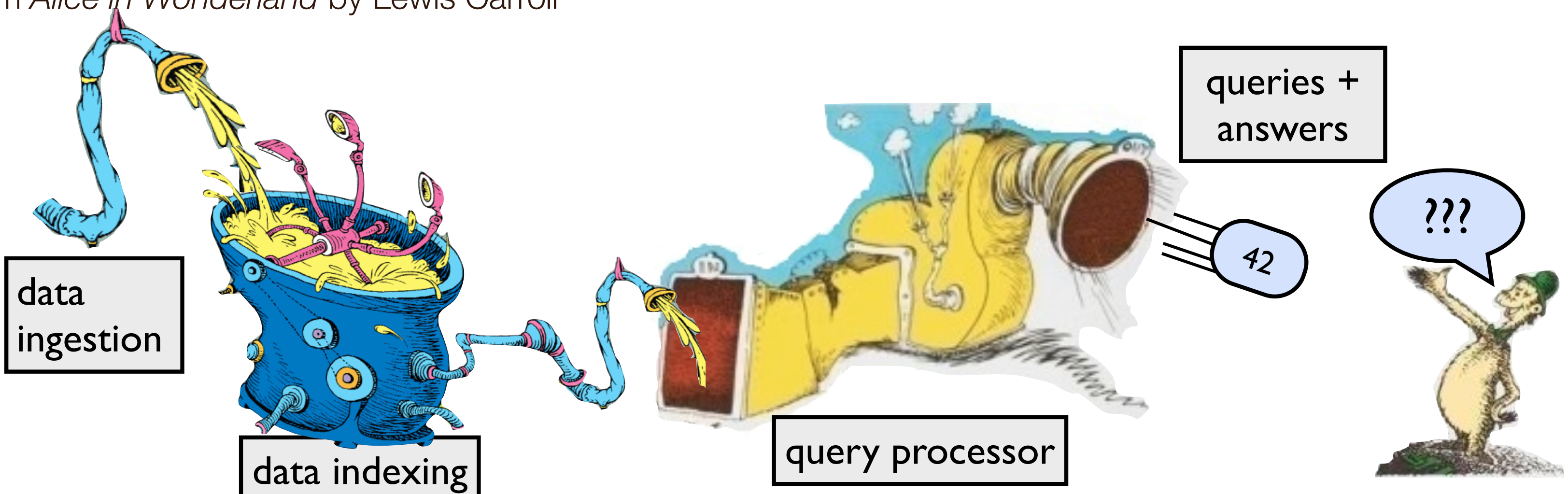
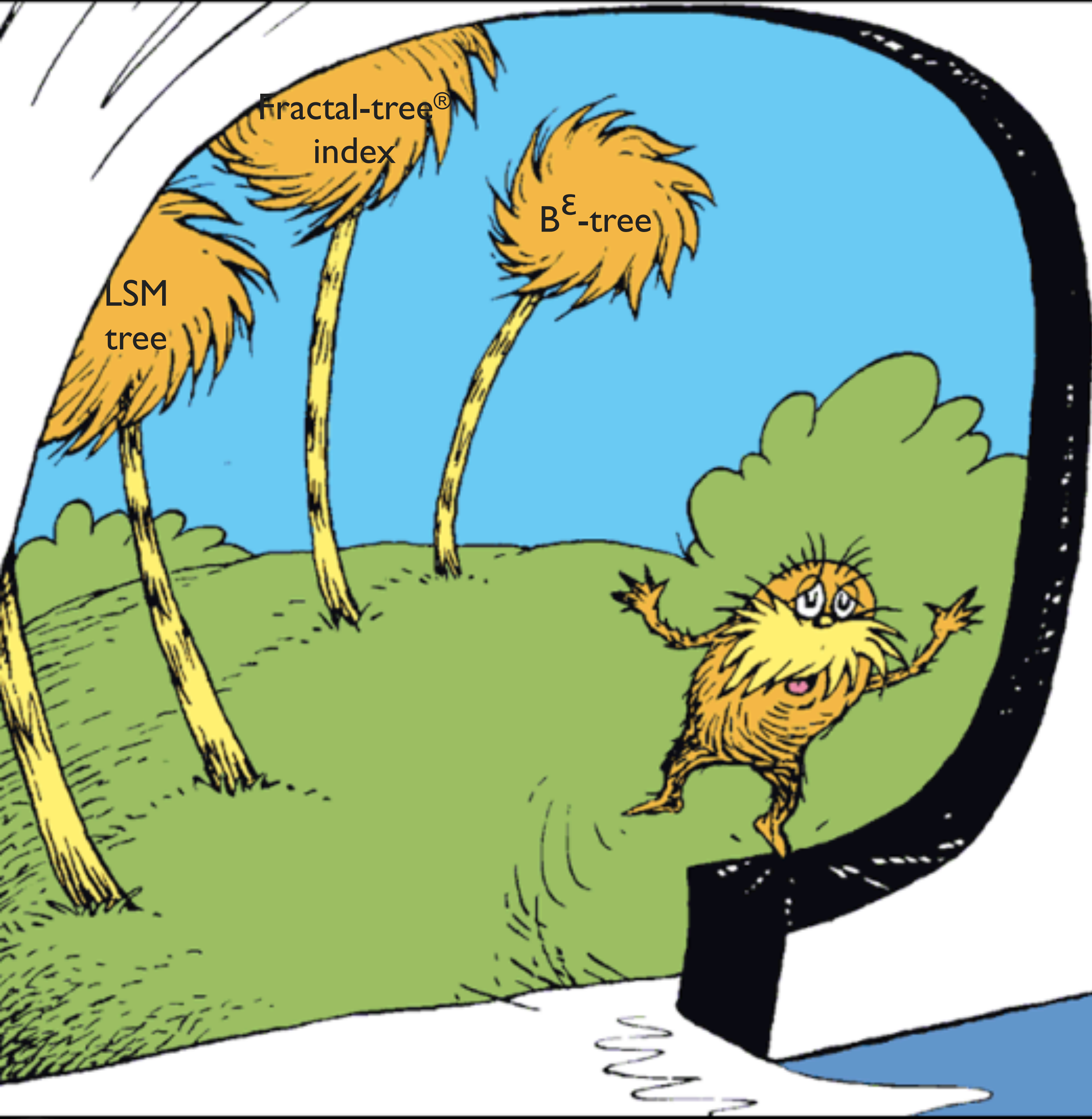




This tutorial

- Better data structures significantly mitigate the insert/query/freshness tradeoff.
- These structures scale to much larger sizes while efficiently using the memory-hierarchy.

What we mean by Big Data

We don't define Big Data in terms of TB, PB, EB.

By Big Data, we mean

- The data is too big to fit in main memory.
- We need data structures on the data.
- Words like "index" or "metadata" suggest that there are underlying data structures.
- These underlying data structures are also too big to fit in main memory.


In this tutorial we study the underlying data structures for managing big data.
File systems
SQL
NewSQL
NoSQL


But enough about databases...
... more about us.

Our Research and Tokutek

A few years ago we started working together on I/O-efficient and cache-oblivious data structures.

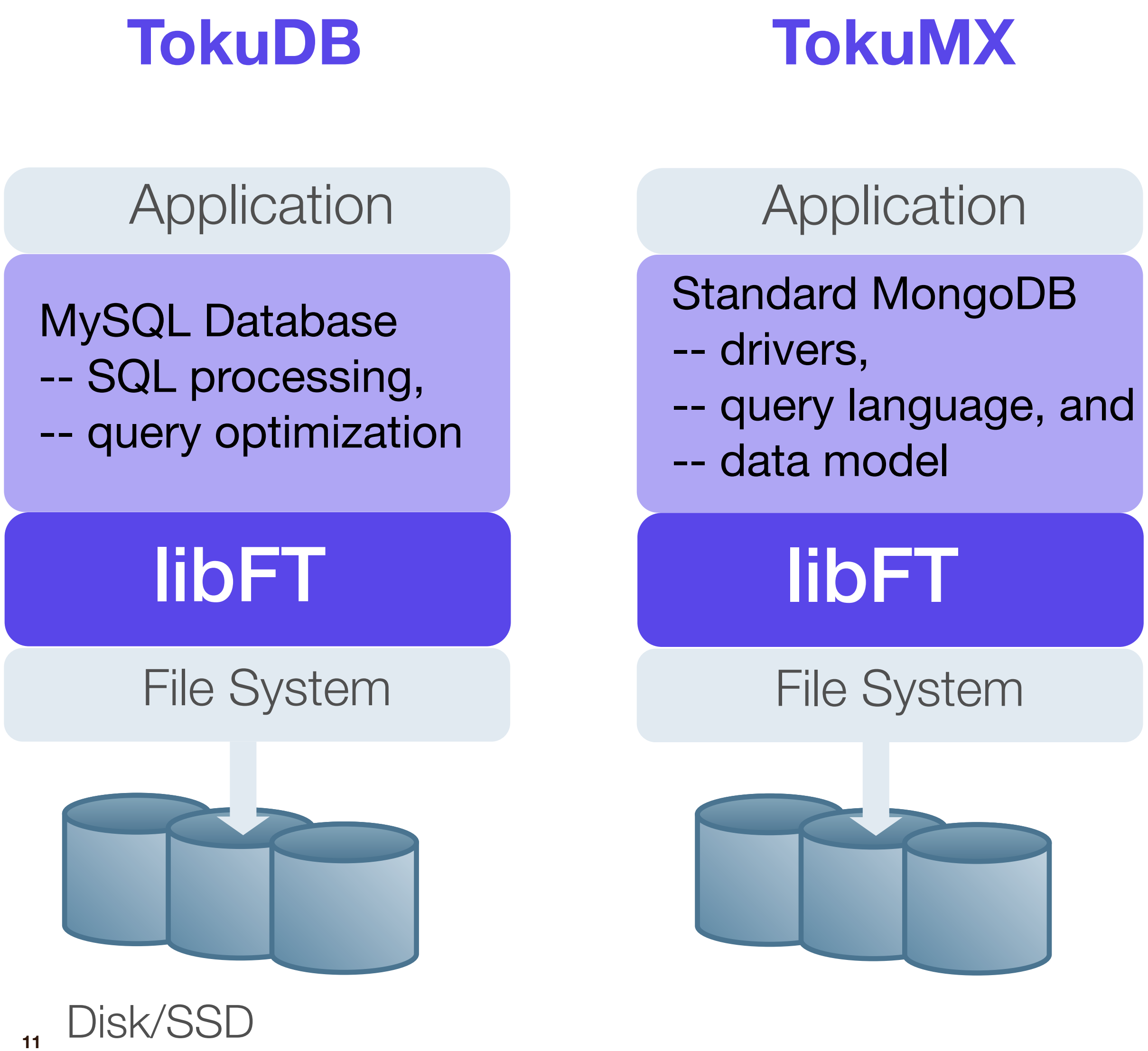
Michael

Martin

Bradley

Along the way, we started Tokutek to commercialize our research.

Tokutek sells open source, ACID compliant, implementations of MySQL and MongoDB.



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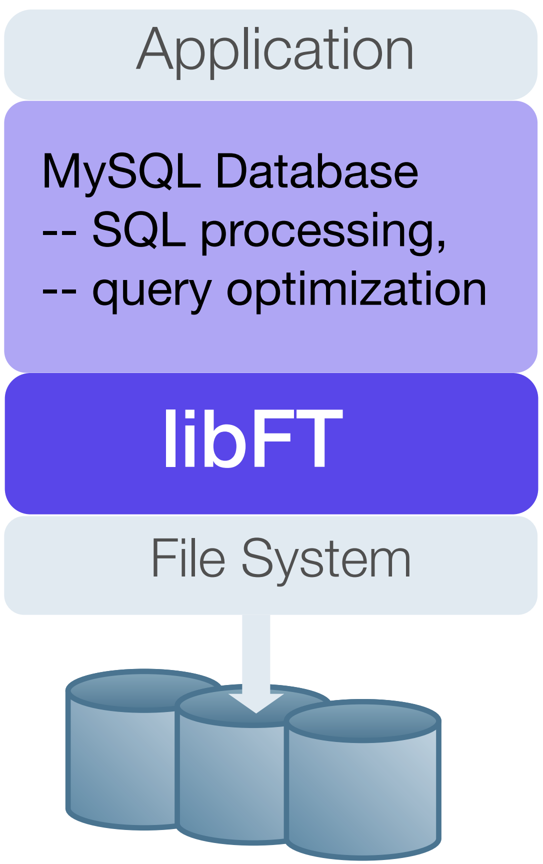
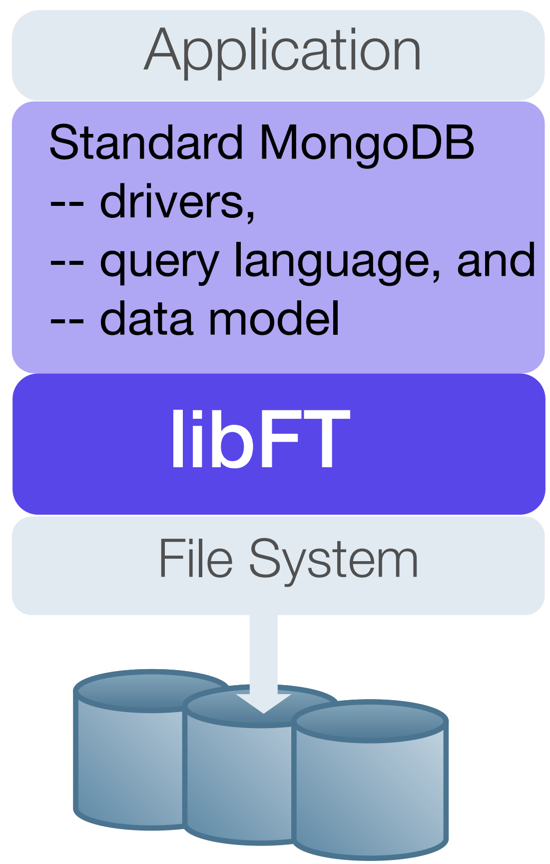
TokuDB



TokuMX



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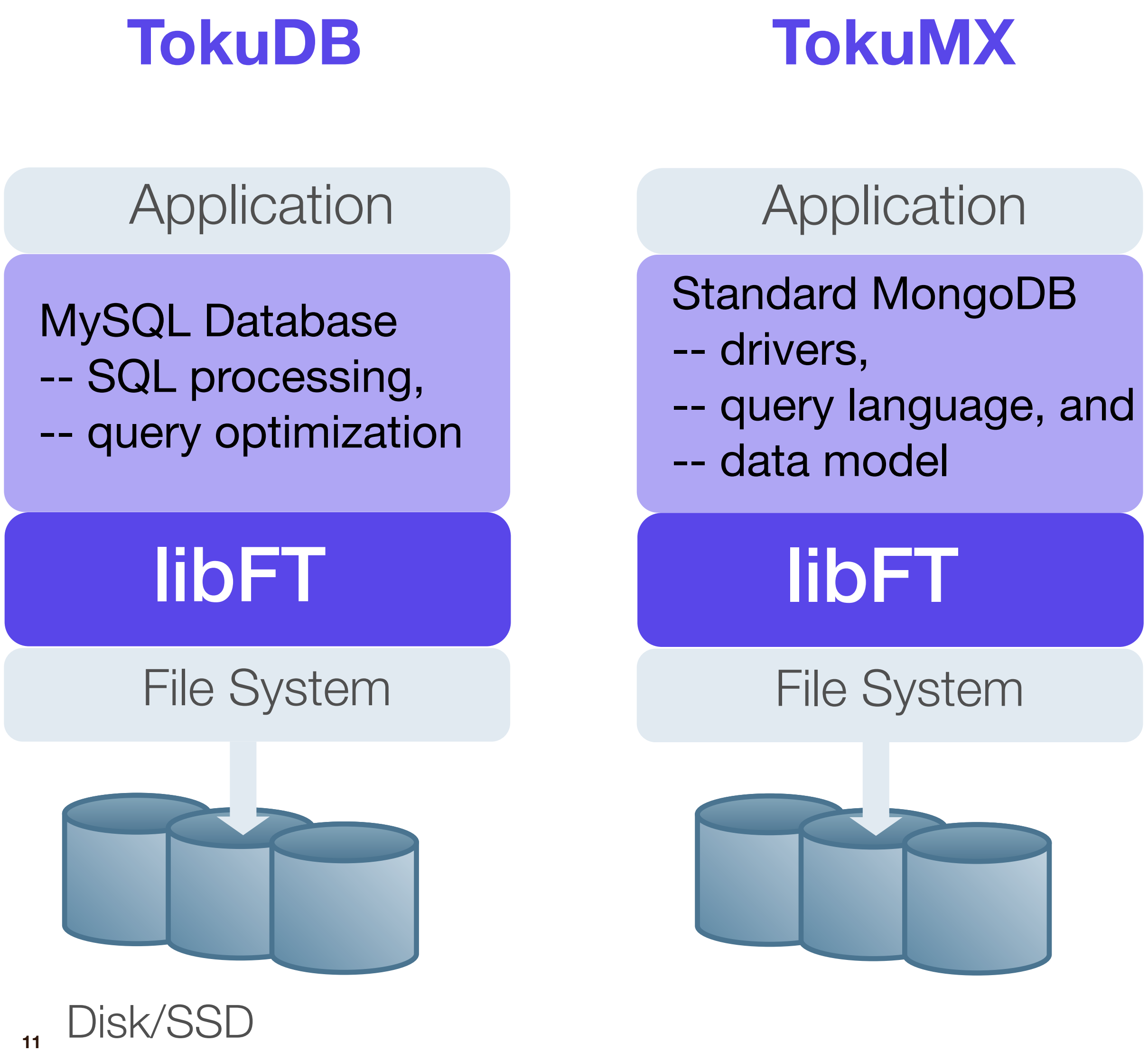

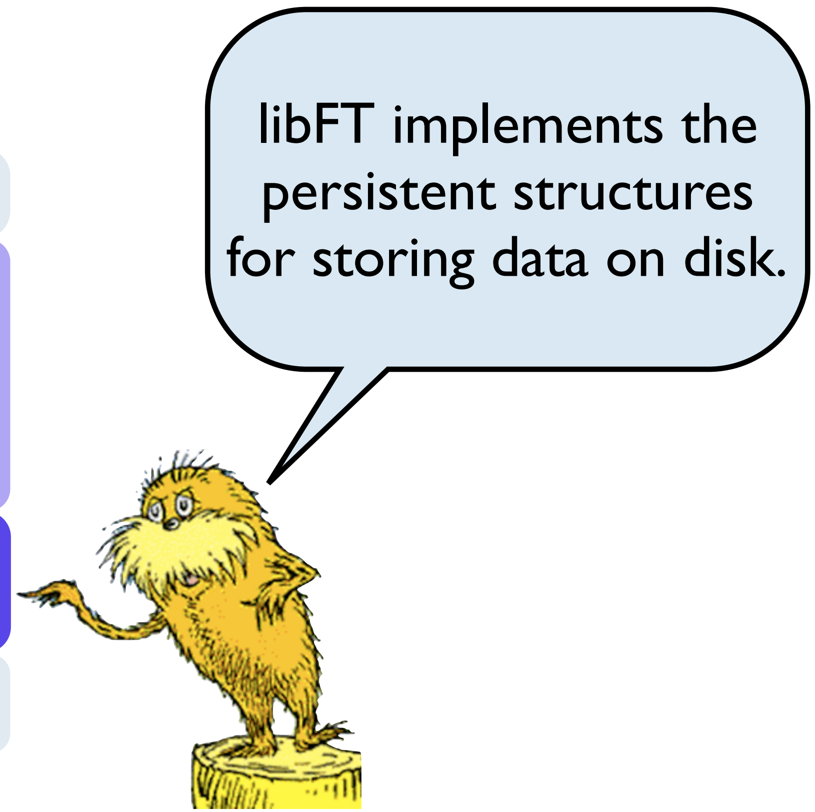

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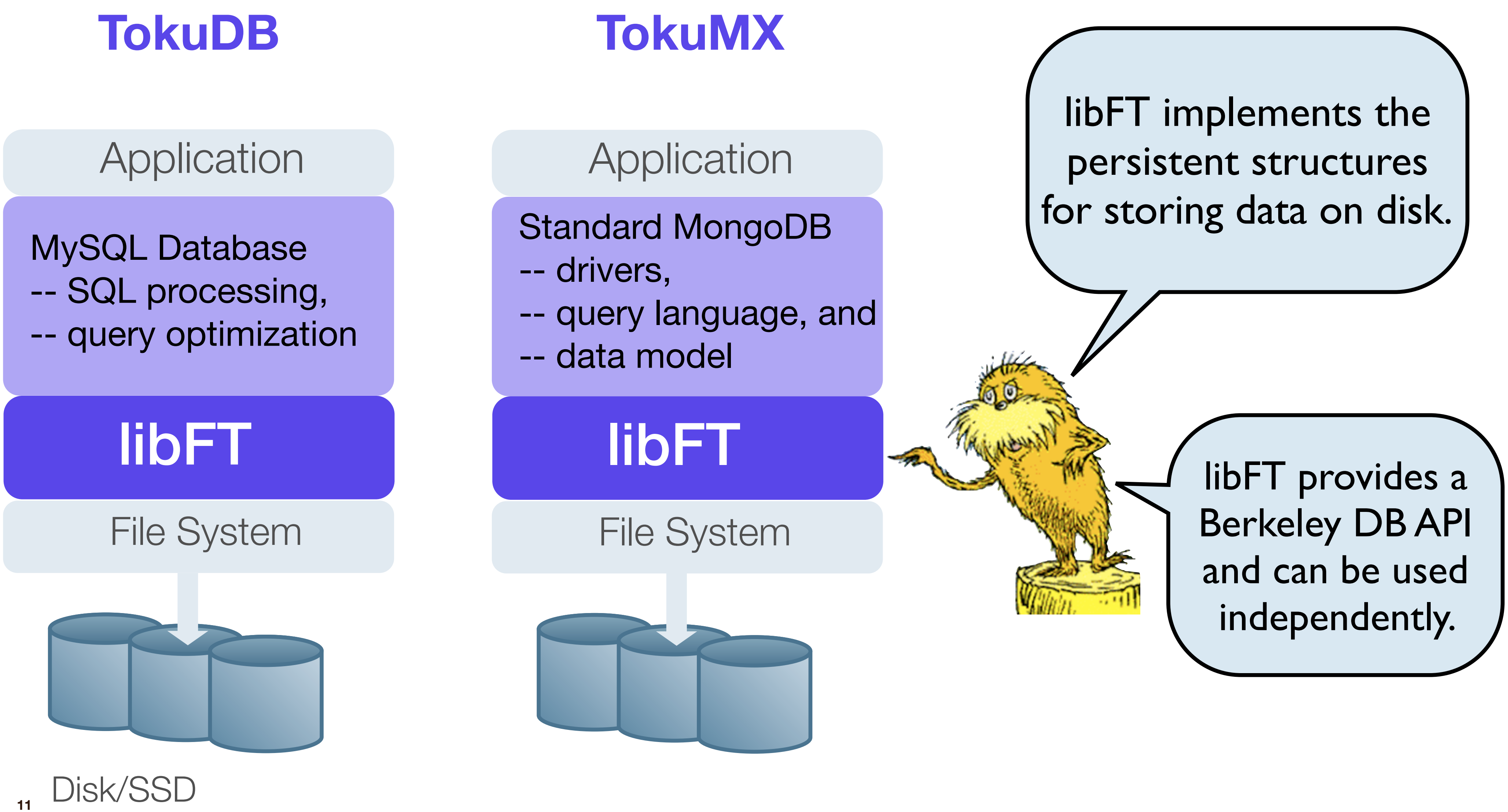

Our Mindset

- This tutorial is self contained.
- We want to teach.
- If something we say isn't clear to you, please ask questions or ask us to clarify/repeat something.
- You should be comfortable using math.
- You should want to listen to data structures for an afternoon.

Topics and Outline for this Tutorial

I/O model.

Write-optimized data structures.

How write-optimized data structures can help file systems.

Cache-oblivious analysis.

Log-structured merge trees.

Indexing strategies.

Block-replacement algorithms.

Sorting Big Data.

Data Structures and Algorithms for Big Data

Module 1: I/O Model and Cache-Oblivious Analysis

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Story for Module

- If we want to understand the performance of data structures within databases we need algorithmic models for understanding I/O.
- There's a long history of memory-hierarchy models. Many are beautiful. Most have found little practical use.
- Two approaches are very powerful, the Disk Access Machine (DAM) model and cache-oblivious analysis.
- We'll present the DAM model in this module to lay a foundation for the rest of the tutorial.
- Cache-oblivious analysis comes later in the tutorial.

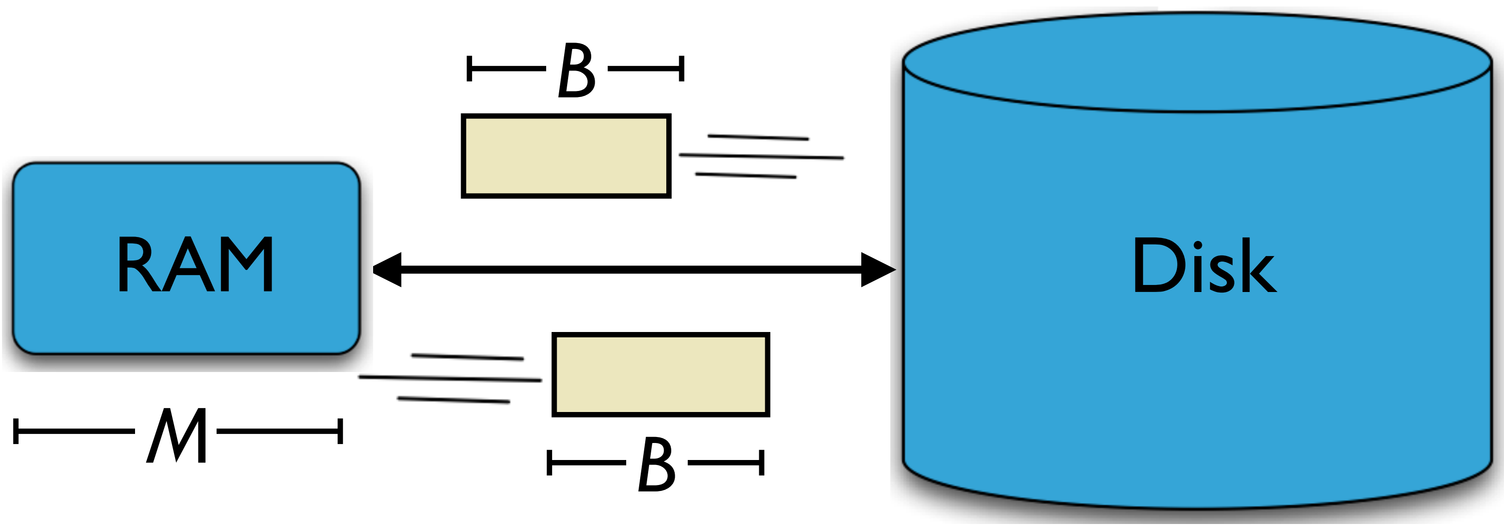
I/O in the Disk Access Machine (DAM) Model

How computation works:

- Data is transferred in blocks between RAM and disk.
- The # of block transfers dominates the running time.

Goal: Minimize # of block transfers

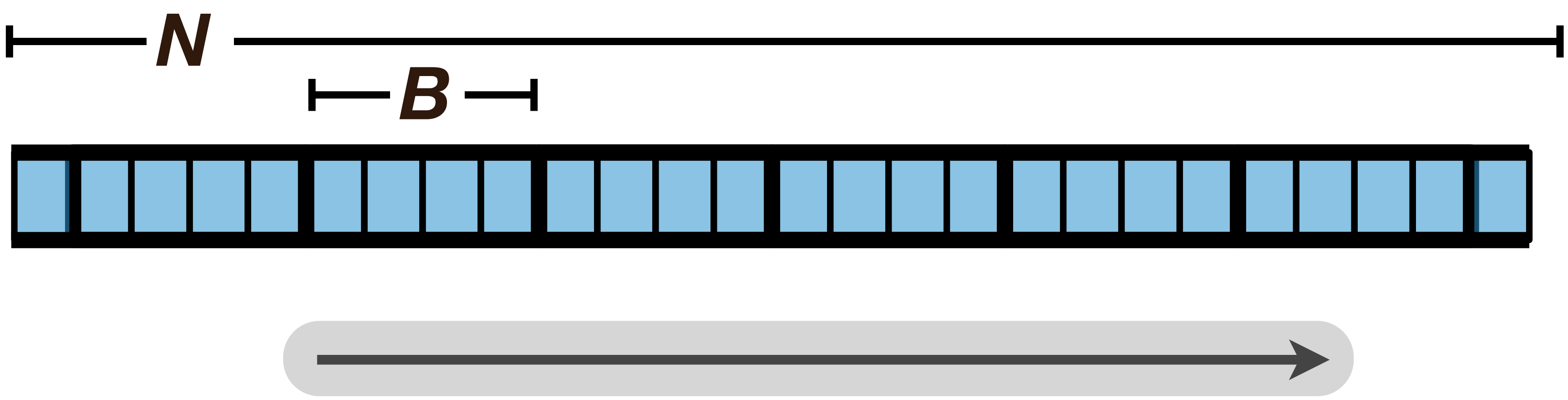
- Performance bounds are parameterized by block size ***B***, memory size ***M***, data size ***N***.



[Aggarwal+Vitter '88]

Example: Scanning an Array

Question: How many I/Os to scan an array of length *N*?



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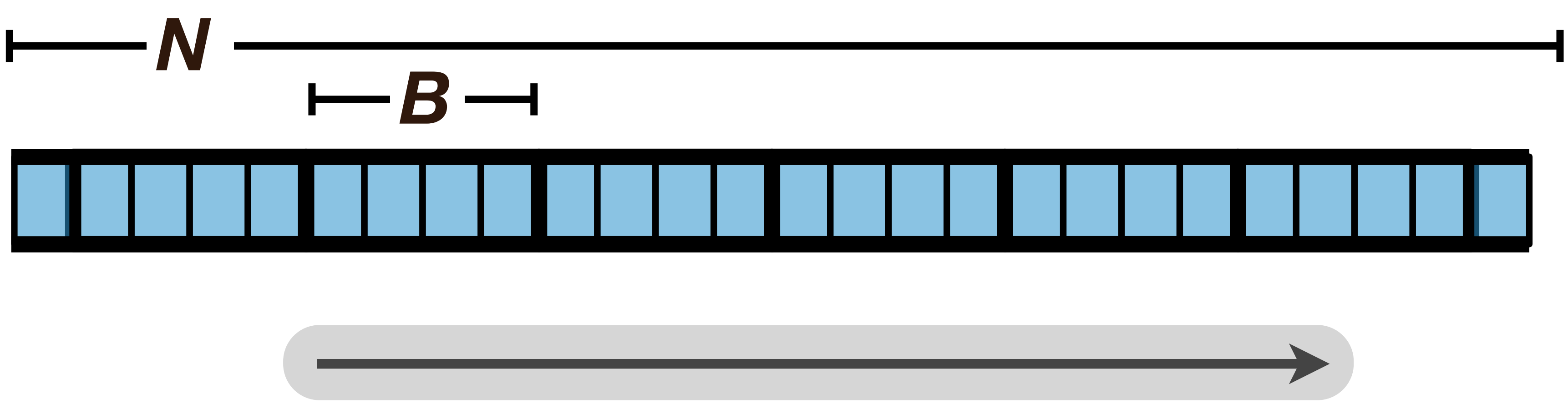

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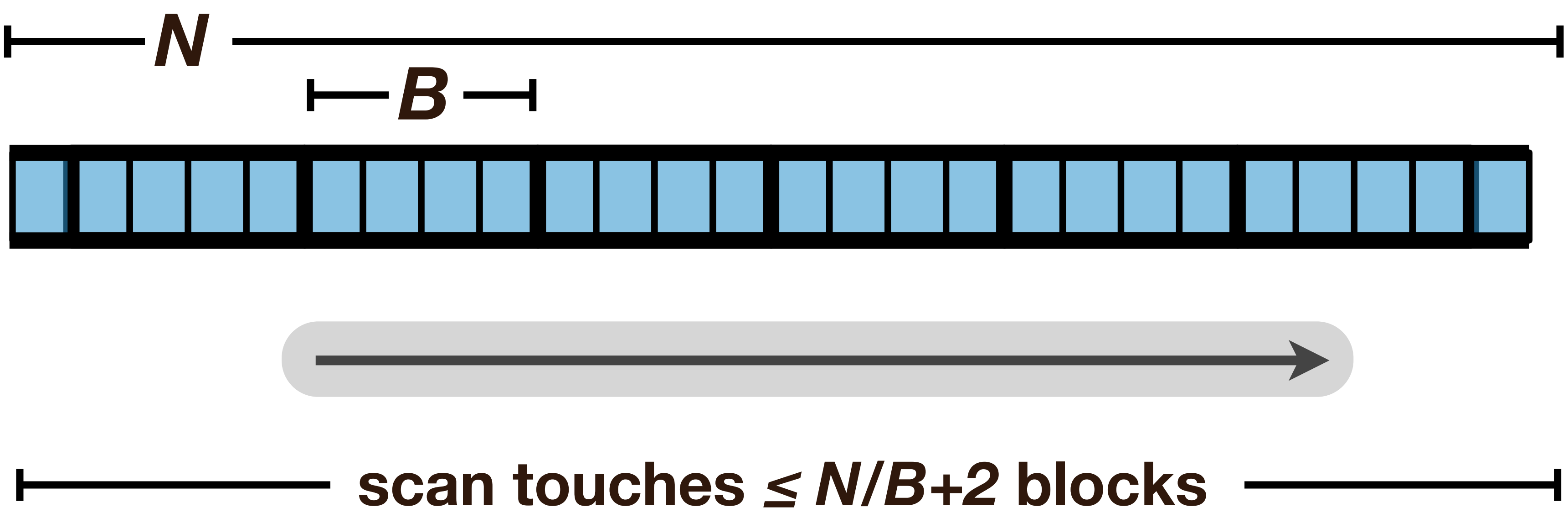

Example: Searching in a B-tree

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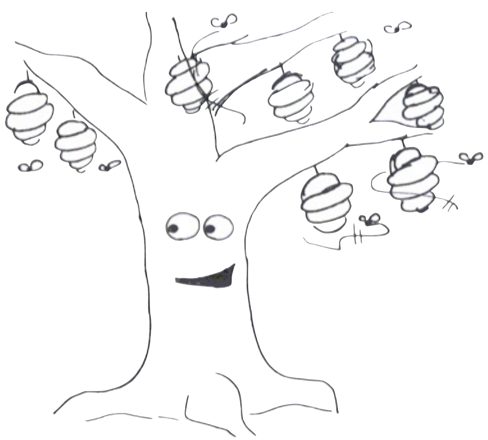

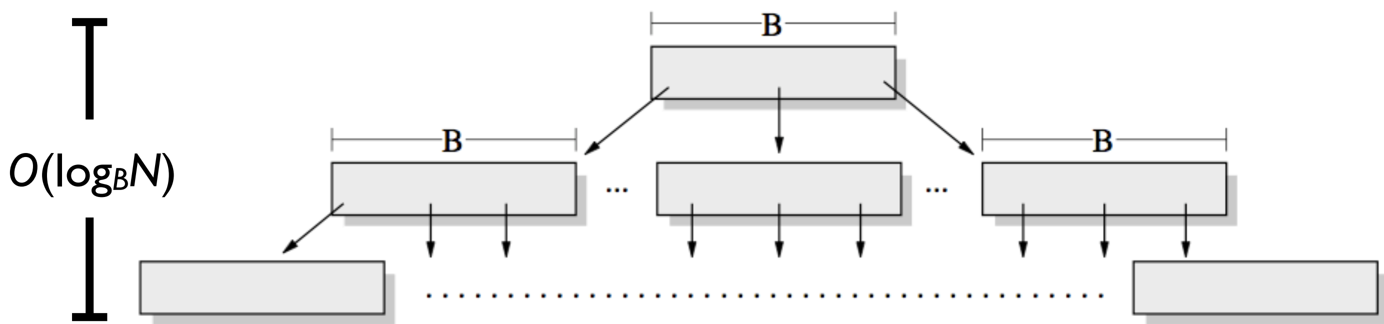

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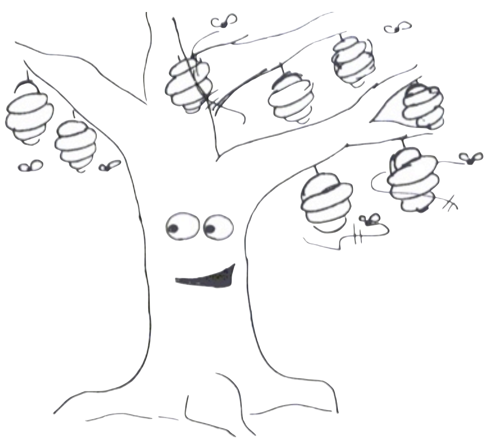
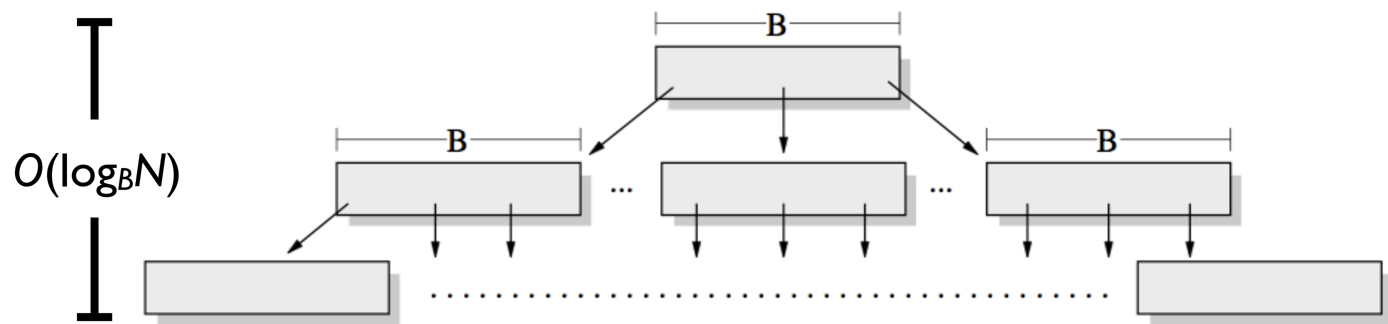
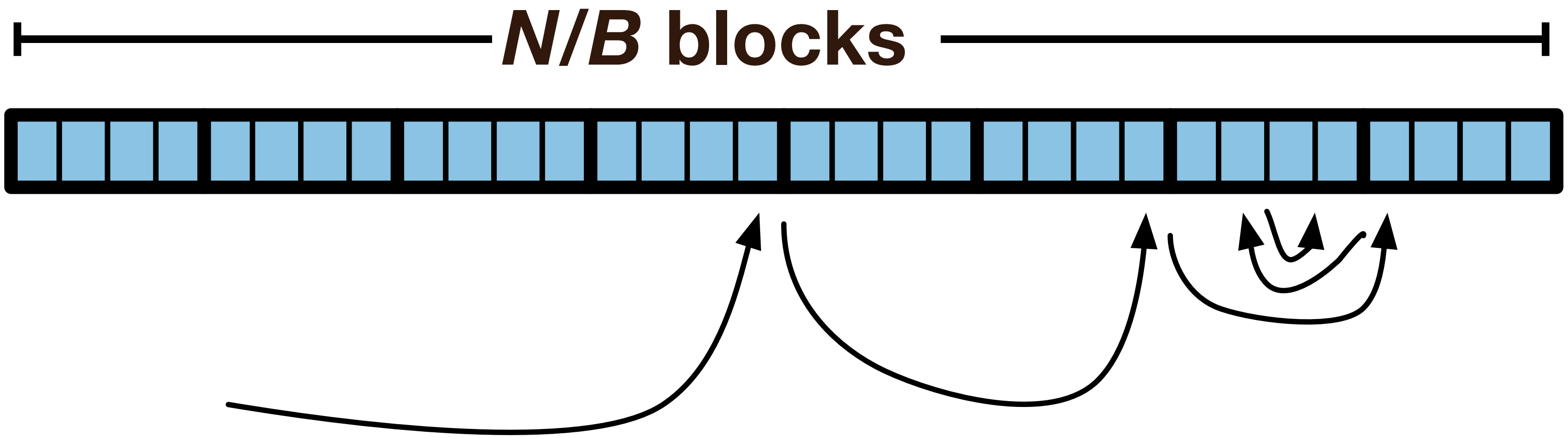
Answer: $O(\log_B N)$





Example: Searching in an Array

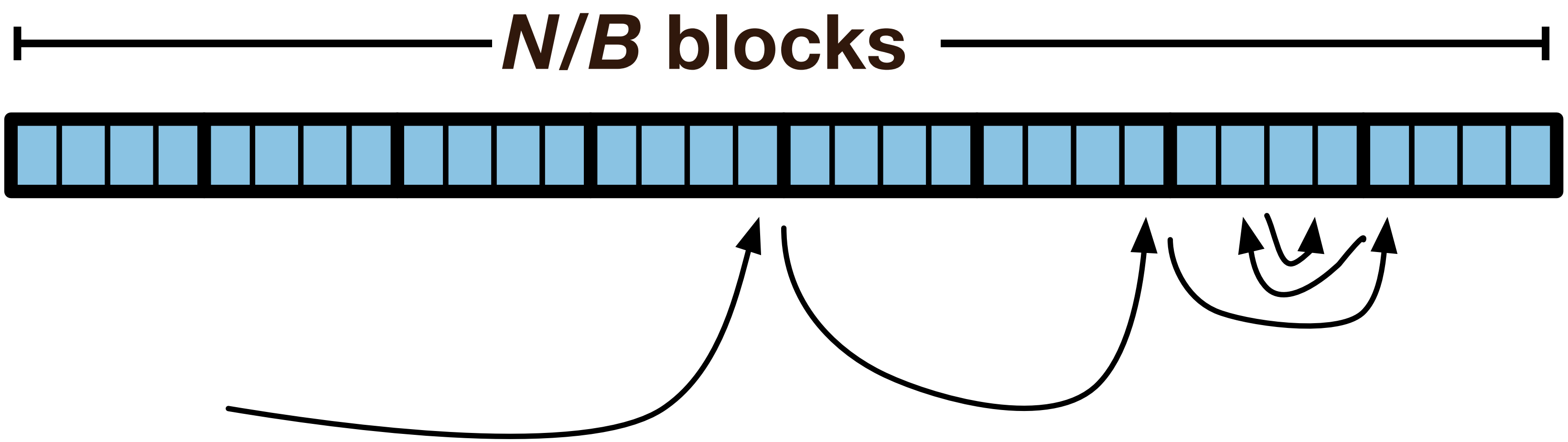
Question: How many I/Os to perform a binary search into an array of size *N*?



Example: Searching in an Array

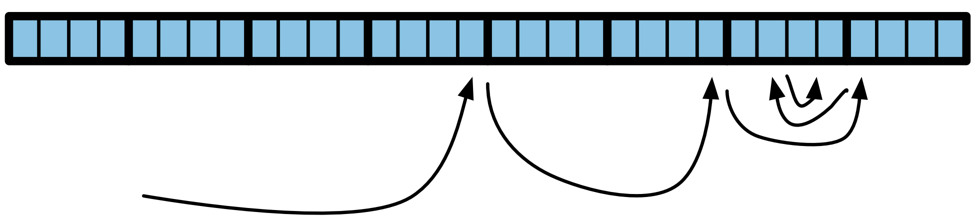
Question: How many I/Os to perform a binary search into an array of size *N*?

Answer: $$O\left(\log_2 \frac{N}{B}\right) \approx O(\log_2 N)$$

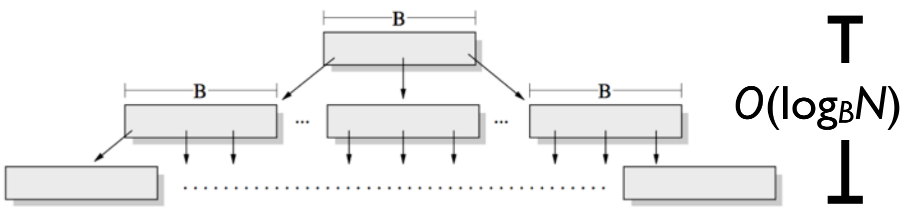


Example: Searching in an Array Versus B-tree

Moral: B-tree searching is a factor of O(log$_2$ *B*) faster than binary searching.



$$O(\log_2 N)$$



$$O(\log_B N) = O\left(\frac{\log_2 N}{\log_2 B}\right)$$

The DAM model is simple and pretty good

The Disk Access Machine (DAM) model

- ignores CPU costs and
- assumes that all block accesses have the same cost.

Is that a good performance model?

- Far from perfect.
- But very powerful nonetheless.
- (We’ll discuss more later in the tutorial.)

Data Structures and Algorithms for Big Data

Module 2: Write-Optimized Data Structures

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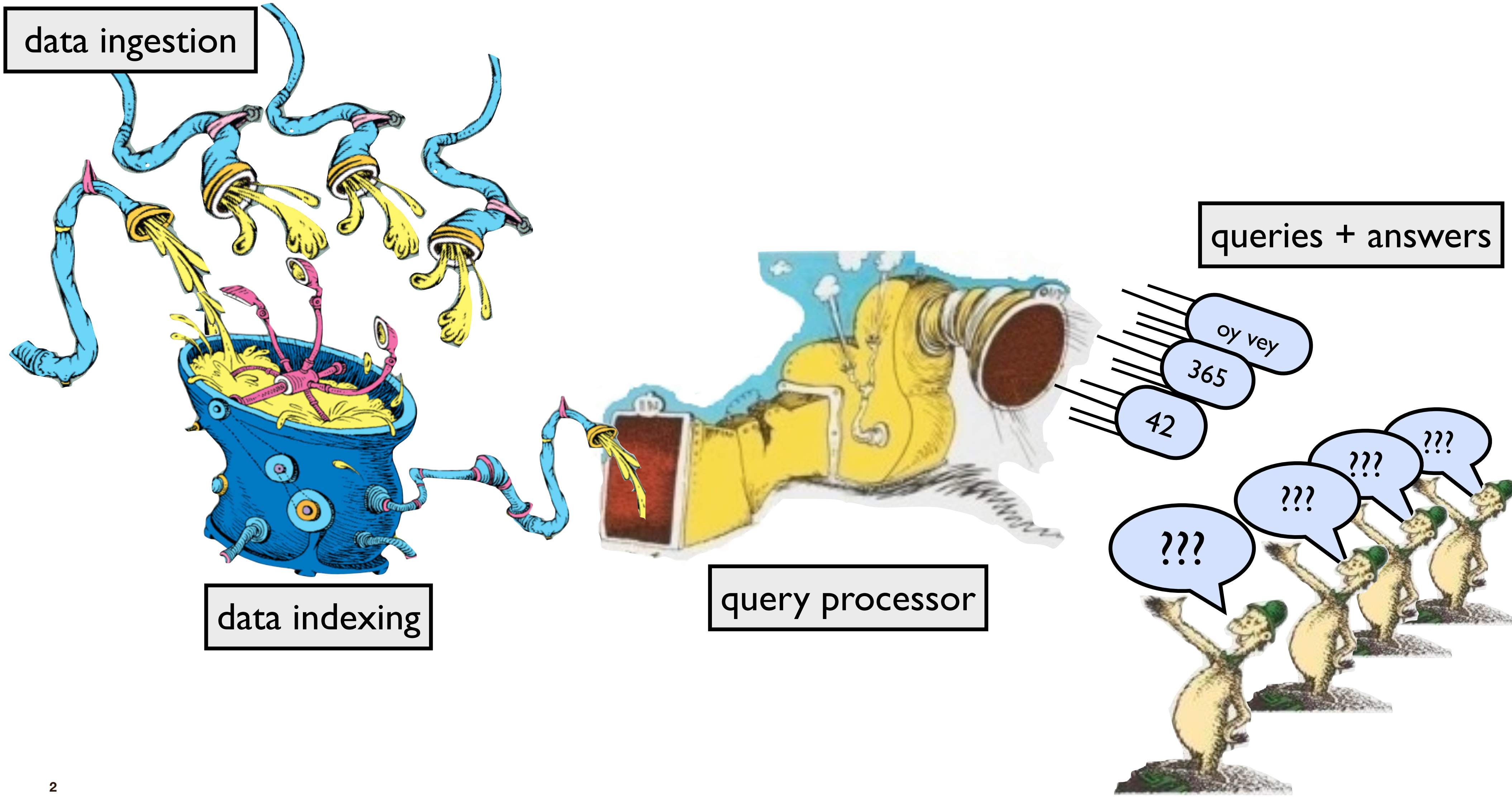
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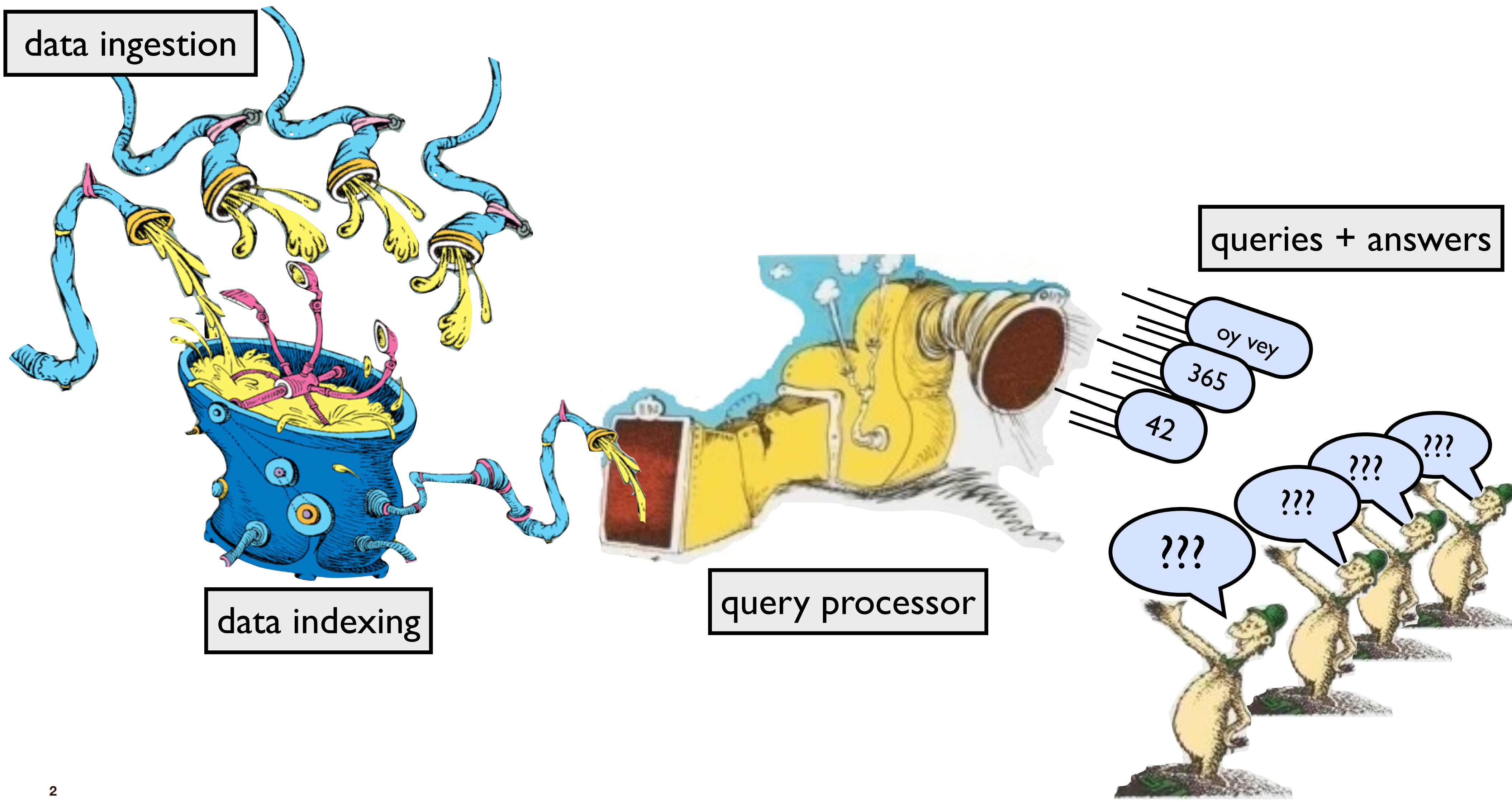

Funny tradeoff in ingestion, querying, freshness

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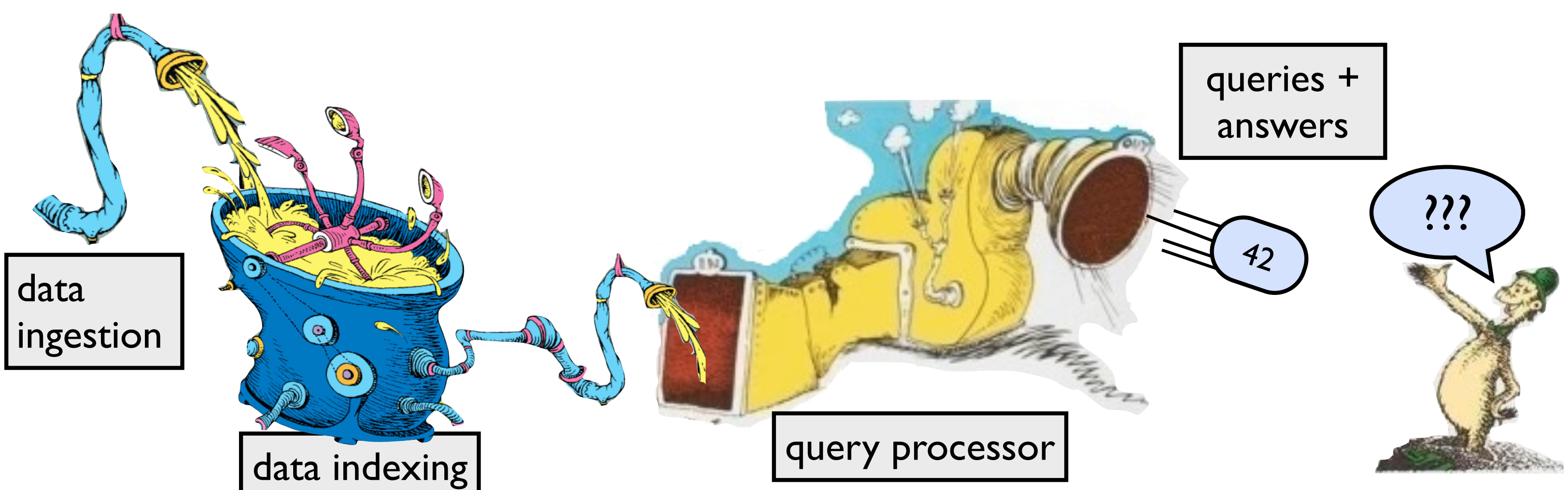

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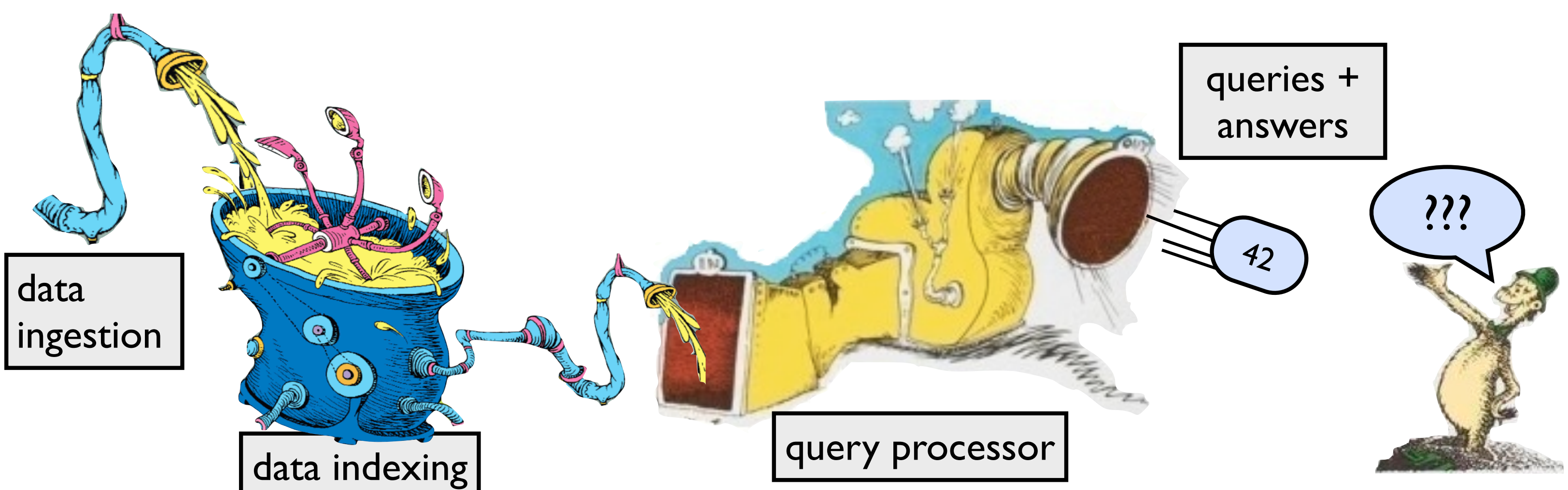

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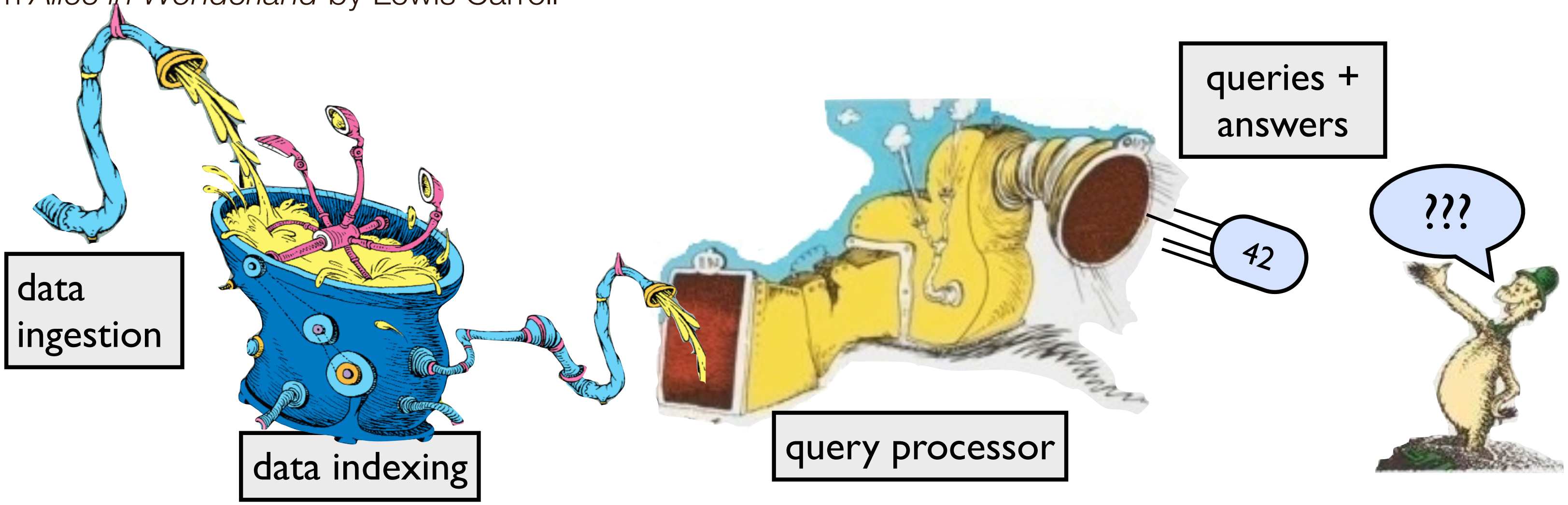
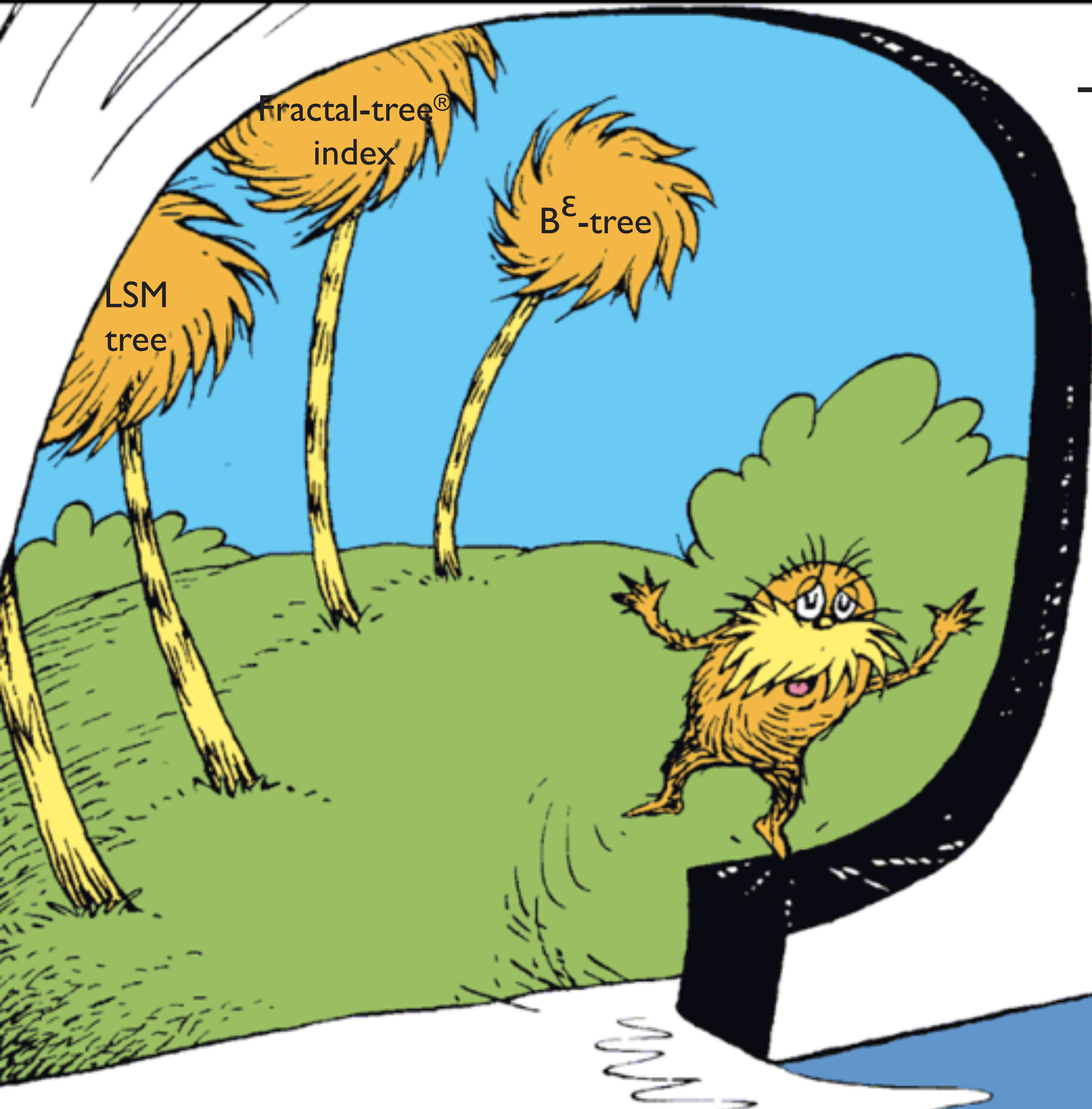




This module

- Write-optimized structures significantly mitigate the insert/query/freshness tradeoff.
- One can insert 10x-100x faster than B-trees while achieving similar point query performance.

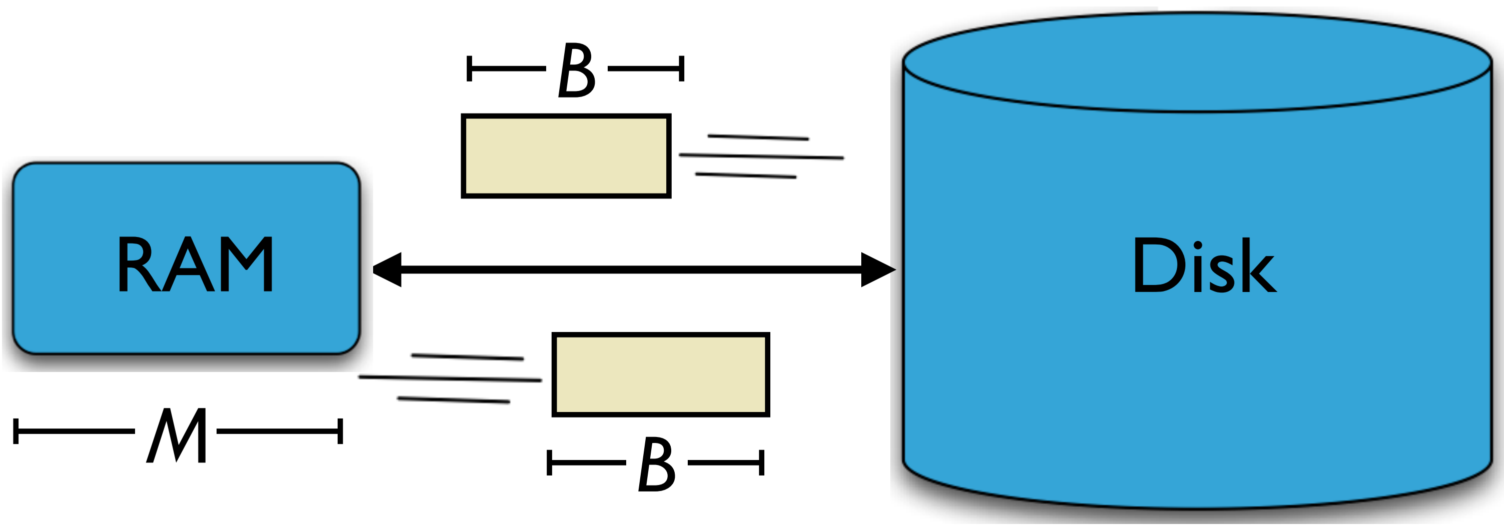
An algorithmic performance model

How computation works:

- Data is transferred in blocks between RAM and disk.
- The number of block transfers dominates the running time.

Goal: Minimize # of block transfers

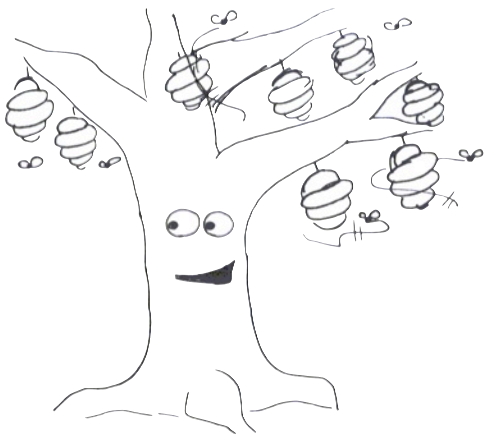
- Performance bounds are parameterized by block size ***B***, memory size ***M***, data size ***N***.

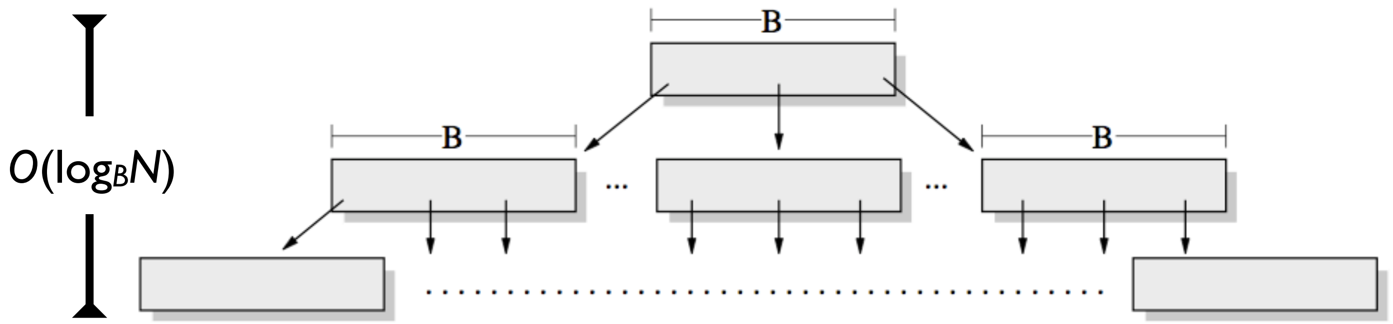


[Aggarwal+Vitter '88]

An algorithmic performance model

B-tree point queries: $O(\log_B N)$ I/Os.





Write-optimized data structures performance

Data structures: [O'Neil,Cheng, Gawlick, O'Neil 96], [Buchsbaum, Goldwasser, Venkatasubramanian, Westbrook 00], [Argel 03], [Graefe 03], [Brodal, Fagerberg 03], [Bender, Farach,Fineman,Fogel, Kuszmaul, Nelson'07], [Brodal, Demaine, Fineman, Iacono, Langerman, Munro 10], [Spillane, Shetty, Zadok, Archak, Dixit 11].
Systems: BigTable, Cassandra, H-Base, LevelDB, TokuDB.

	B-tree	Some write-optimized structures
Insert/delete	$O(\log_B N)=O\left(\frac{\log N}{\log B}\right)$	$O\left(\frac{\log N}{B}\right)$

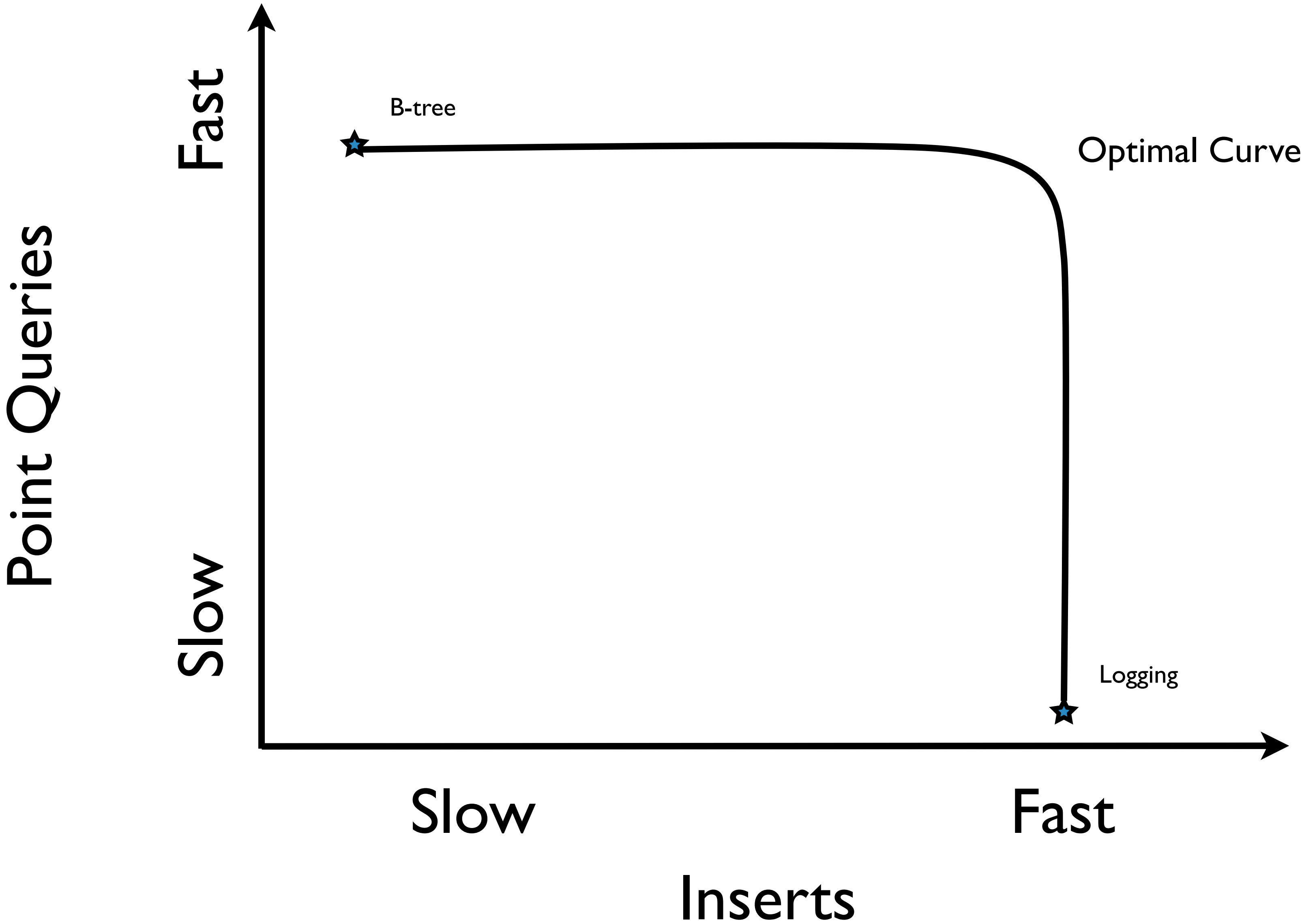
- If B=1024, then insert speedup is $B/\log B \approx 100$.
- Hardware trends mean bigger B, bigger speedup.
- Less than 1 I/O per insert.

Optimal Search-Insert Tradeoff [Brodal, Fagerberg 03]

	insert	**point query**
Optimal tradeoff (function of ε=0...1)	$O\left(\frac{\log_{1+B^\varepsilon} N}{B^{1-\varepsilon}}\right)$	$O\left(\log_{1+B^\varepsilon} N\right)$
B-tree (ε=1)	$O\left(\log_B N\right)$	$O\left(\log_B N\right)$
ε=1/2	$O\left(\frac{\log_B N}{\sqrt{B}}\right)$	$O\left(\log_B N\right)$
ε=0	$O\left(\frac{\log N}{B}\right)$	$O\left(\log N\right)$

10x-100x faster inserts

Illustration of Optimal Tradeoff [Brodal, Fagerberg 03]



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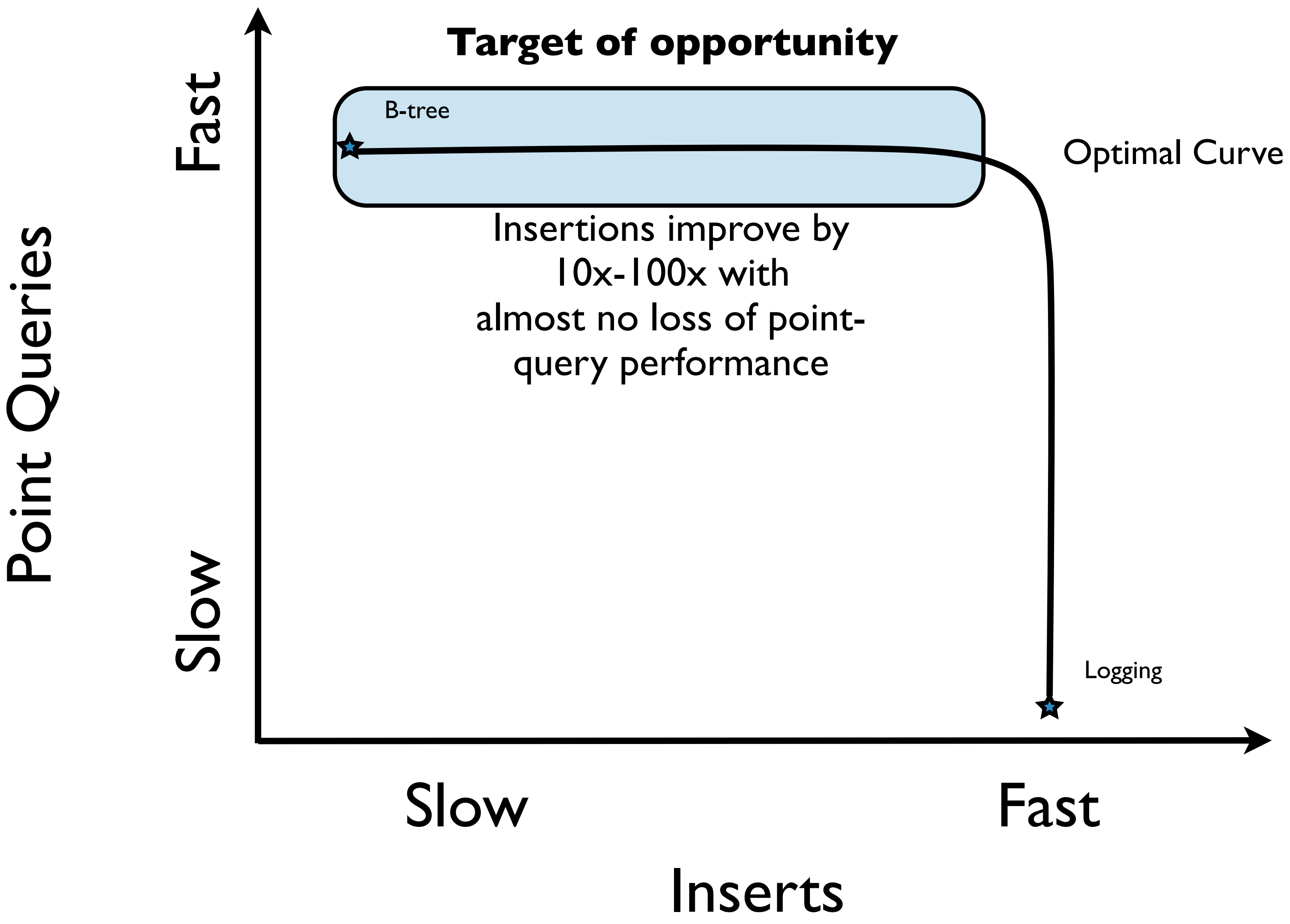

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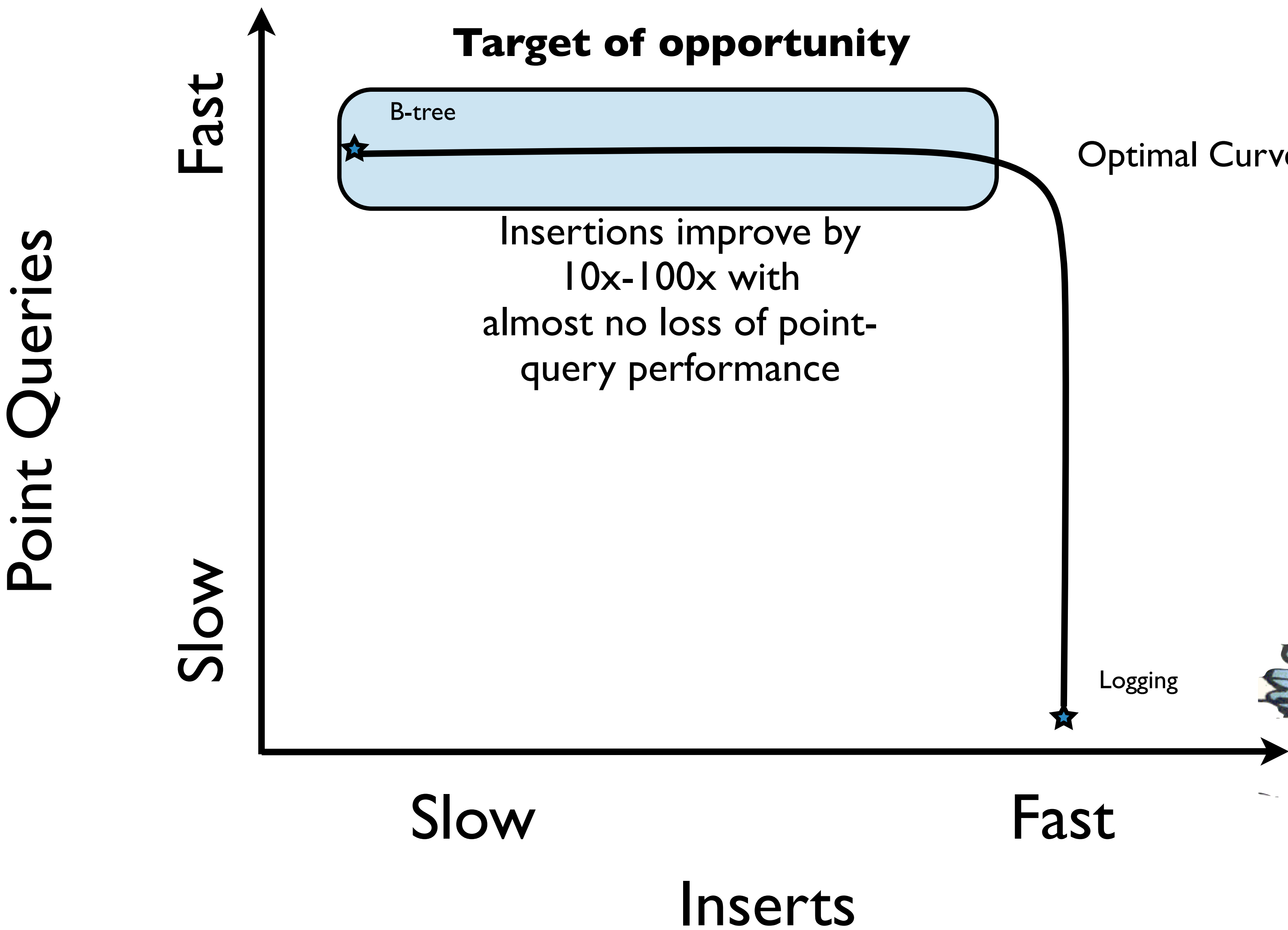


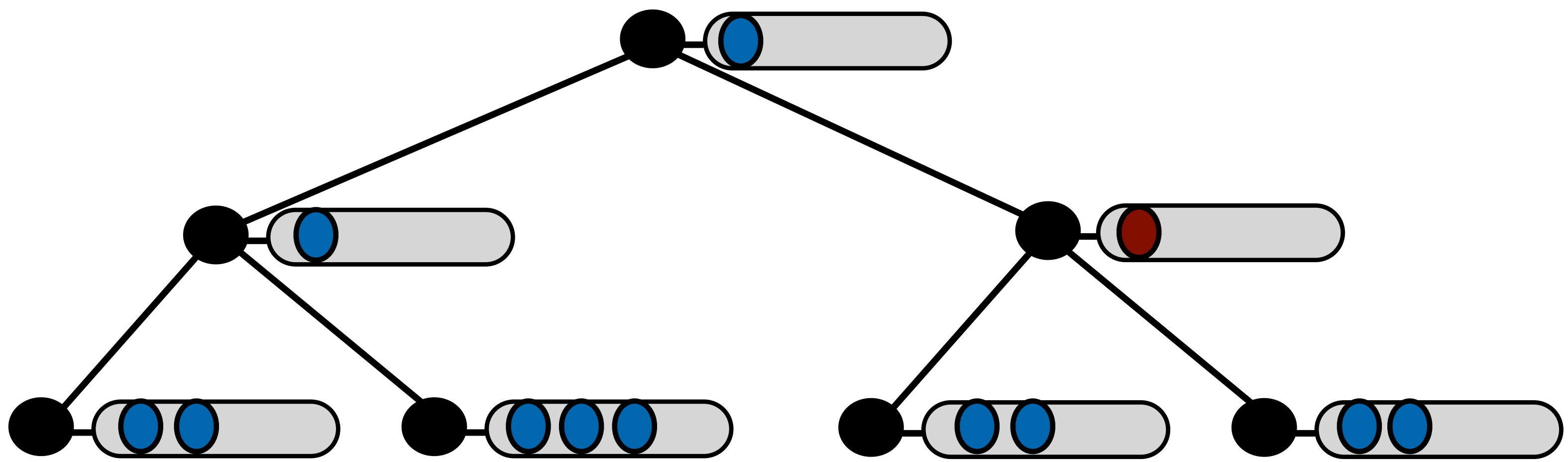
One way to Build Write-Optimized Structures

(other approaches later in tutorial)

A simple write-optimized structure

O(log *N*) queries and O((log *N*)/*B*) inserts:

- A balanced binary tree with buffers of size *B*



Inserts + deletes:

- Send insert/delete messages down from the root and store them in buffers.
- When a buffer fills up, flush.

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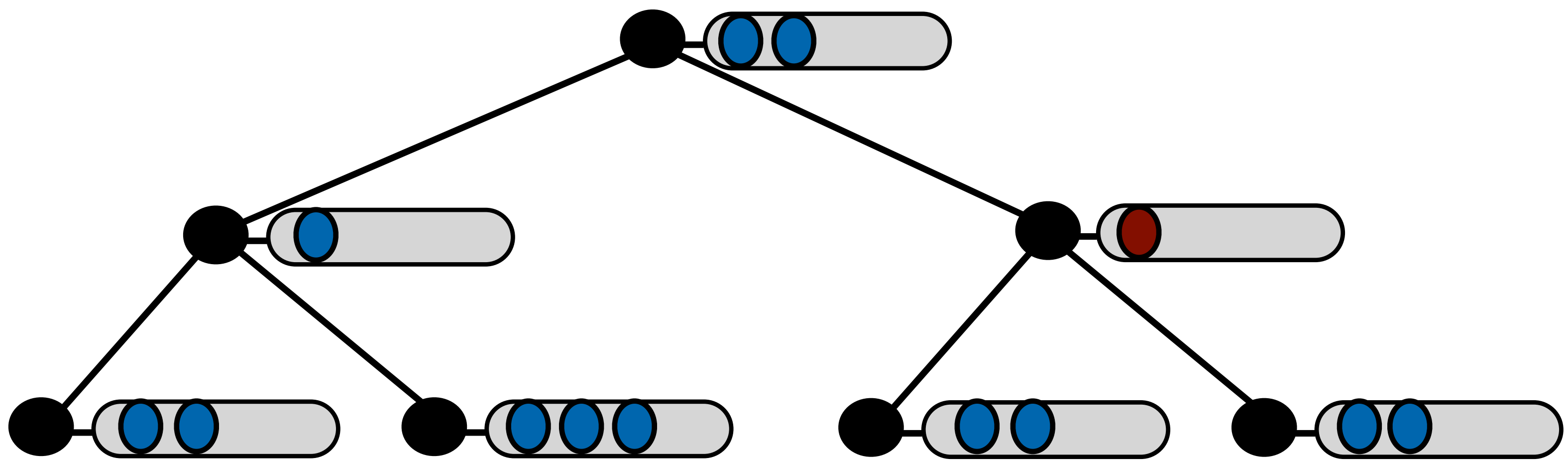

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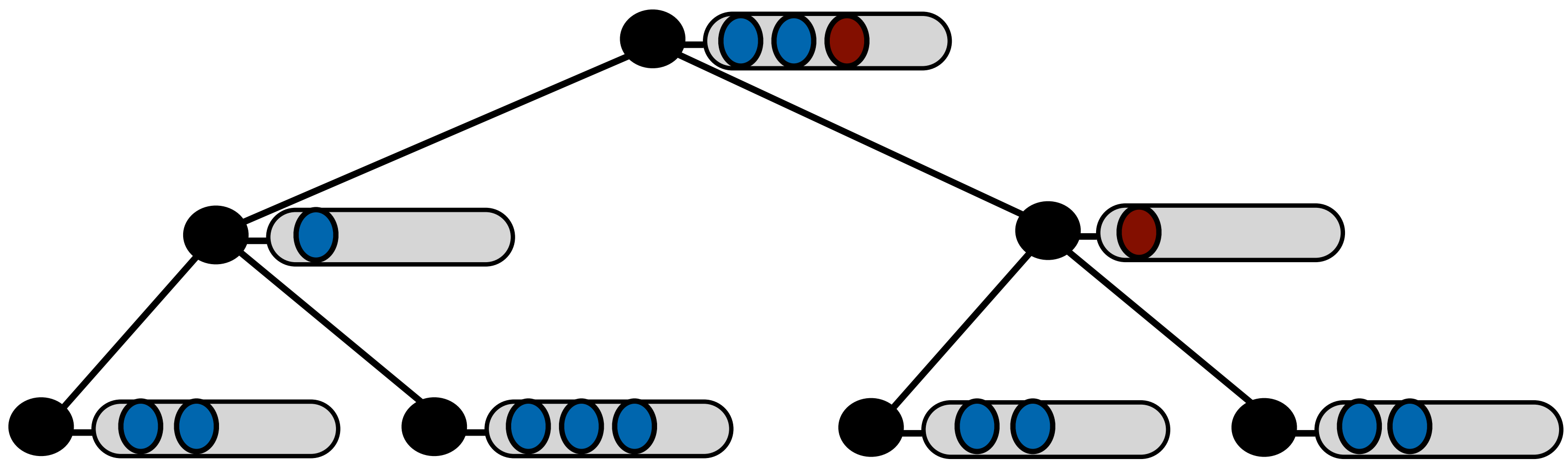

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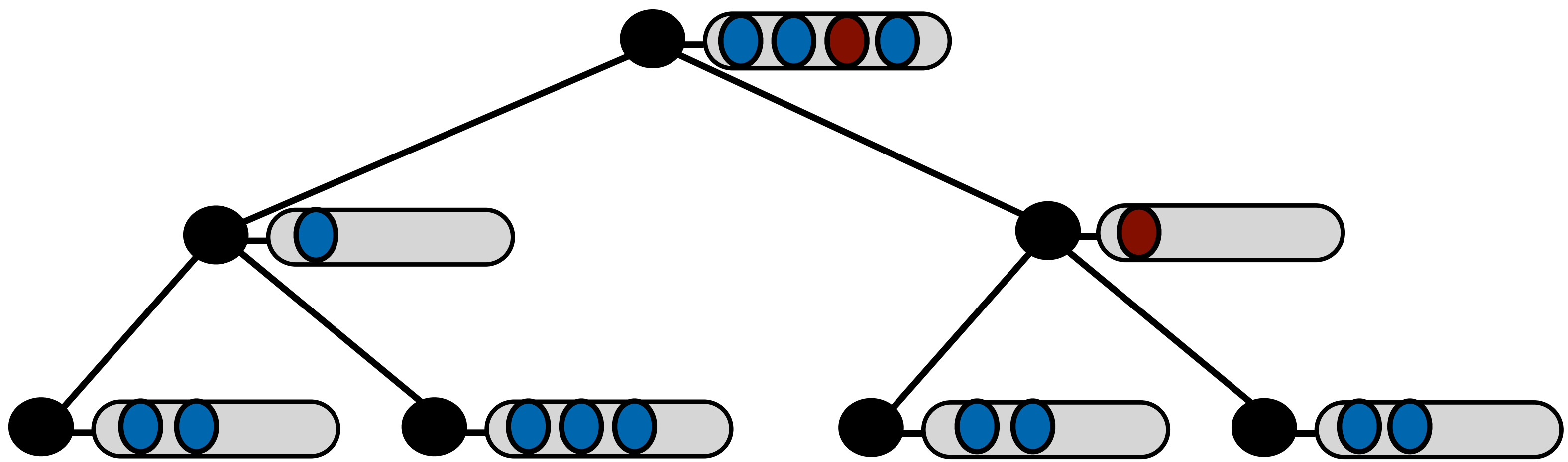

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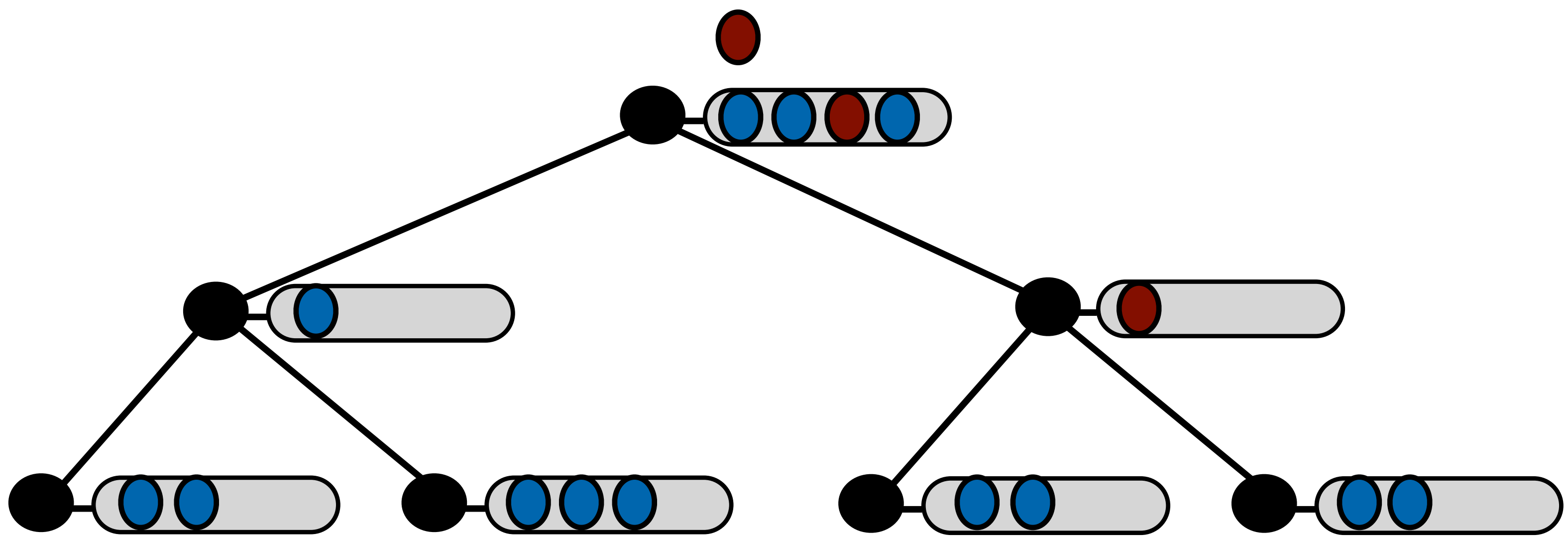

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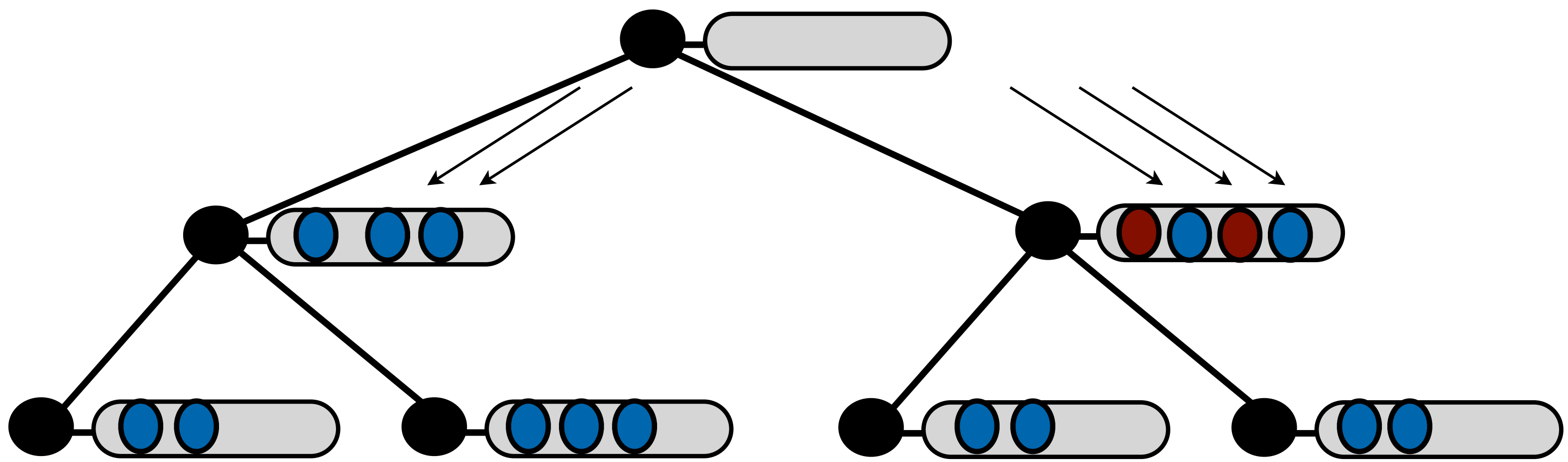

Inserts + deletes:

- Send insert/delete messages down from the root and store them in buffers.
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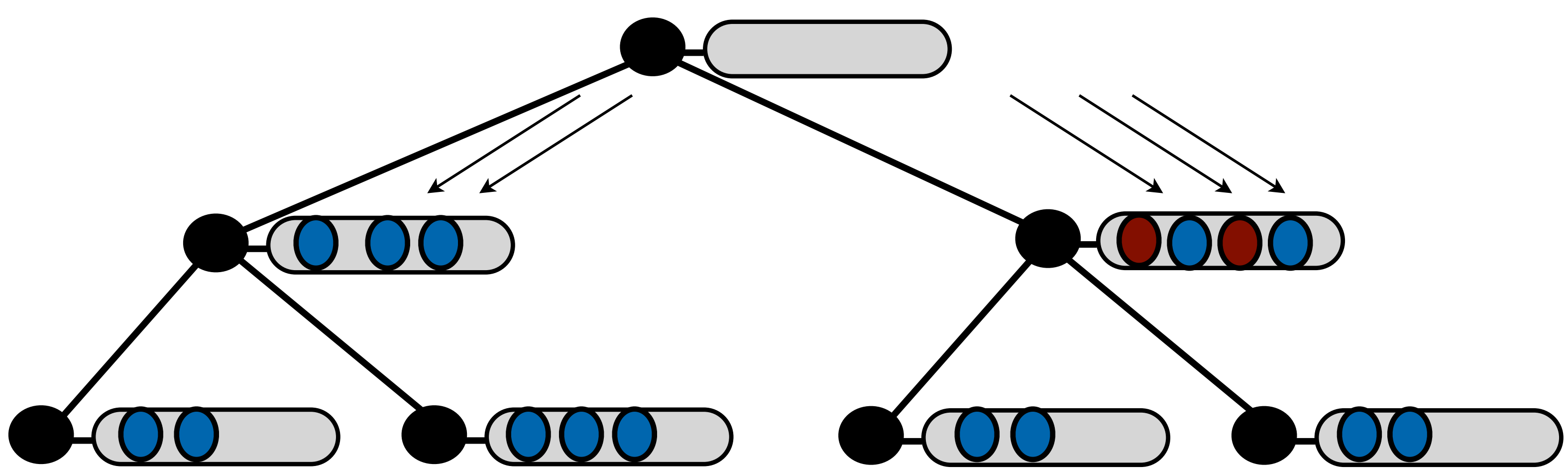
Analysis of writes

An insert/delete costs amortized O((log *N*)/B) per insert or delete

- A buffer flush costs O(1) & sends *B* elements down one level.
- It costs O(1/*B*) to send element down one level of the tree.
- There are O(log *N*) levels in a tree.



Difficulty of Key Accesses

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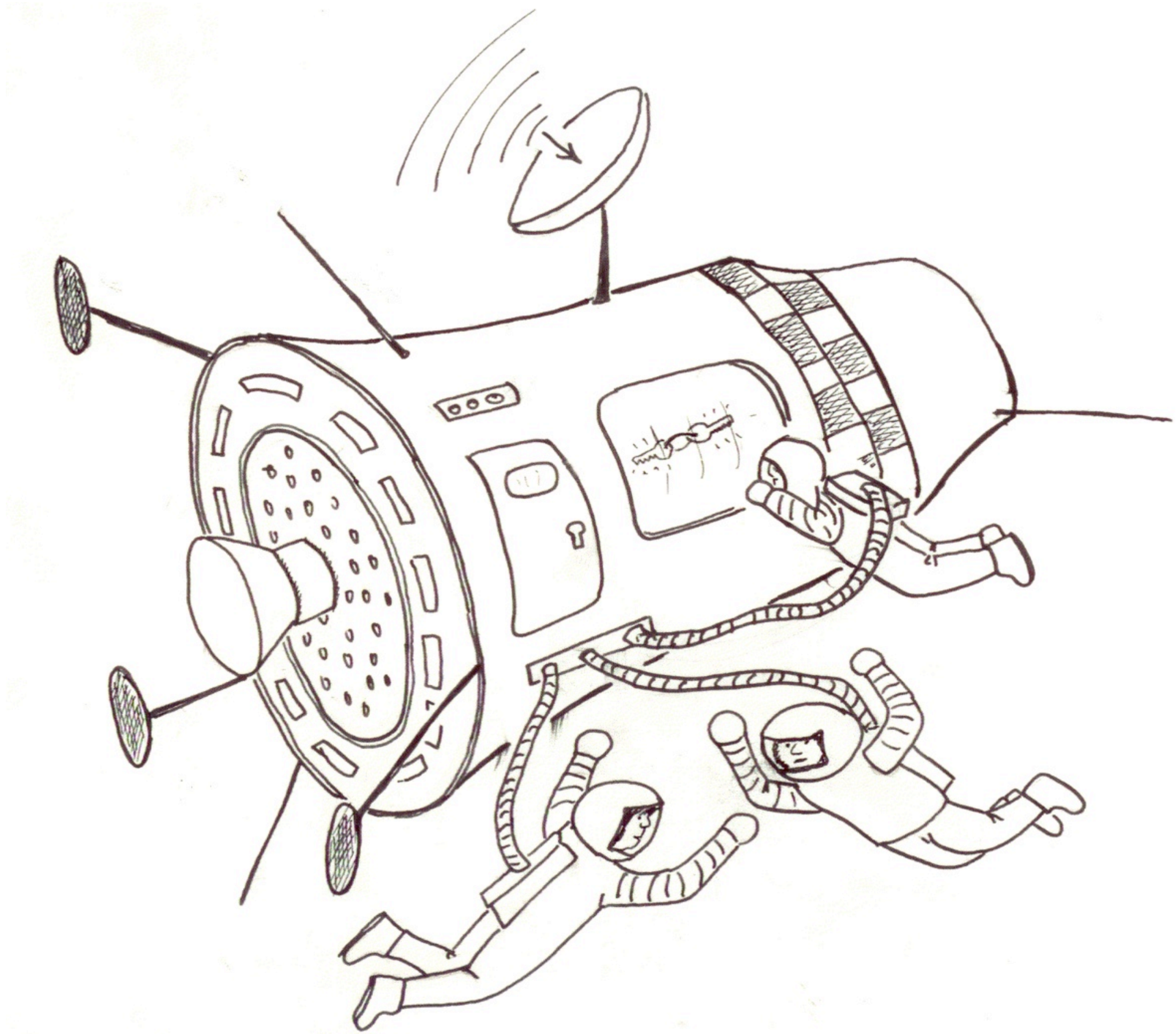
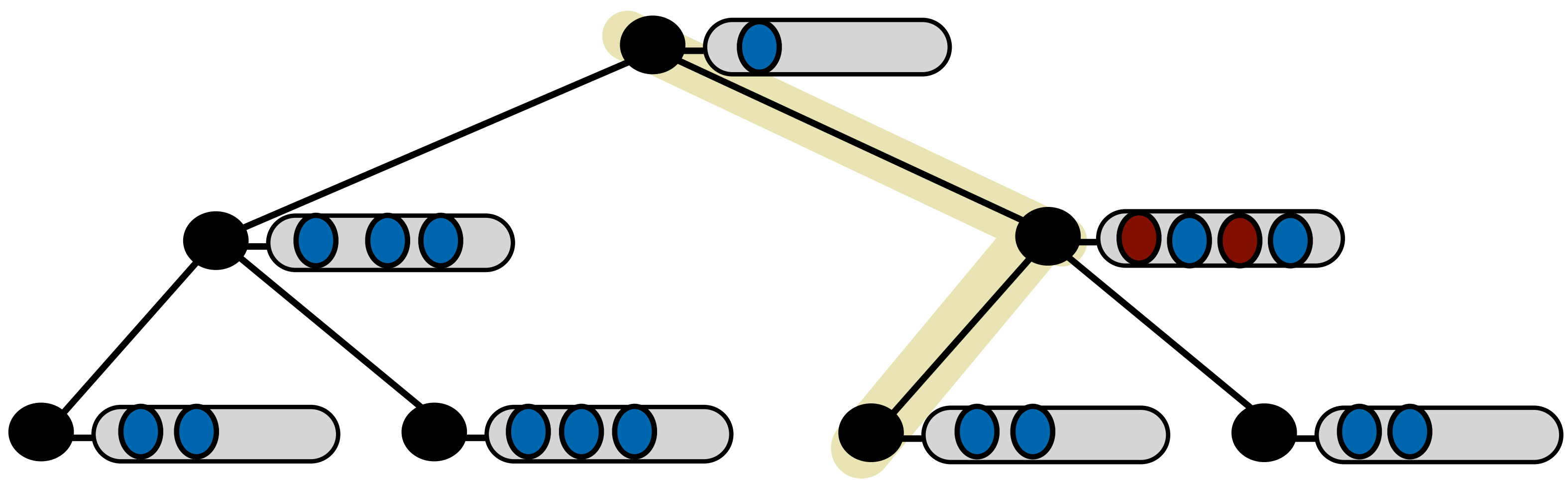


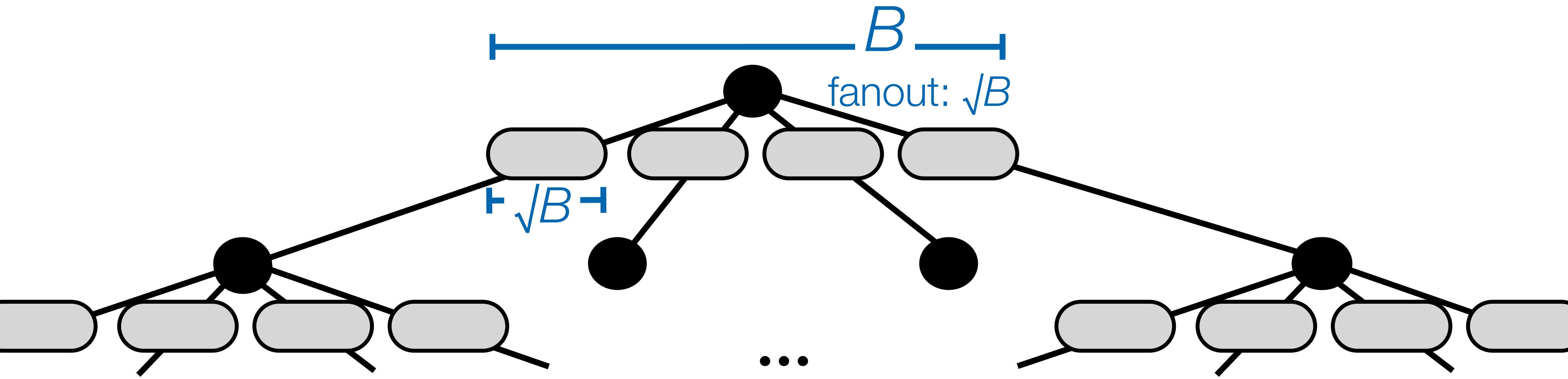
Analysis of point queries

To search:

- examine each buffer along a single root-to-leaf path.
- This costs O(log *N*).



Obtaining optimal point queries + very fast inserts



Point queries cost $O(\log_{\sqrt{B}} N) = O(\log_B N)$

- This is the tree height.

Inserts cost $O((\log_B N)/\sqrt{B})$

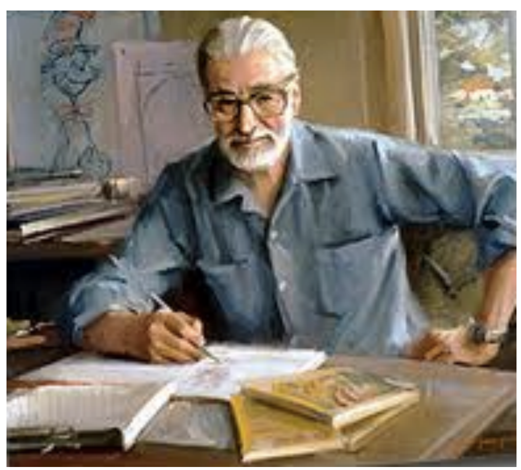
- Each flush cost O(1) I/Os and flushes $\sqrt{B}$ elements.

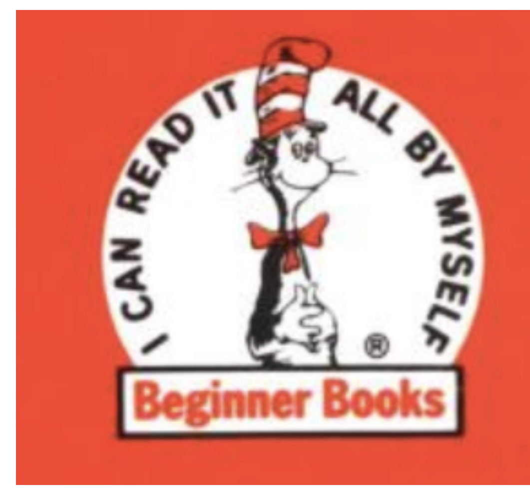
What the world looks like

Insert/point query asymmetry

- Inserts can be fast: >50K high-entropy writes/sec/disk.
- Point queries are necessarily slow: <200 high-entropy reads/sec/disk.

We are used to reads and writes having about the same cost, but writing is easier than reading.

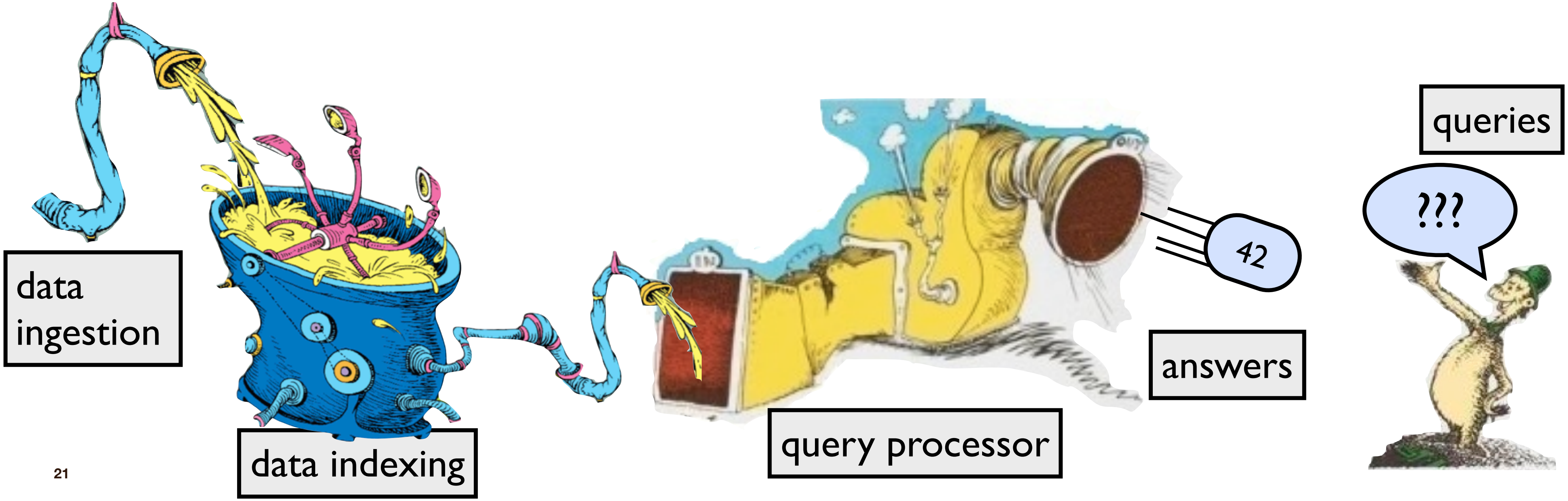




The right read-optimization is write-optimization

The right index makes queries run fast.

- Write-optimized structures maintain indexes efficiently.

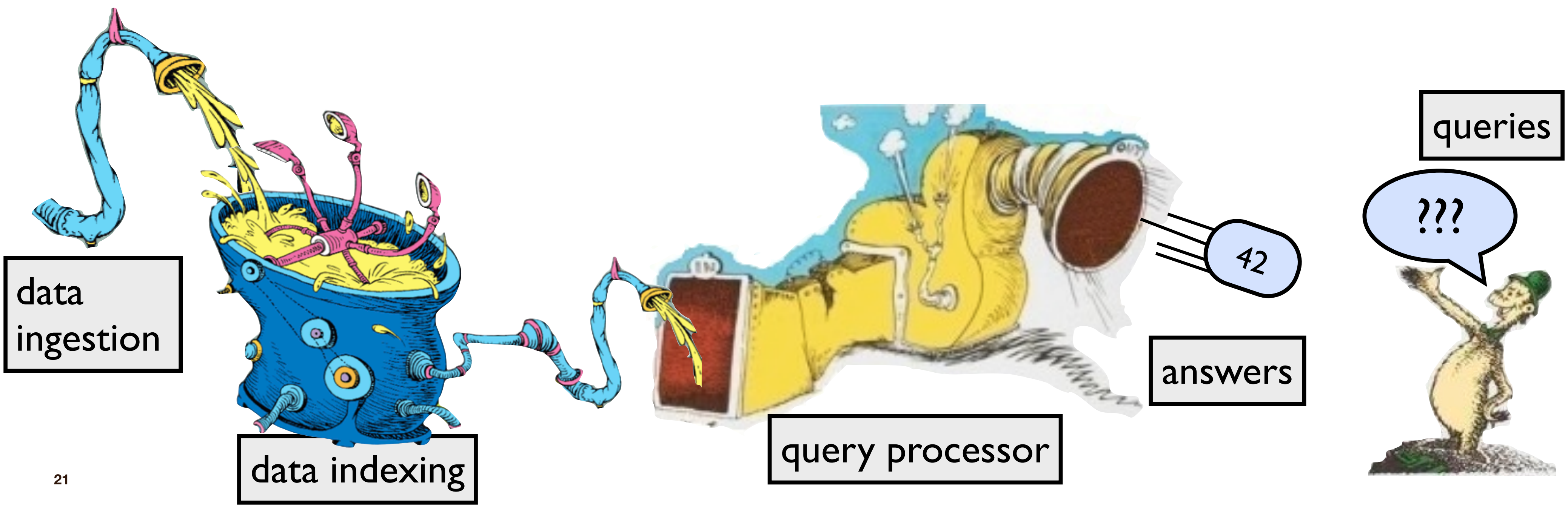


The right read-optimization is write-optimization

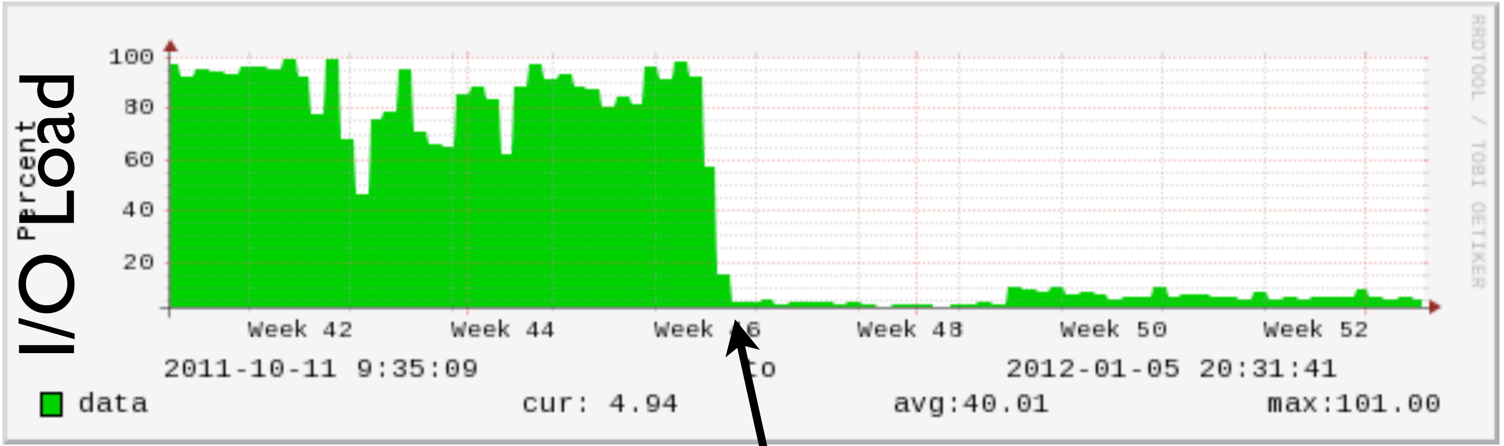
The right index makes queries run fast.

- Write-optimized structures maintain indexes efficiently.

Fast writing is a currency we use to accelerate queries. Better indexing means faster queries.



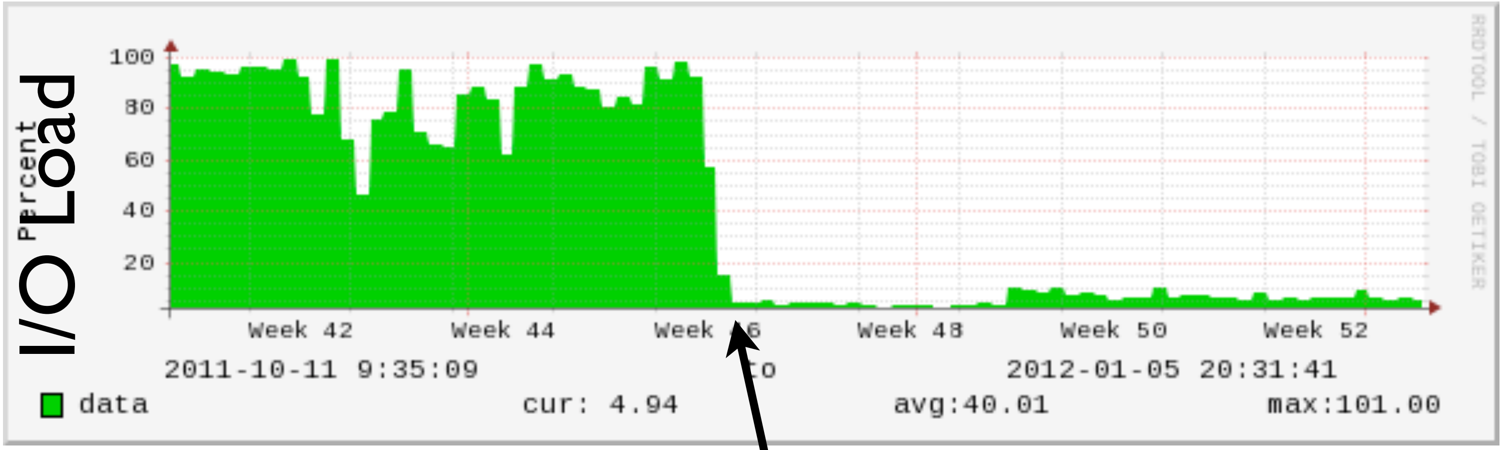
The right read-optimization is write-optimization



Add selective indexes.

(We can now afford to maintain them.)

The right read-optimization is write-optimization



Add selective indexes.

(We can now afford to maintain them.)

Write-optimized structures can significantly mitigate the insert/query/freshness tradeoff.

Implementation Issues

Write optimization. ✔ What's missing?

Optimal read-write tradeoff: Easy

Full featured: Hard

- Variable-sized rows
- Concurrency-control mechanisms
- Multithreading
- Transactions, logging, ACID-compliant crash recovery
- Optimizations for the special cases of sequential inserts and bulk loads
- Compression
- Backup

Systems often assume search cost = insert cost

Some inserts/deletes have hidden searches.

Example:

- return error when a duplicate key is inserted.
- return # elements removed on a delete.

These "cryptosearches" throttle insertions down to the performance of B-trees.

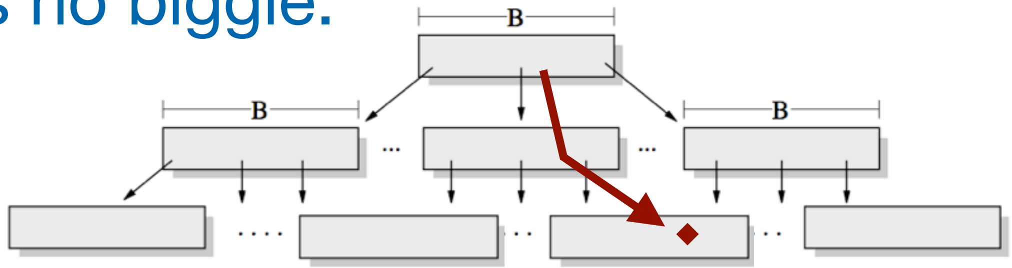
Cryptosearches in uniqueness checking

Uniqueness checking has a hidden search:

```
If Search(key) == True
    Return Error;
Else
    Fast_Insert(key,value);
```

In a B-tree uniqueness checking comes for free

- On insert, you fetch a leaf.
- Checking if key exists is no biggie.



Cryptosearches in uniqueness checking

Uniqueness checking has a hidden search:

```
If Search(key) == True
   Return Error;
Else
   Fast_Insert(key,value);
```

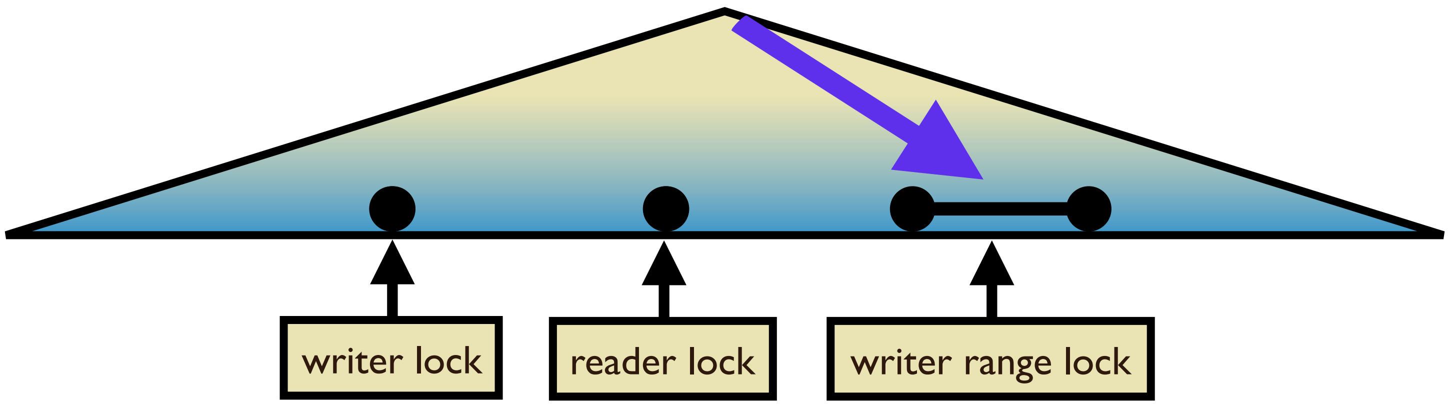
In a write-optimized structure, that crypto-search can throttle performance

- Insertion messages are injected.
- These eventually get to “bottom” of structure.
- Insertion w/Uniqueness Checking 100x slower.
- Bloom filters, Cascade Filters, etc help.

[Bender, Farach-Colton, Johnson, Kraner, Kuszmaul, Medjedovic, Montes, Shetty, Spillane, Zadok 12]

A locking scheme with cryptosearches

One implementation of pessimistic locking:
maintain locks in leaves



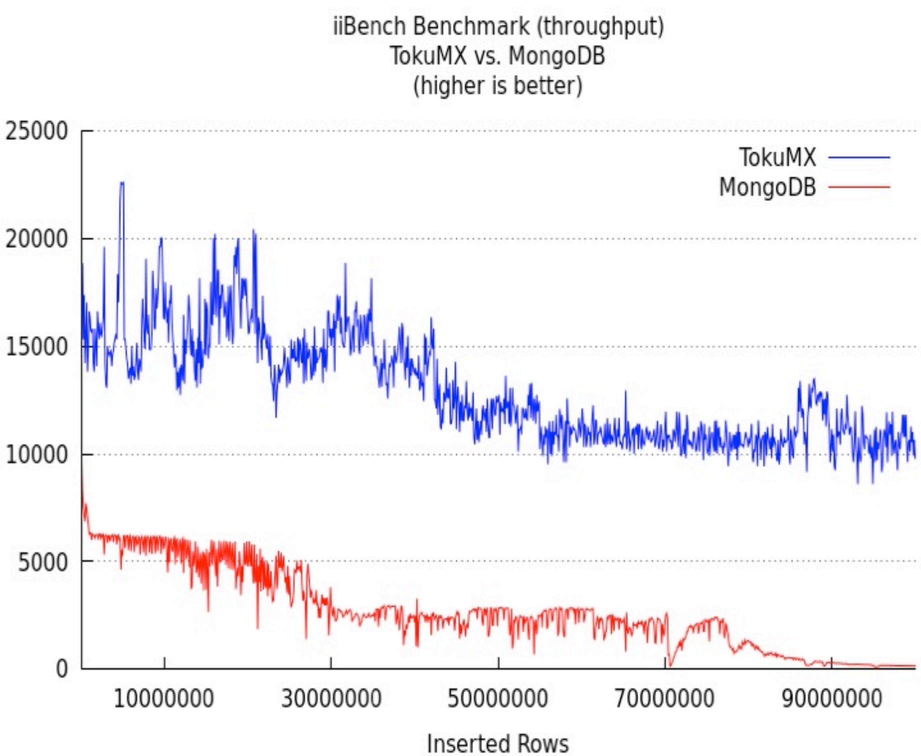
To insert a new row *v*, determine whether there is already a lock on *v* at a leaf.

This is also a cryptosearch.

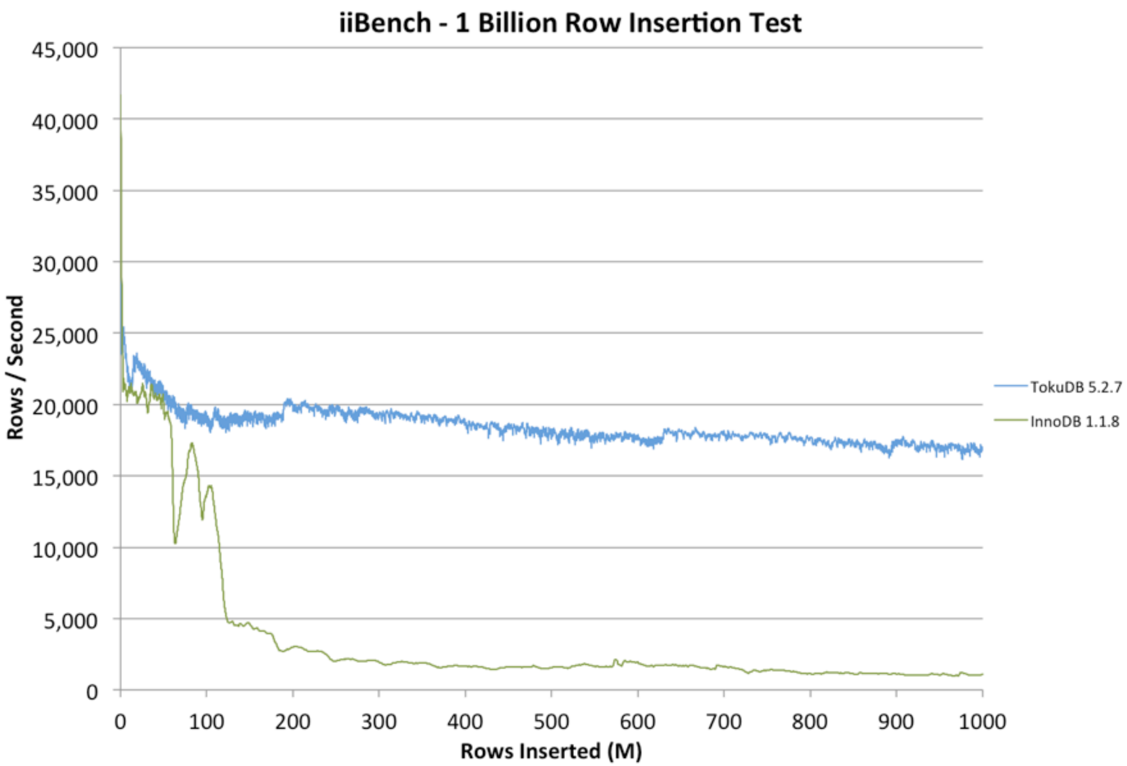
Performance

iiBench Insertion Benchmark

Write performance on large data

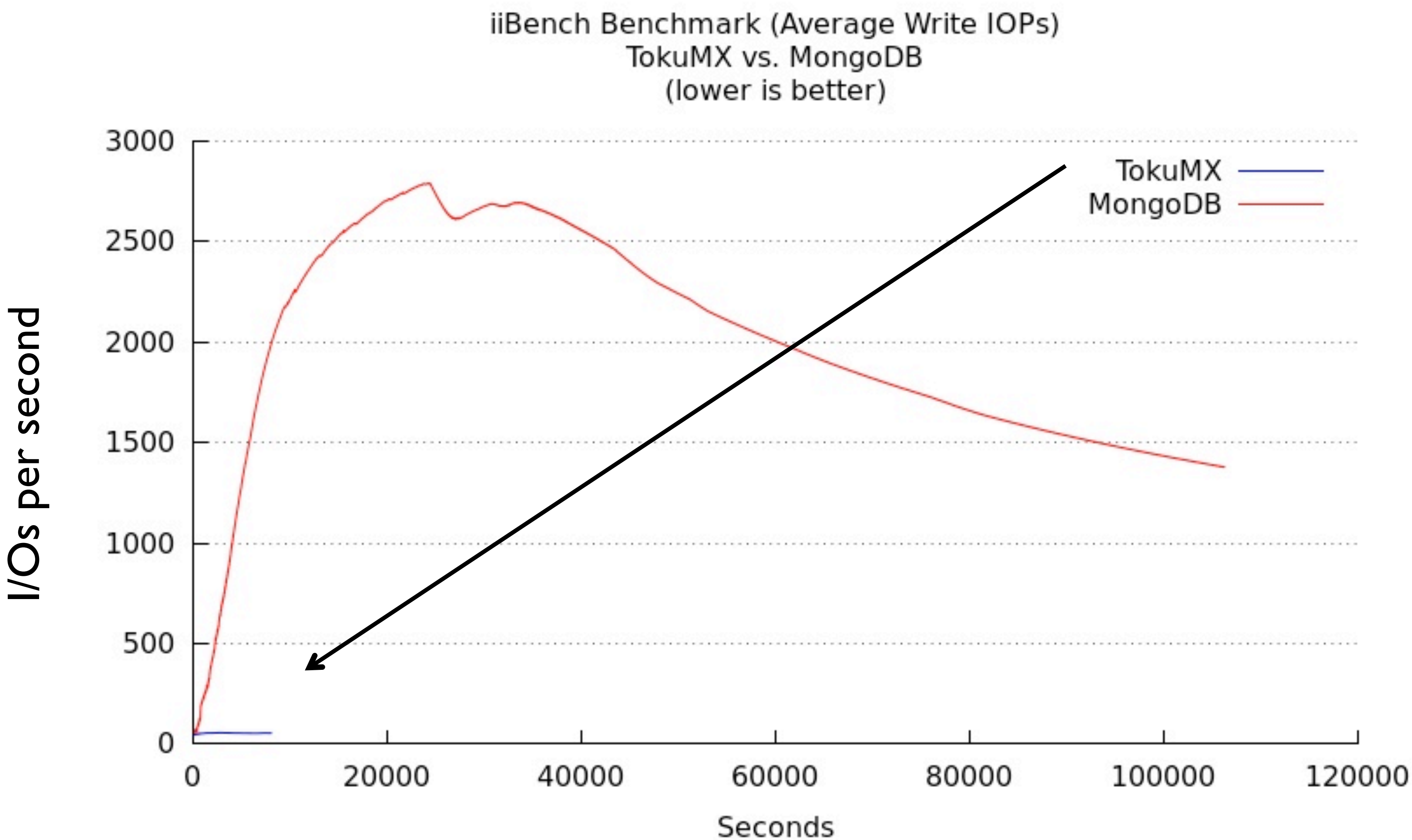


MongoDB



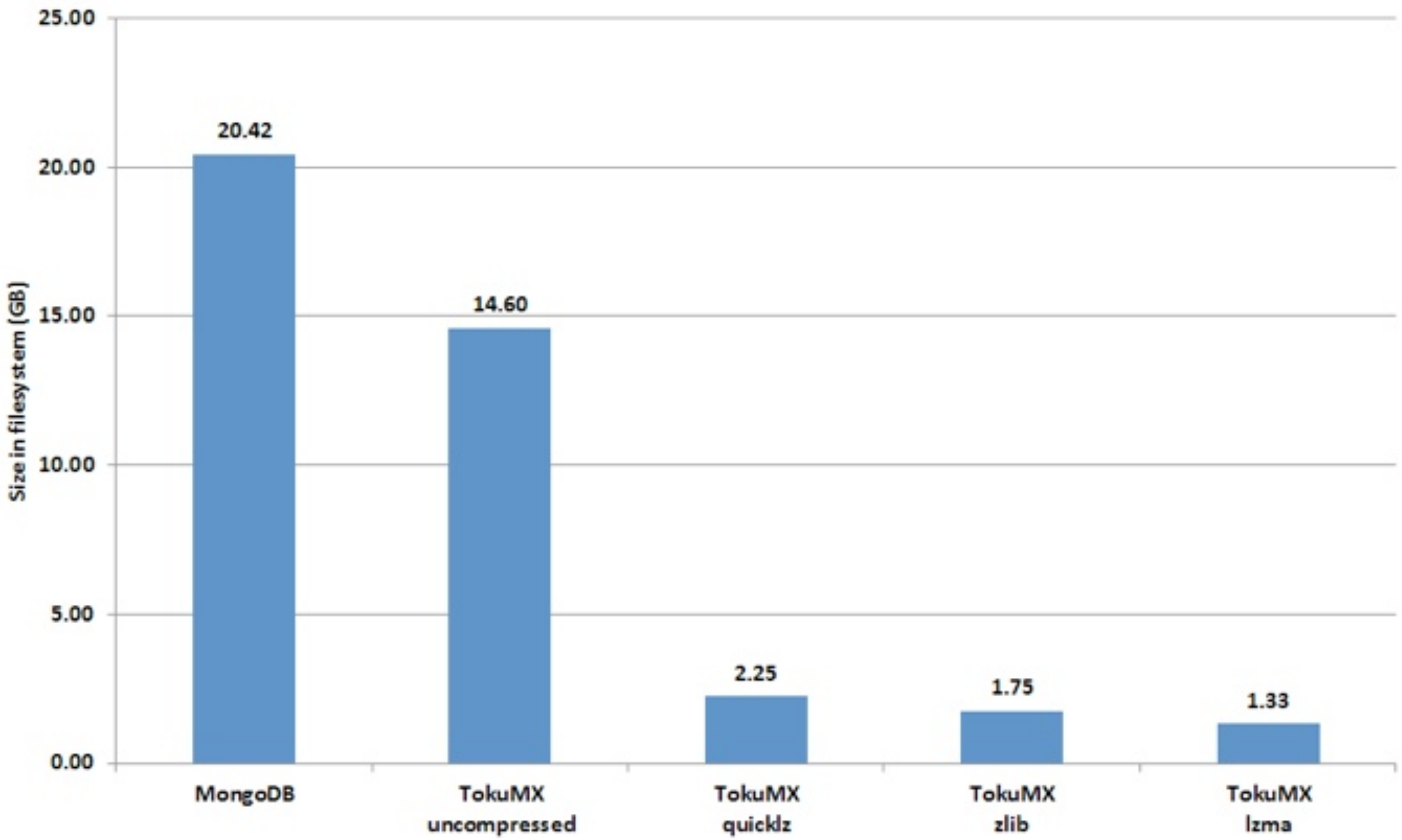
MySQL

TokuMX runs fast because it uses less I/O

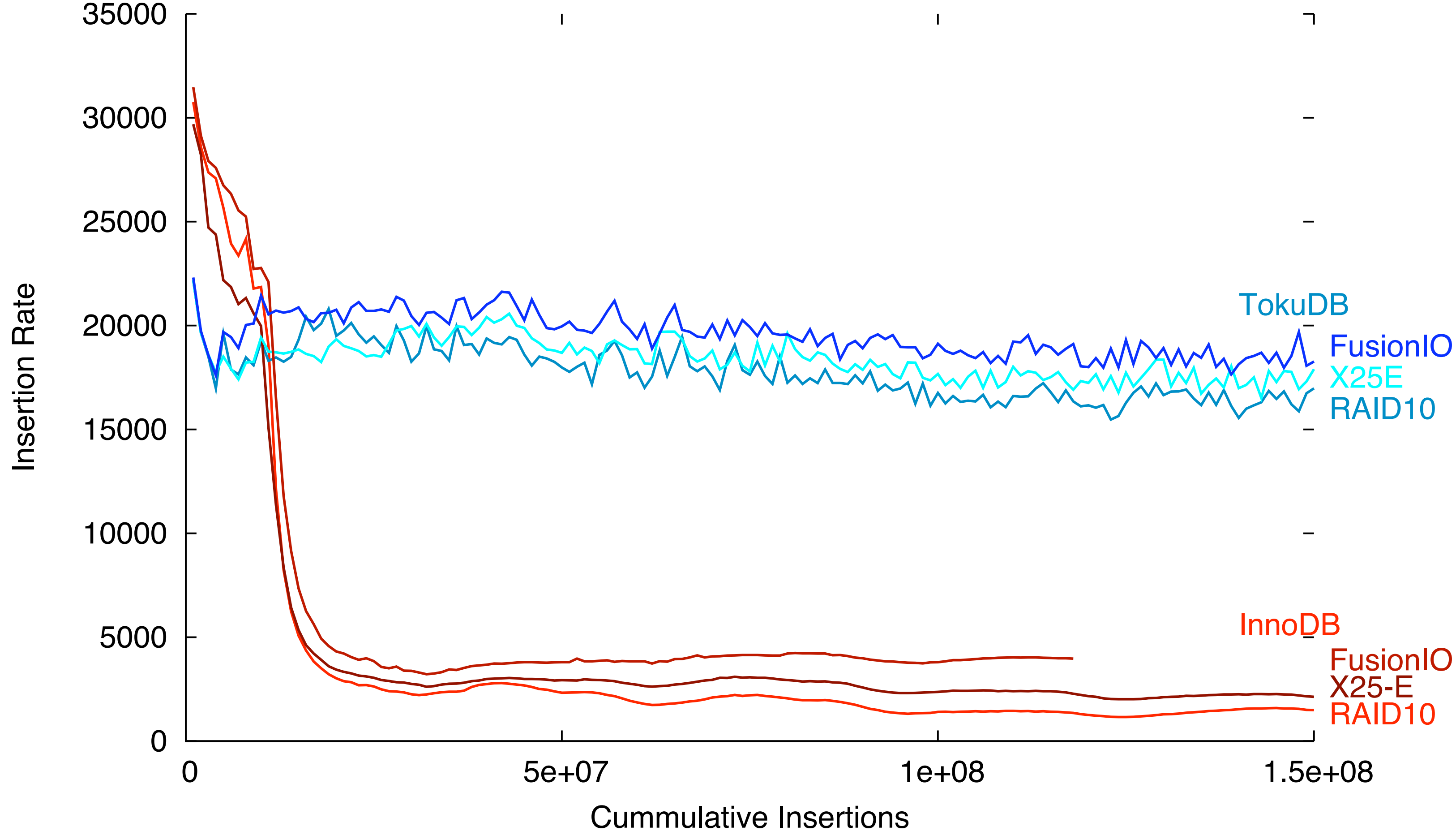


100M inserts into a collection with 3 secondary indexes

Compression (up to 25x)



iiBench on SSD



TokuDB on rotating disk beats InnoDB on SSD.

Write-optimization Can Help Schema Changes

InnoDB Index Creation		TokuDB Hot Indexing
00:31:34	⇨	00:00:02
InnoDB Column Addition		**TokuDB Hot Column Addition**
17:44:41	⇨	00:00:03

Scaling into the Future

Write-optimization going forward

Example: Time to fill a disk in 1973, 2010, 2022.

- log high-entropy data sequentially versus index data in B-tree.

Year	Size	Bandwidth	Access Time	Time to log data on disk	Time to fill disk using a B-tree (row size 1K)
1973	35MB	835KB/s	25ms	39s	975s
2010	3TB	150MB/s	10ms	5.5h	347d
2022	220TB	1.05GB/s	10ms	2.4d	70y

Better data structures may be a luxury now, but they will be essential by the decade's end.

Write-optimization going forward

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* Projected times for fully multi-threaded version

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Better data structures may be a luxury now, but they will be essential by the decade's end.

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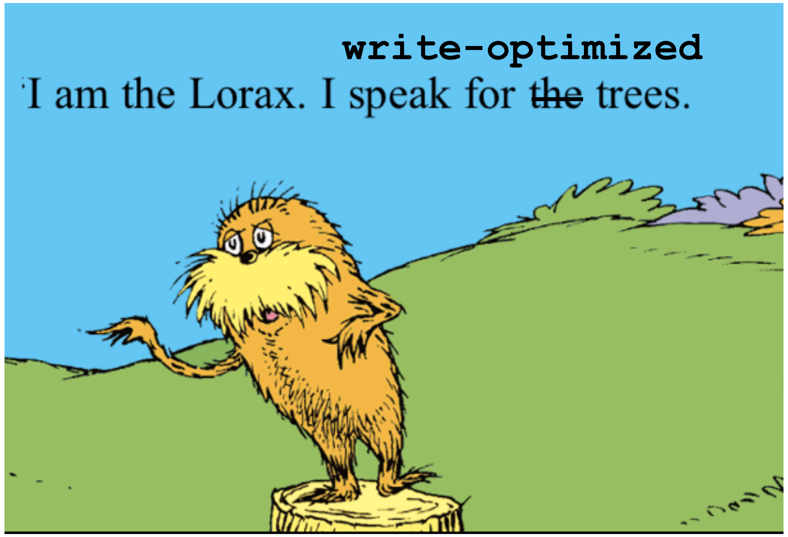
Summary of Module

Write-optimization can solve many problems.

- There is a provable point-query insert tradeoff. We can insert 10x-100x faster without hurting point queries.
- We can avoid much of the funny tradeoff between data ingestion, freshness, and query speed.
- We can avoid tuning knobs.



Data Structures and Algorithms for Big Data

Module 3: (Case Study) TokuFS--How to Make a Write-Optimized File System

Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

Story for Module

Algorithms for Big Data apply to all storage systems, not just databases.

Some big-data users store use a file system.

The problem with Big Data is Microdata...

HEC FSIO Grand Challenges

Store 1 trillion files

Create tens of thousands of files per second

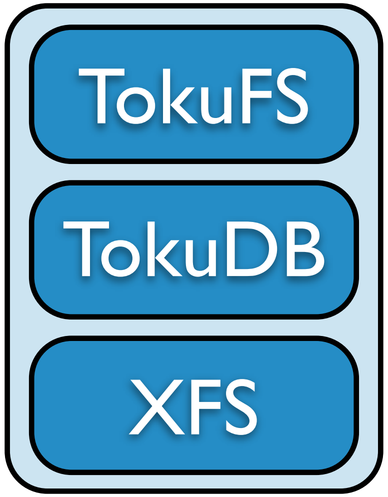
Traverse directory hierarchies fast (`ls -R`)

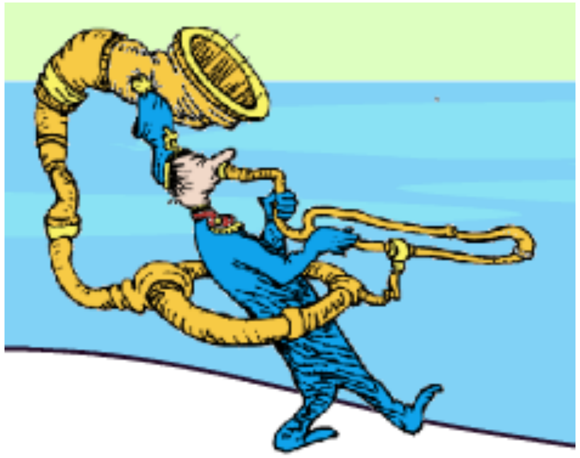
B-trees would require at least hundreds of disk drives.

TokuFS

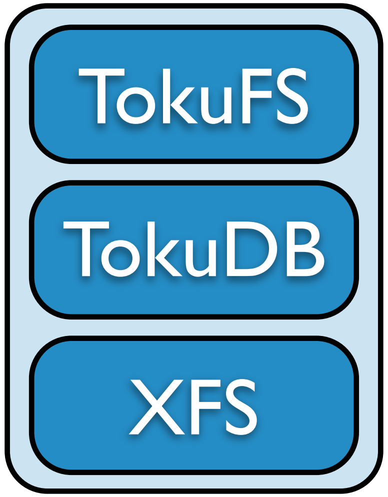
TokuFS [Esmet, Bender, Farach-Colton, Kuszmaul HotStorage12]

- A file-system prototype
- >20K file creates/sec
- very fast `ls -R`
- HEC grand challenges on a cheap disk (except 1 trillion files)



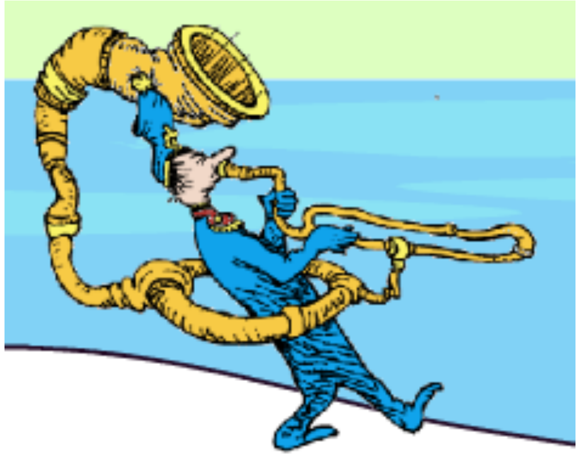


TokuFS [Esmet, Bender, Farach-Colton, Kuszmaul HotStorage12]



- A file-system prototype
- >20K file creates/sec
- very fast `ls -R`
- HEC grand challenges on a cheap disk (except 1 trillion files)

- TokuFS offers orders-of-magnitude speedup on *microdata* workloads.
  - Aggregates microwrites while indexing.
  - So it can be faster than the underlying file system.



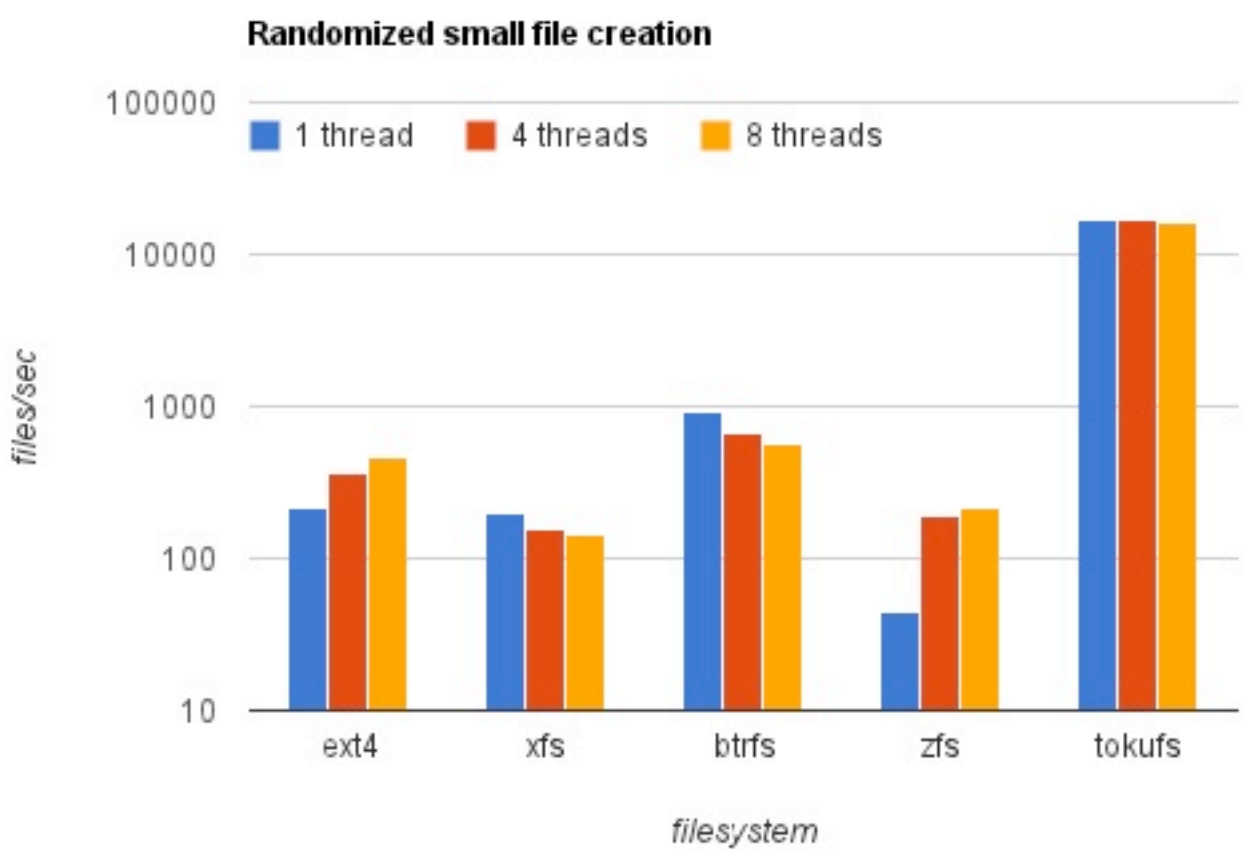
Big speedups on microwrites

We ran microdata-intensive benchmarks

- Compared TokuFS to ext4, XFS, Btrfs, ZFS.
- Stressed metadata and file data.
- Used commodity hardware:
  - 2 core AMD, 4GB RAM
  - Single 7200 RPM disk
  - Simple, cheap setup. No hardware tricks.
- In all tests, we observed orders of magnitude speed up.

Faster on small file creation

Create 2 million 200-byte files in a directory tree



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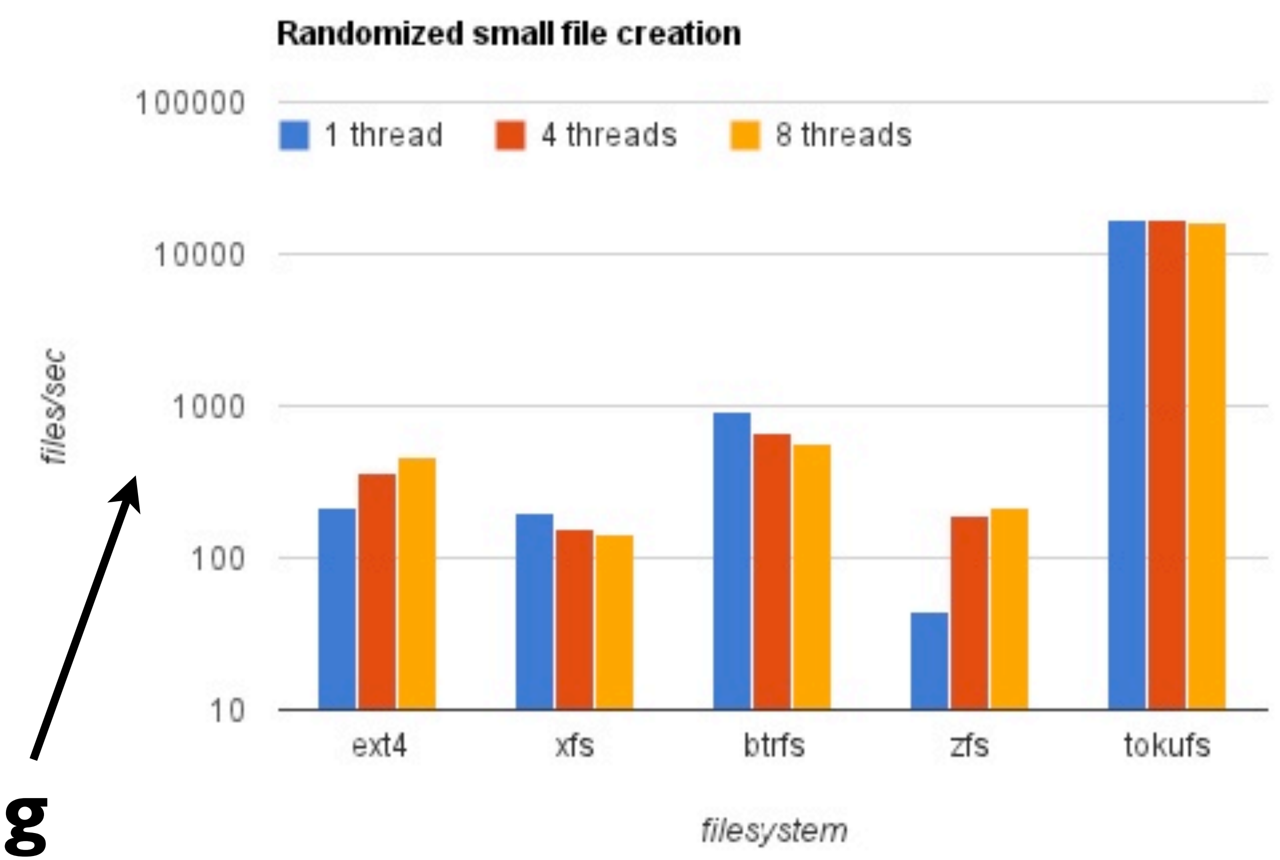
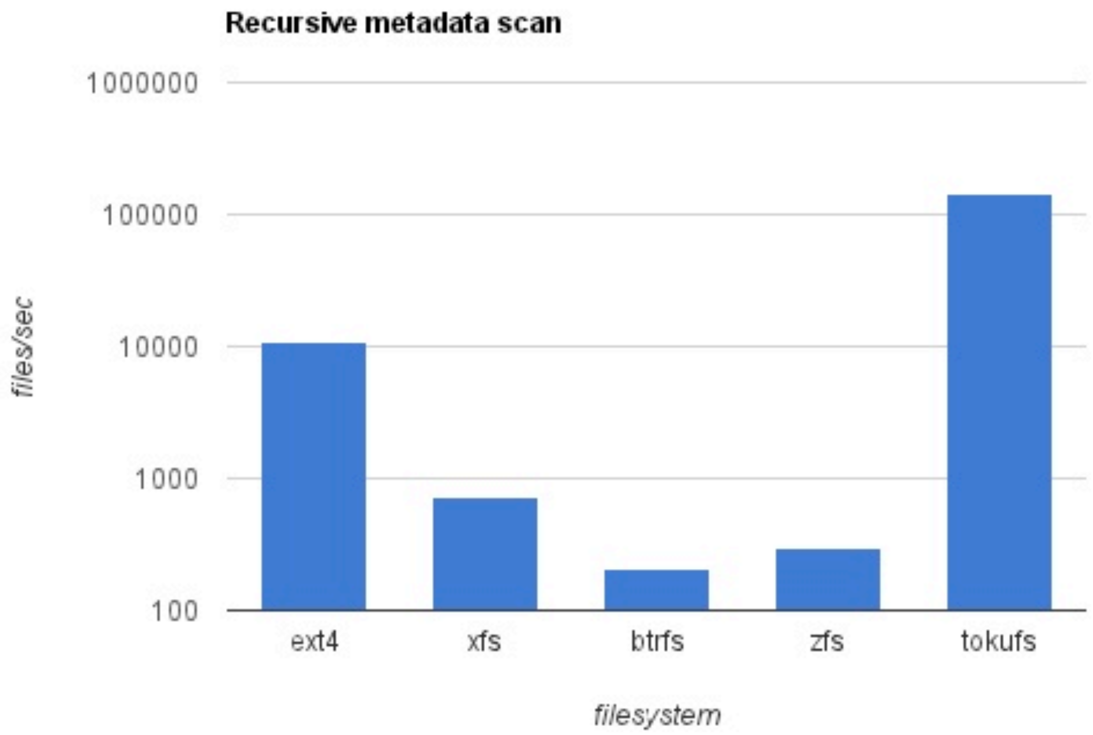


Log scale

Faster on metadata scan

Recursively scan directory tree for metadata

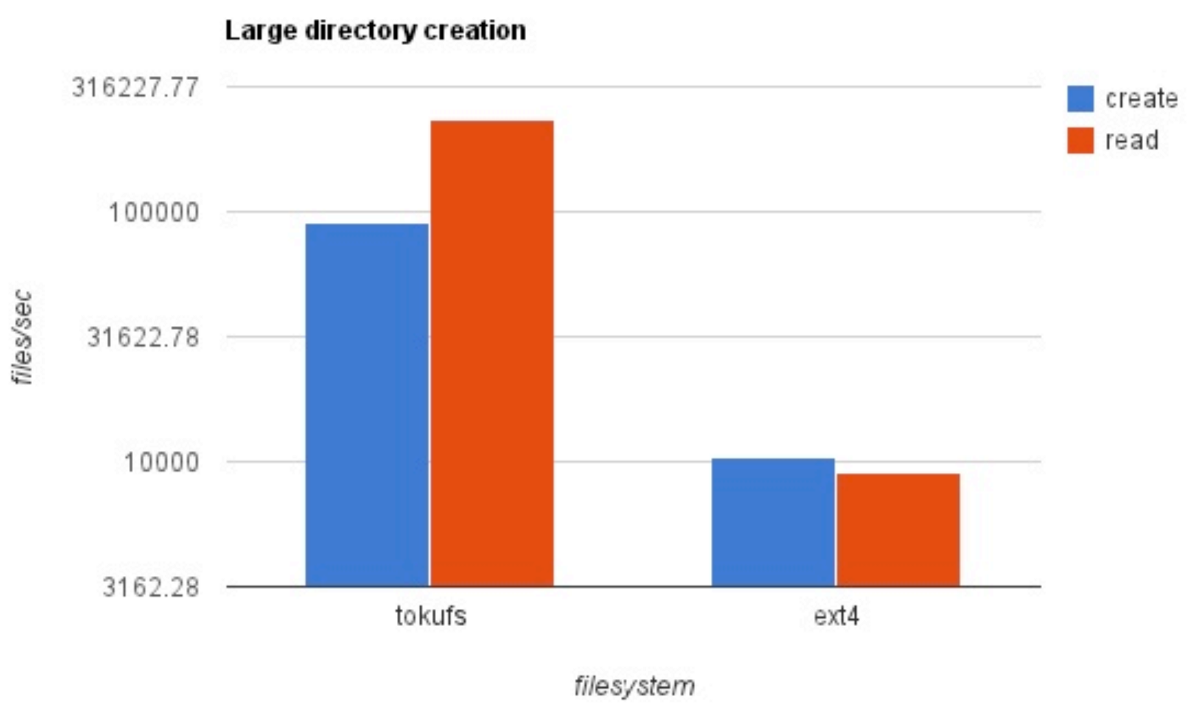
- Use the same 2 million files created before.
- Start on a cold cache to measure disk I/O efficiency



Faster on big directories

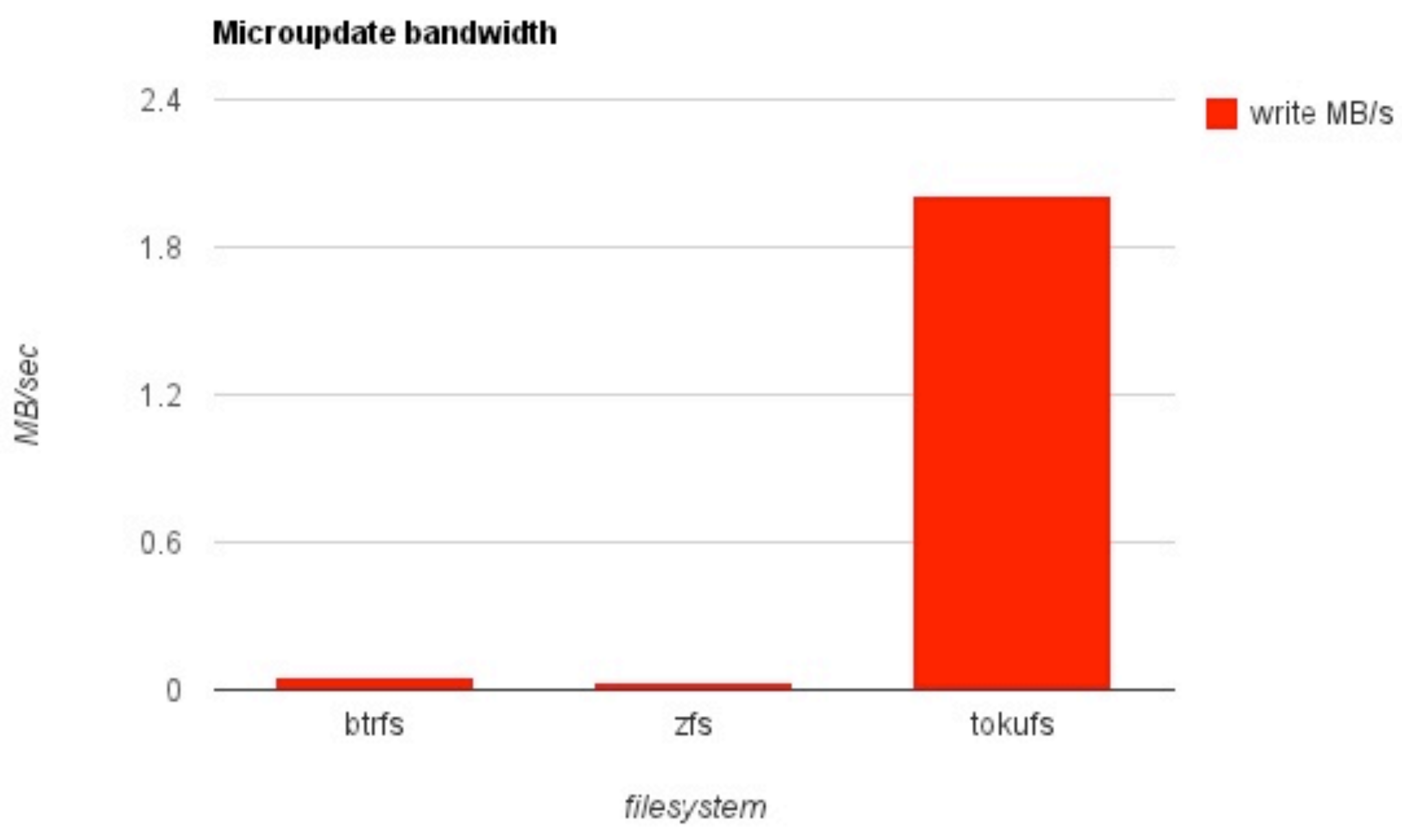
Create one million empty files in a directory

- Create files with random names, then read them back.
- Tests how well a single directory scales.



Faster on microwrites in a big file

Randomly write out a file in small, unaligned pieces



TokuFS Implementation

TokuFS employs two indexes

Metadata index:

- The metadata index maps pathname to file metadata.
  - /home/esmet   mode, file size, access times, ...
  - /home/esmet/tokufs.c   mode, file size, access times, ...

Data index:

- The data index maps pathname, blocknum to bytes.
  - /home/esmet/tokufs.c, 0   [block of bytes]
  - /home/esmet/tokufs.c, 1   [block of bytes]
- Block size is a compile-time constant: 512.
  - good performance on small files, moderate on large files

Common queries exhibit locality

Metadata index keys: full path as string

- All the children of a directory are contiguous in the index
- Reading a directory is simple and fast

Data block index keys: 【full path, blocknum】

- So all the blocks for a file are contiguous in the index
- Reading a file is simple and fast

TokuFS compresses indexes

Reduces overhead from full path keys

- Pathnames are highly “prefix redundant”
- They compress very, very well in practice

Reduces overhead from zero-valued padding

- Uninitialized bytes in a block are set to zero
- Good portions of the metadata struct are set to zero

Compression between 7-15x on real data

- For example, a full MySQL source tree

TokuFS is fully functional

TokuFS is a prototype, but fully functional.

- Implements files, directories, metadata, etc.
- Interfaces with applications via shared library, header.

We wrote a FUSE implementation, too.

- FUSE lets you implement filesystems in user space.
- But there's overhead, so performance isn't optimal.
- The best way to run is through our POSIX-like file API.

Microdata is the Problem

Data Structures and Algorithms for Big Data

Module 4: Cache-Oblivious Analysis

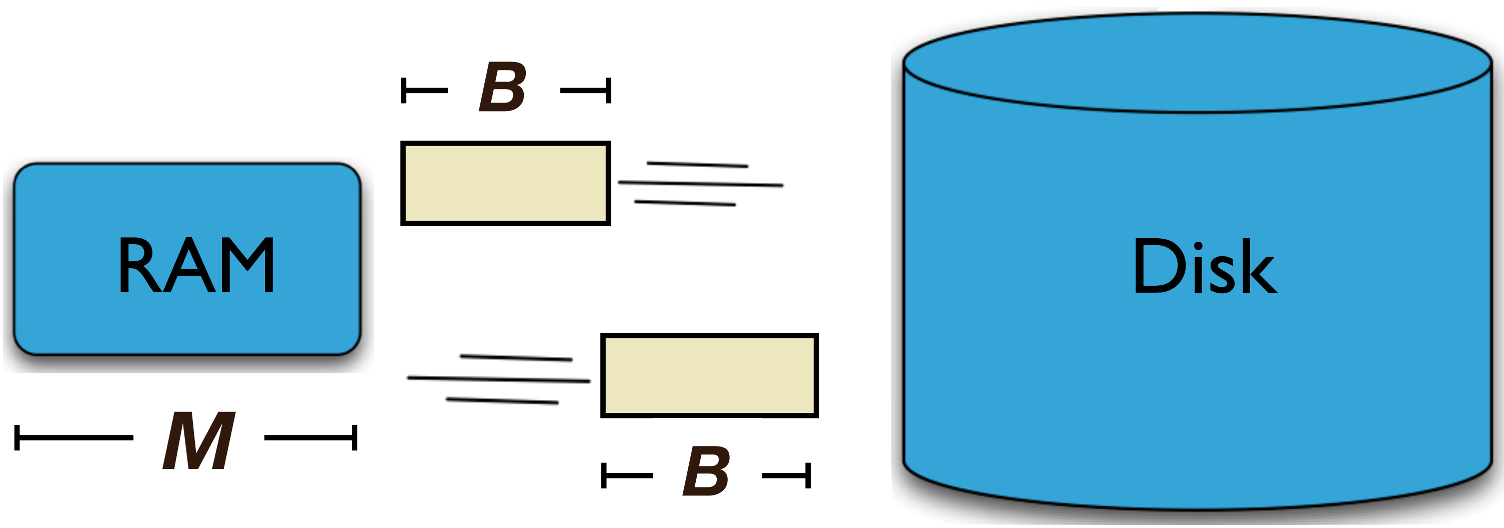
Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

Recall the Disk Access Machine

External-memory model:

- Time bounds are parameterized by ***B***, ***M***, ***N***.
- Goal: Minimize # of block transfers ≈ time.



Cache-Oblivious (CO) Algorithms [Frigo, Leiserson, Prokop, Ramachandran '99]

External-memory model:

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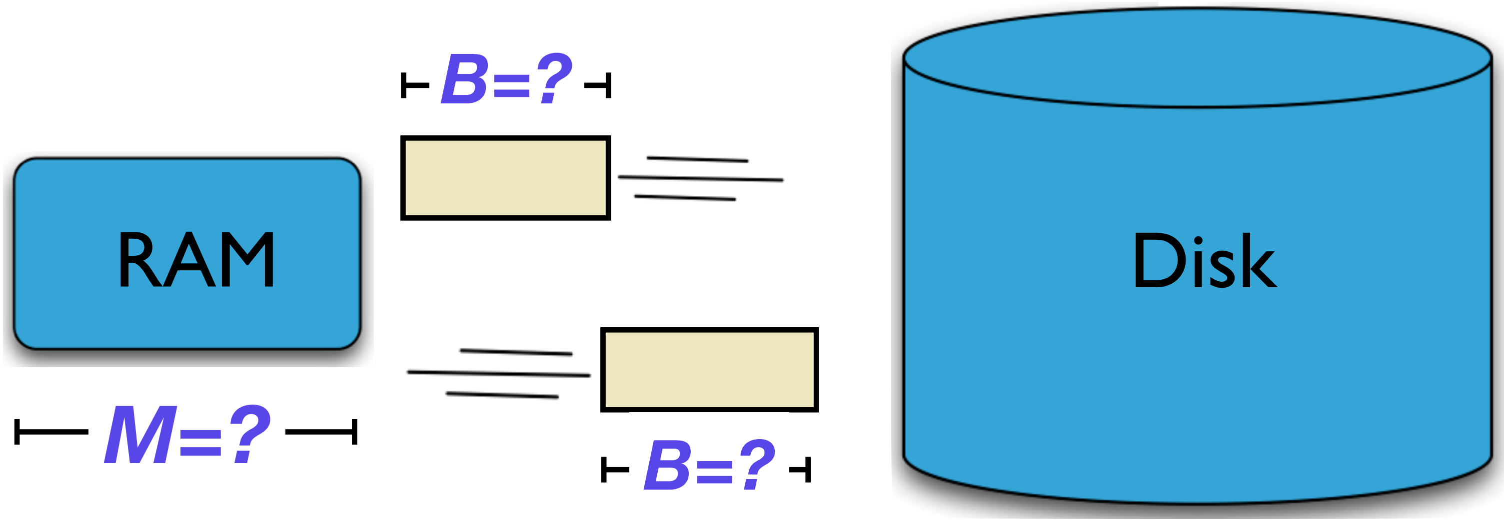
Beautiful restriction:

- Parameters ***B***, ***M*** are unknown to the algorithm or coder.



Cache-Oblivious (CO) Algorithms [Frigo, Leiserson, Prokop, Ramachandran '99]

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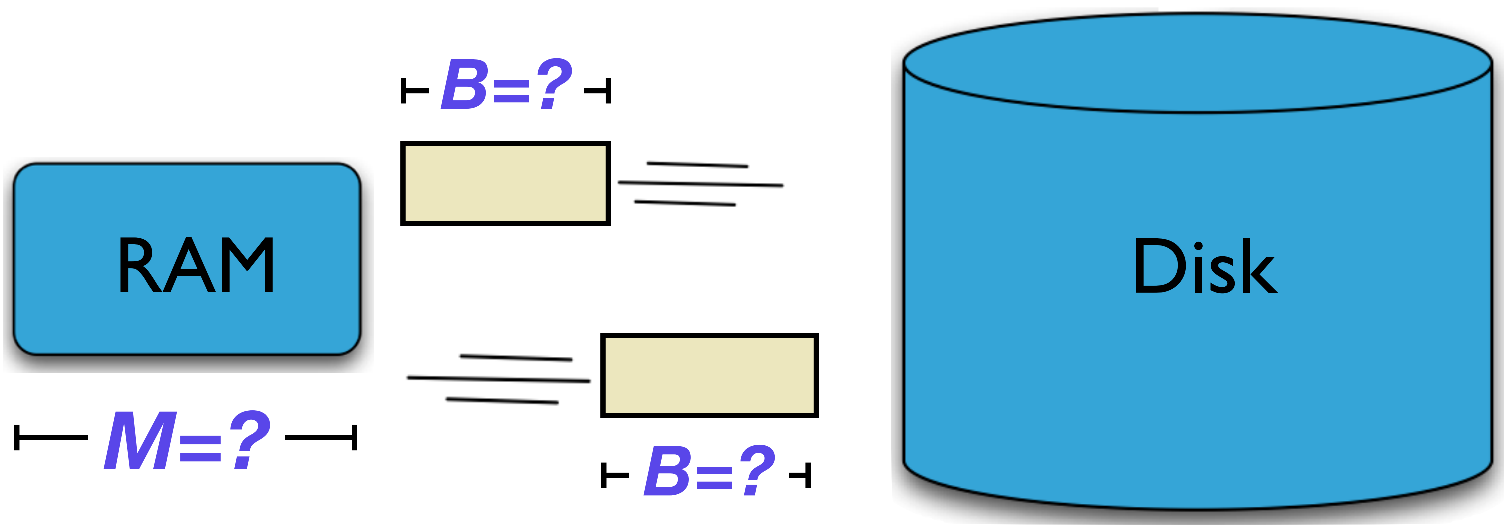
- Goal: Minimize # of block transfers ≈ time.

Beautiful restriction:

- Parameters ***B***, ***M*** are unknown to the algorithm or coder.

An optimal CO algorithm is universal for all *B*, *M*, *N*.



Overview of Module

Cache-oblivious definition

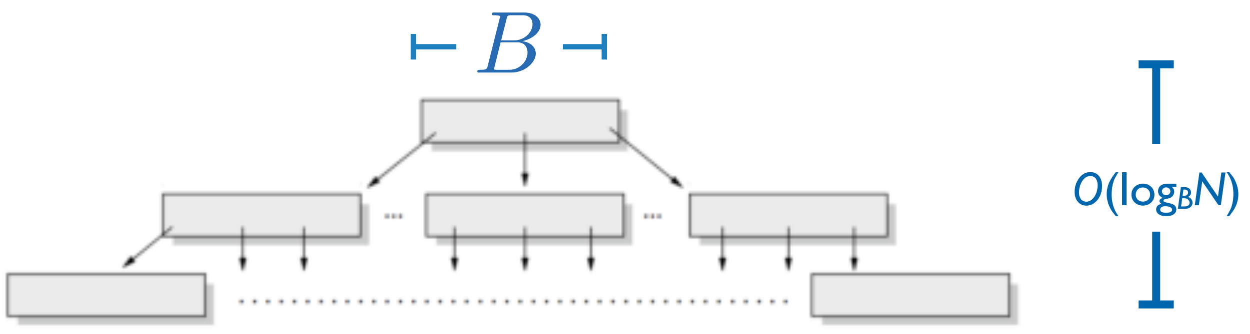
Cache-oblivious B-tree

Cache-oblivious performance advantages

Cache-oblivious write-optimized data structure (COLA)

Cache-adaptive algorithms

Traditional B-trees aren't cache-oblivious



The fan-out is a function of *B*.

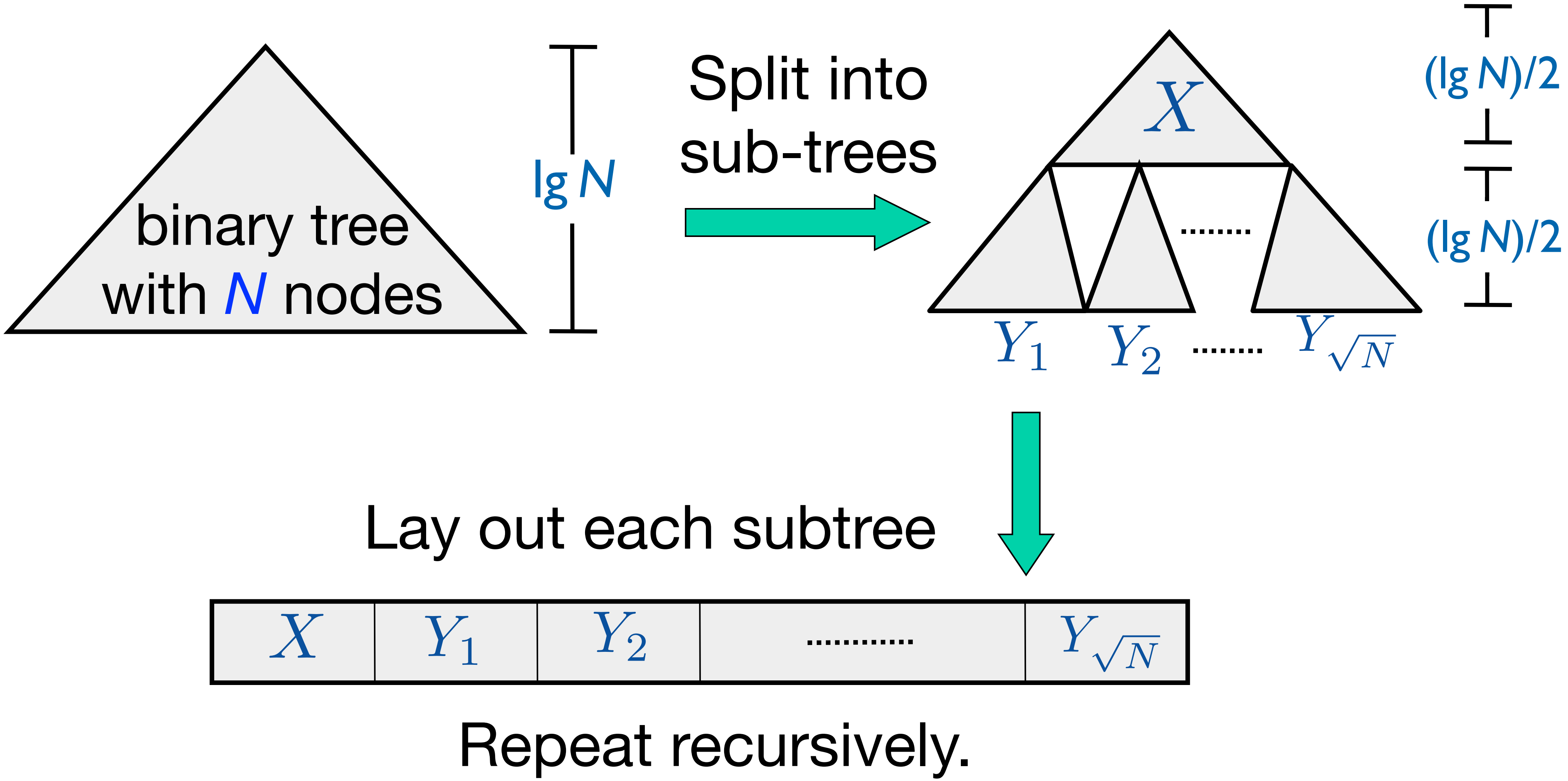
There do exist cache-oblivious B-trees.

- We can still achieve $O(\log_B N)$ I/Os per operation, even without parameterizing by B or M.

[Bender, Demaine, Farach-Colton '00] [Bender, Duan, Iacono, Wu '02]
[Brodal, Fagerberg, Jacob '02]

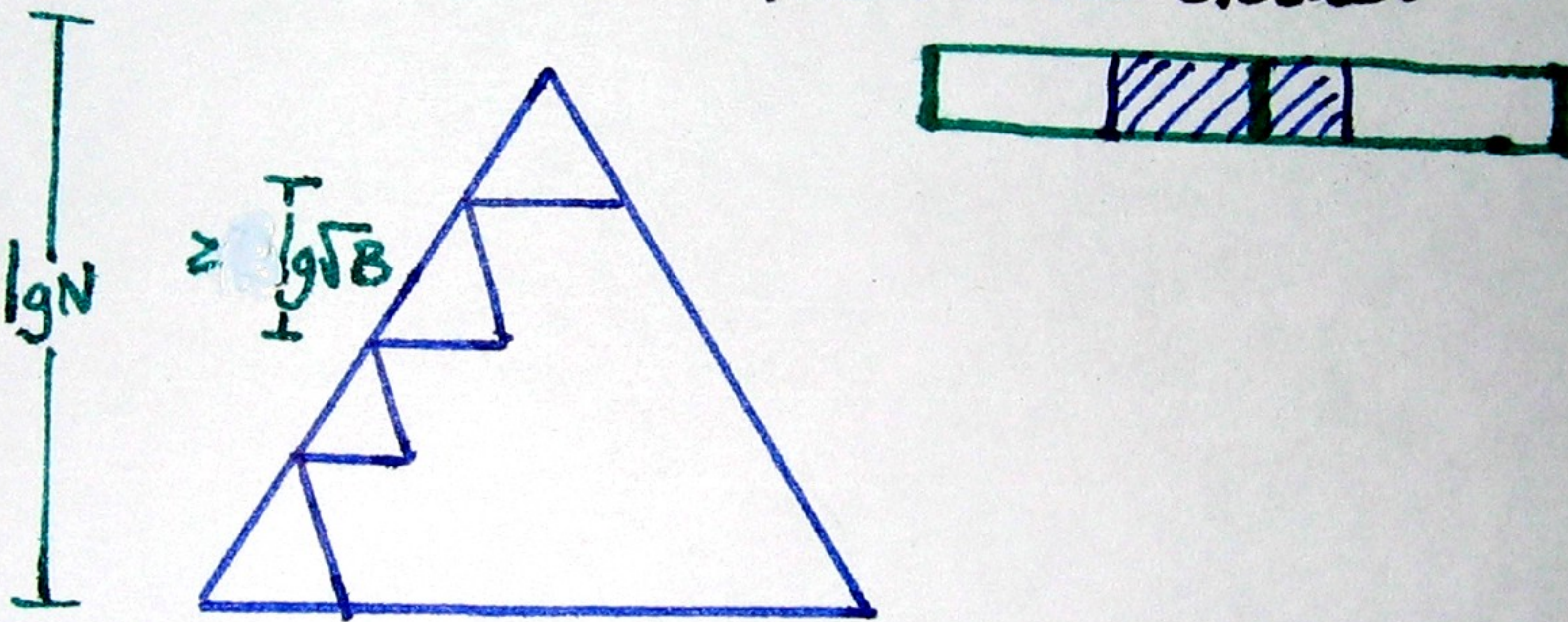
Static cache-oblivious B-Tree (no inserts) [Prokop 99]

- Technique: divide & conquer (Van Emde Boas layout)



Analysis of vEB Layout

Conceptually stop recursion when recursive subtrees $\leq B$ $\Rightarrow$ subtree fits in ≤ 2 blocks.



$\Rightarrow$ A search visits $\leq \lg N / \lg \sqrt{B} = 2 \log_B N$ subtrees

$\Rightarrow$ $\leq 4 \log_B N$ memory transfers

Dynamic Cache-Oblivious B-trees

[Bender, Demaine, Farach-Colton 00]

We won't describe how to dynamize....

After all, the cache-oblivious dynamic B-tree isn't write-optimized.

We believe that write-optimized data structures win out over B-trees (even cache-oblivious ones) in the majority of cases.

Overview of Module

Cache-oblivious definition

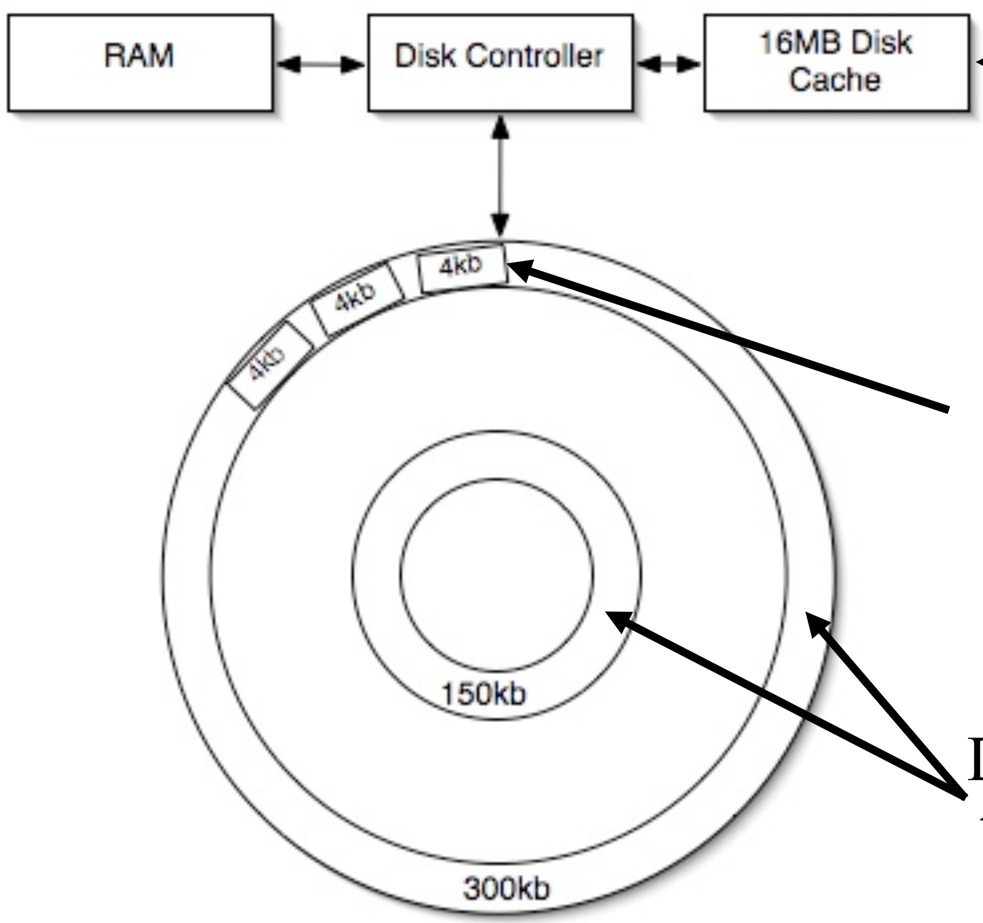
Example: cache-oblivious B-tree

Cache-oblivious performance advantages

Cache-oblivious write-optimized data structure (COLA)

Cache-adaptive algorithms

The DAM model is a simplification



Tracks get prefetched into the disk cache, which holds ~100 tracks.

Fixed-size blocks are fetched.

Disks are organized into tracks of different sizes

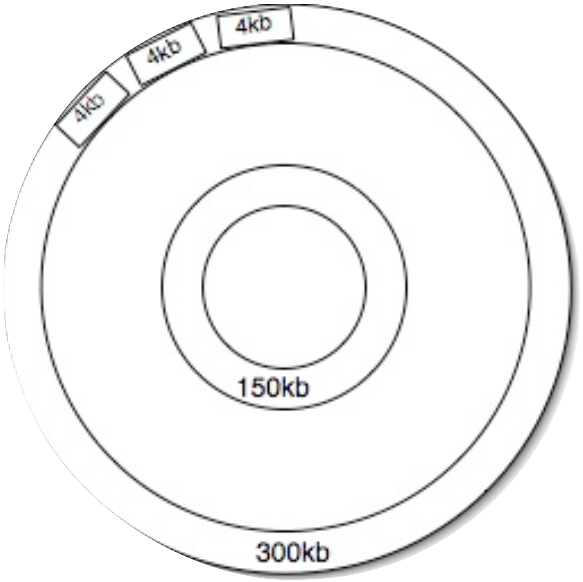
The DAM model is a simplification

2kB or 4kB is too small for the model.

- B-tree nodes in Berkeley DB & InnoDB have this size.
- Issue: sequential block accesses run 10x faster than random block accesses, which doesn't fit the model.

There is no single best block size.

- The best node size for a B-tree depends on the operation
  (insert/delete/point query).



Time for 1000 Random B-tree Searches

[Bender, Farach-Colton, Kuszmaul '06]

B	Small	Big
4K	17.3ms	22.4ms
16K	13.9ms	22.1ms
32K	11.9ms	17.4ms
64K	12.9ms	17.6ms
128K	13.2ms	16.5ms
256K	18.5ms	14.4ms
512K		16.7ms

	Small	Big
CO B-tree	12.3ms	13.8ms

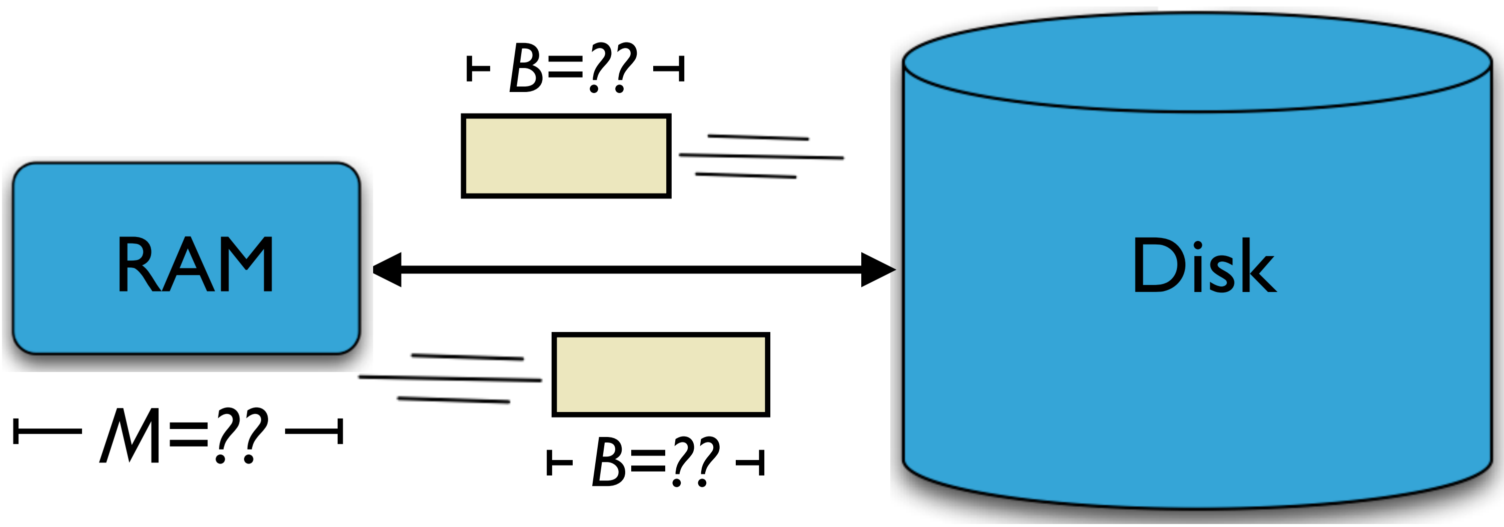
There's no best block size.

The optimal block size for inserts is very different.

Cache-Oblivious Analysis

- Cache-oblivious algorithms work for all B and M...
- ... and all levels of a multi-level hierarchy.

It's better to optimize approximately for all B, M than to pick the best B and M.



[Frigo, Leiserson, Prokop, Ramachandran '99]

Overview of Module

Cache-oblivious definition

Example: cache-oblivious B-tree

Cache-oblivious performance advantages

Cache-oblivious write-optimized data structure (COLA)

- You can even make write-optimized data structures cache-oblivious

[Bender, Farach-Colton, Fineman, Fogel, Kuszmaul, Nelson, SPAA 07]

[Brodal, Demaine, Fineman, Iacono, Langerman, Munro, SODA 10]

Cache-adaptive algorithms

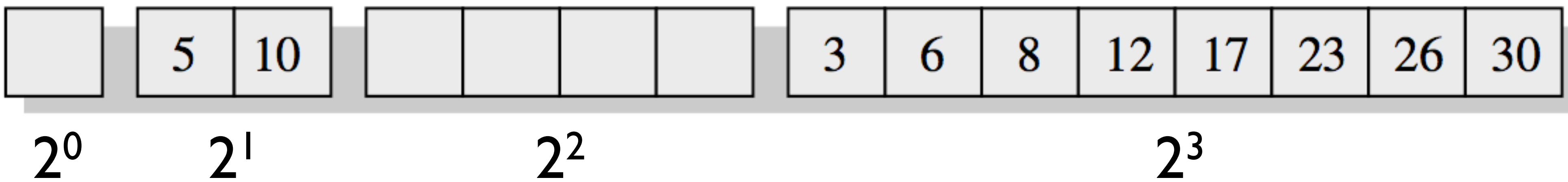
Recall optimal search-insert tradeoff [Brodal, Fagerberg 03]

	insert	point query
Optimal tradeoff (function of ε=0...1)	$O\left(\frac{\log_{1+B^\varepsilon} N}{B^{1-\varepsilon}}\right)$	$O\left(\log_{1+B^\varepsilon} N\right)$
B-tree (ε=1)	$O\left(\log_B N\right)$	$O\left(\log_B N\right)$
ε=1/2	$O\left(\frac{\log_B N}{\sqrt{B}}\right)$	$O\left(\log_B N\right)$
ε=0	$O\left(\frac{\log N}{B}\right)$	$O\left(\log N\right)$

10x-100x faster inserts

We give a cache-oblivious solution for ε=0.

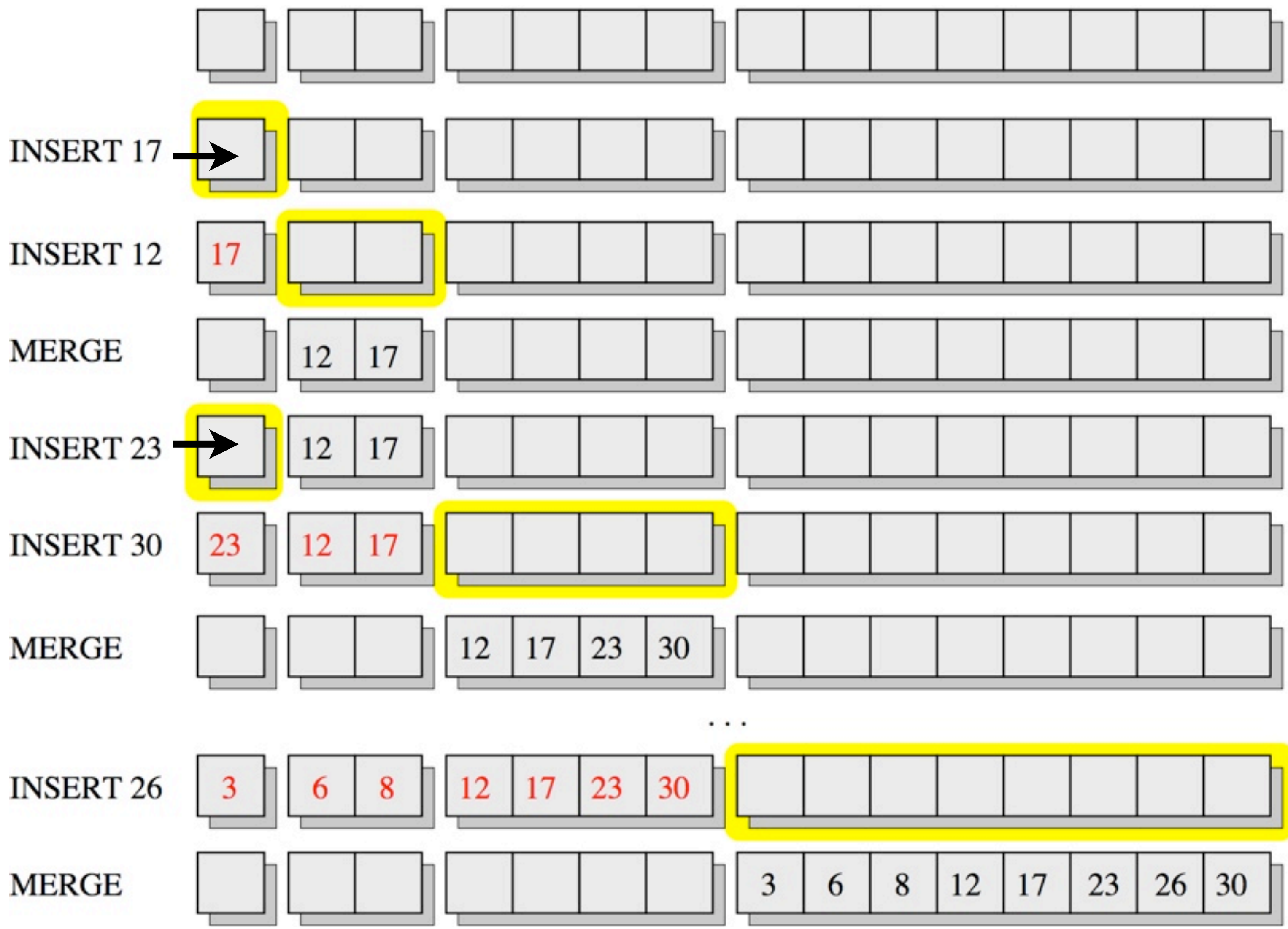
Simplified CO write-optimized structure (COLA)



O((log*N*)/*B*) insert cost & O(log^2*N*) search cost

- Sorted arrays of exponentially increasing size.
- Arrays are completely full or completely empty (depends on the bit representation of # of elmts).
- Insert into the smallest array.
  Merge arrays to make room.

Simplified CO write-optimized structure (COLA)



Simplified CO write-optimized structure (COLA)

17	5	10	13	41	57	90	3	6	8	12	17	23	26	30

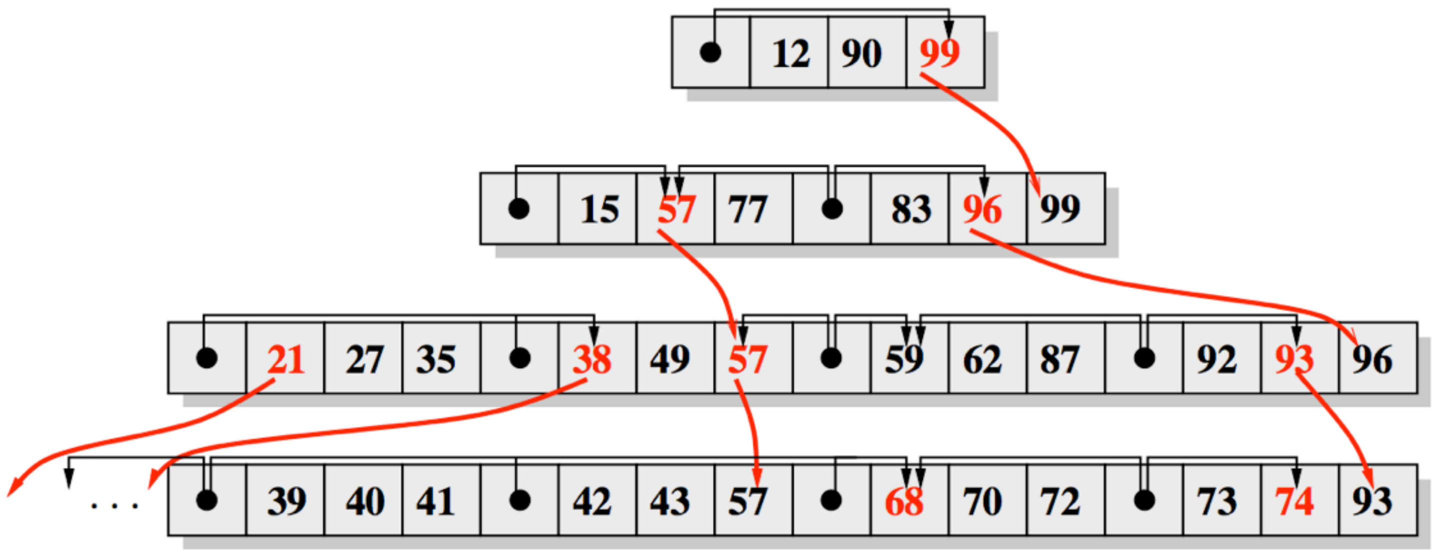
Insert Cost:

- cost to flush level of size X = $O(X/B)$
- cost per element to flush level = $O(1/B)$
- max # of times each element is flushed = $\log N$
- insert cost = $O((\log N))/B)$ amortized memory transfers

Search Cost

- Binary search at each level
- $\log(N/B) + \log(N/B) - 1 + \log(N/B) - 2 + \ldots + 2 + 1$
  $= O(\log^2(N/B))$

Cache-oblivious write-optimized structure (COLA)



O((log*N*)/*B*) insert cost & O(log*N*) search cost

- Some redundancy of elements between levels
- Arrays can be partially full
- Horizontal and vertical pointers to redundant elements
- (Fractional Cascading)

Overview of Module

Cache-oblivious definition

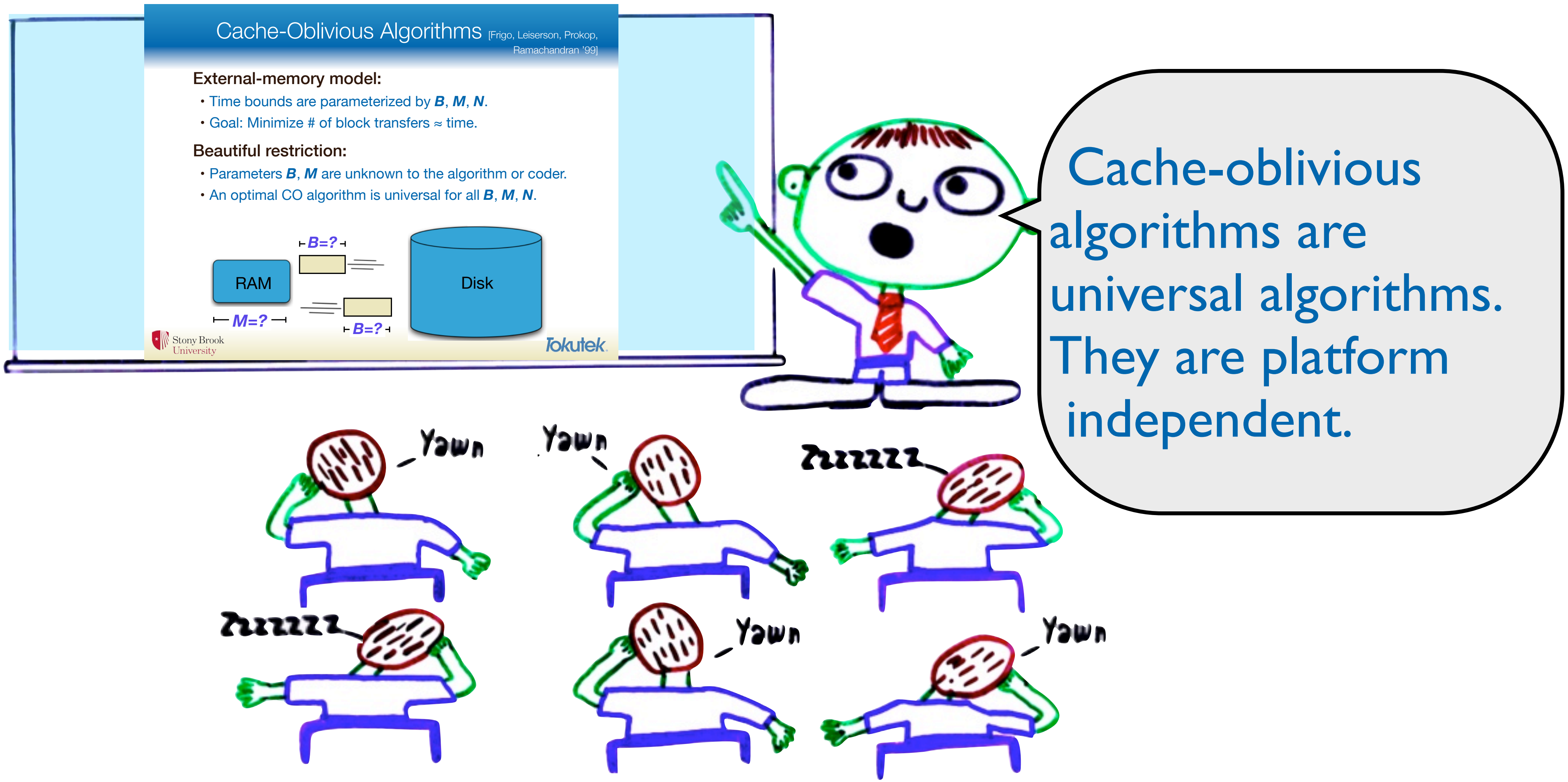
Example: cache-oblivious B-tree

Cache-oblivious performance advantages

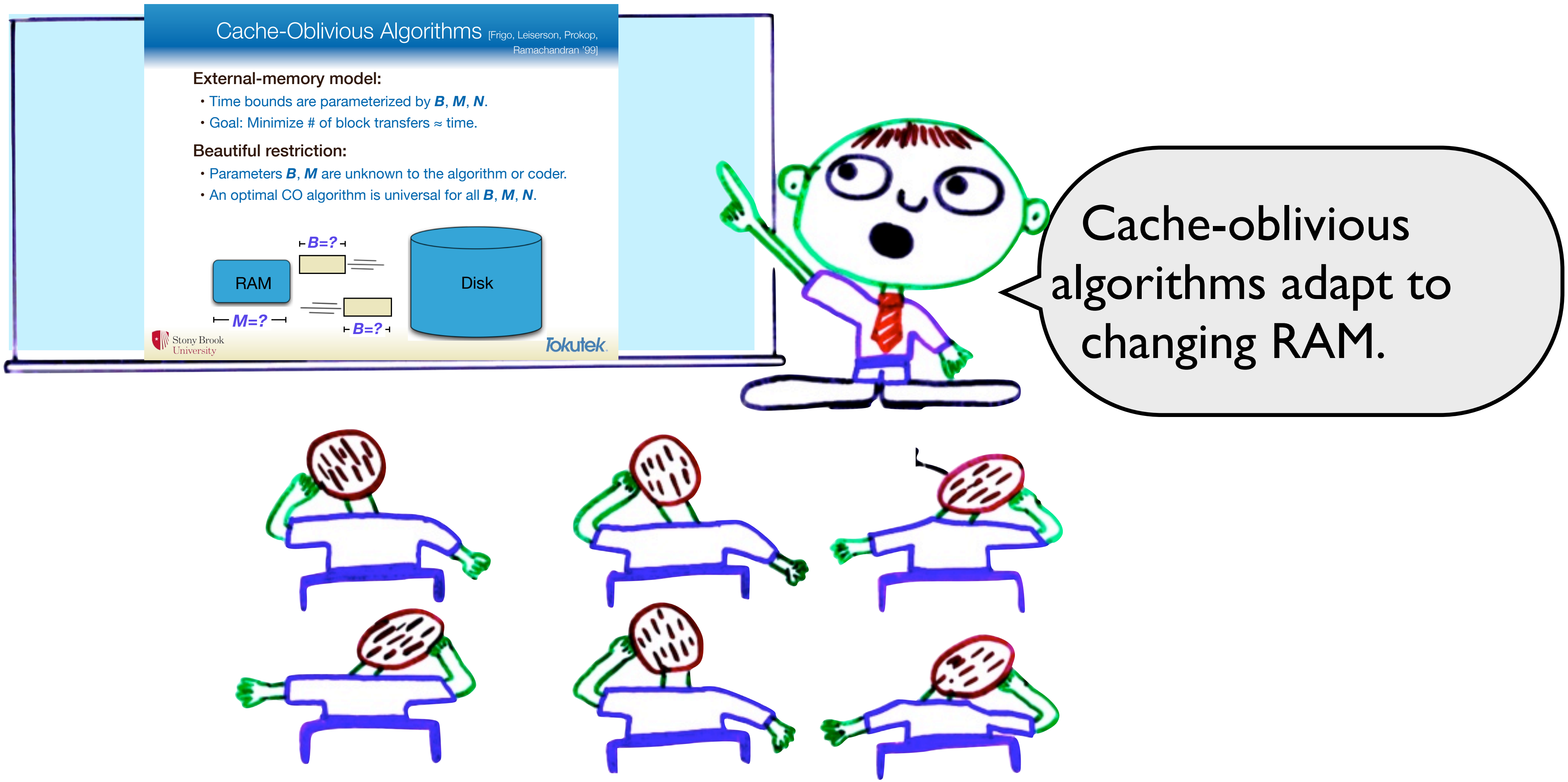
Cache-oblivious write-optimized data structure (COLA)

Cache-adaptive algorithms

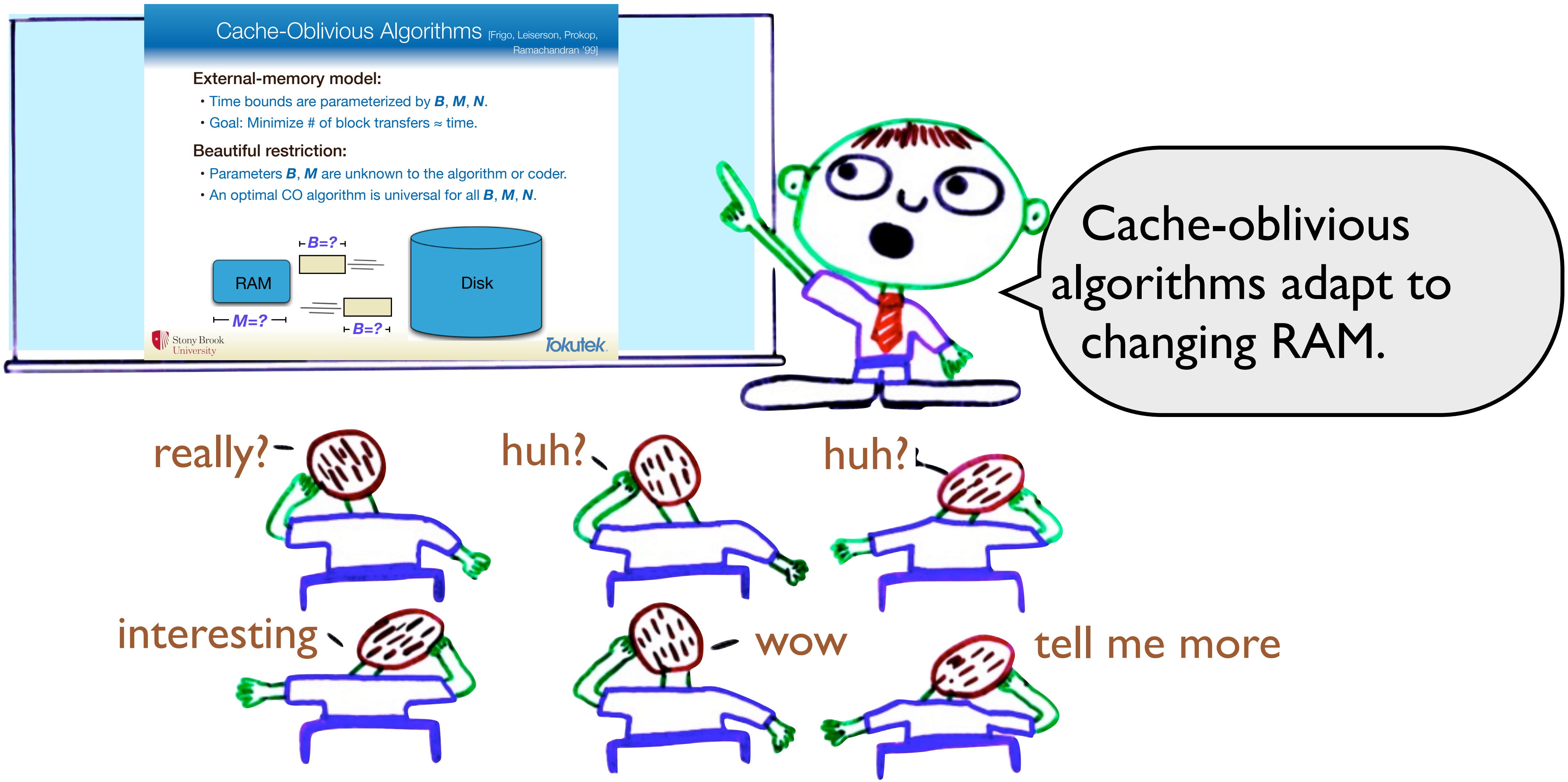
Michael at a Dagstuhl Workshop on Database Workload Management



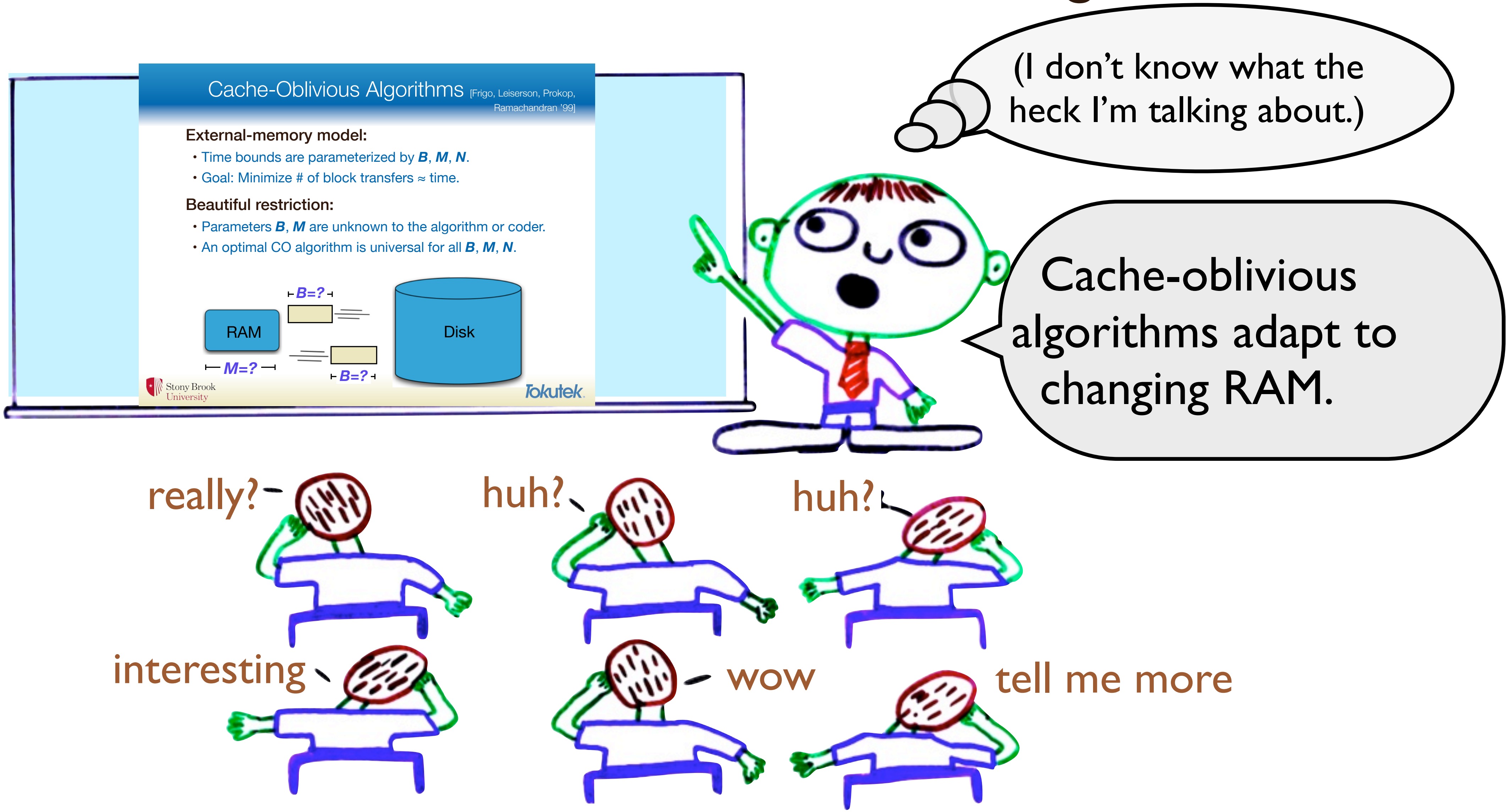
Michael at a Dagstuhl Workshop on Database Workload Management



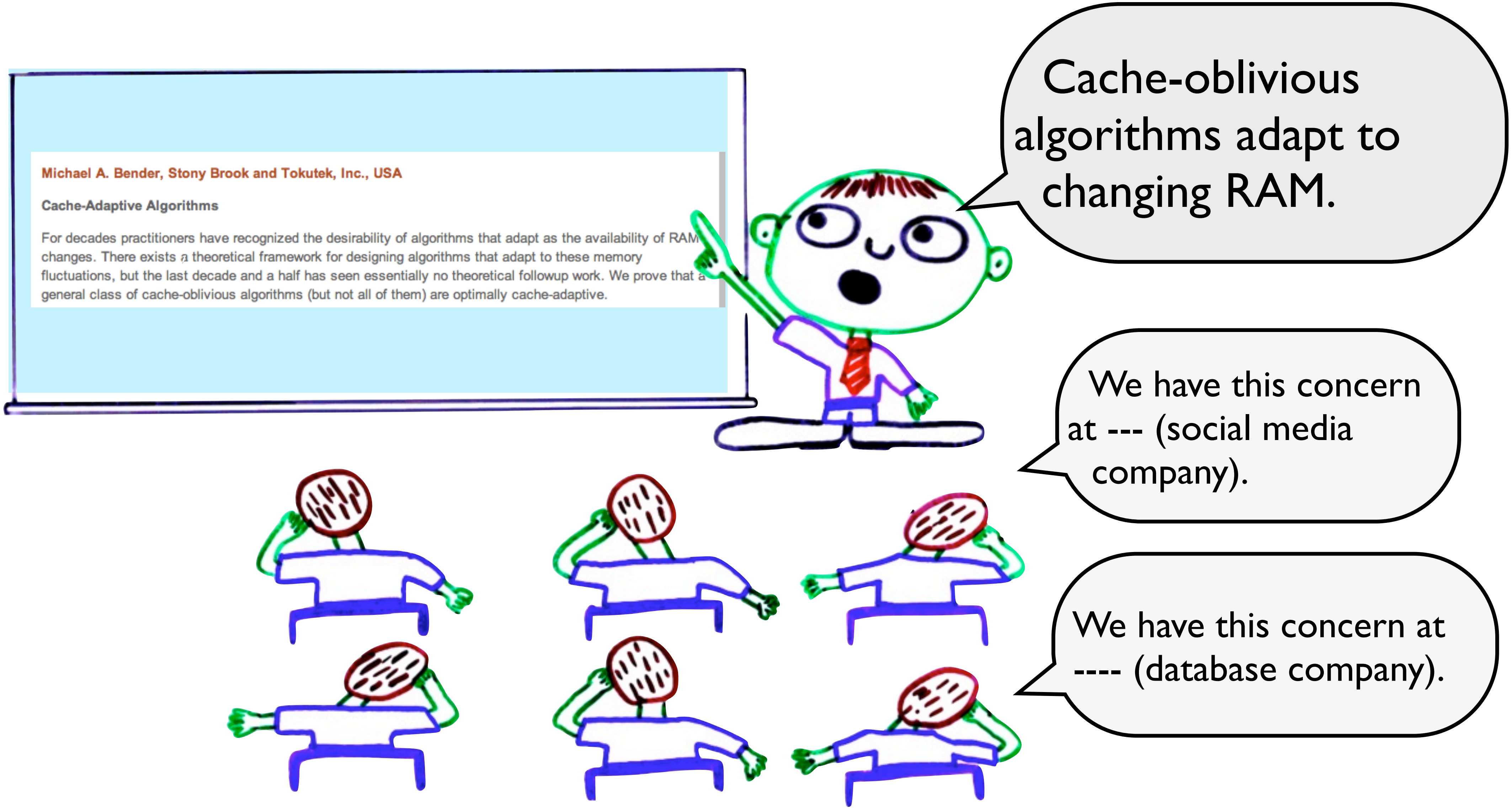
Michael at a Dagstuhl Workshop on Database Workload Management



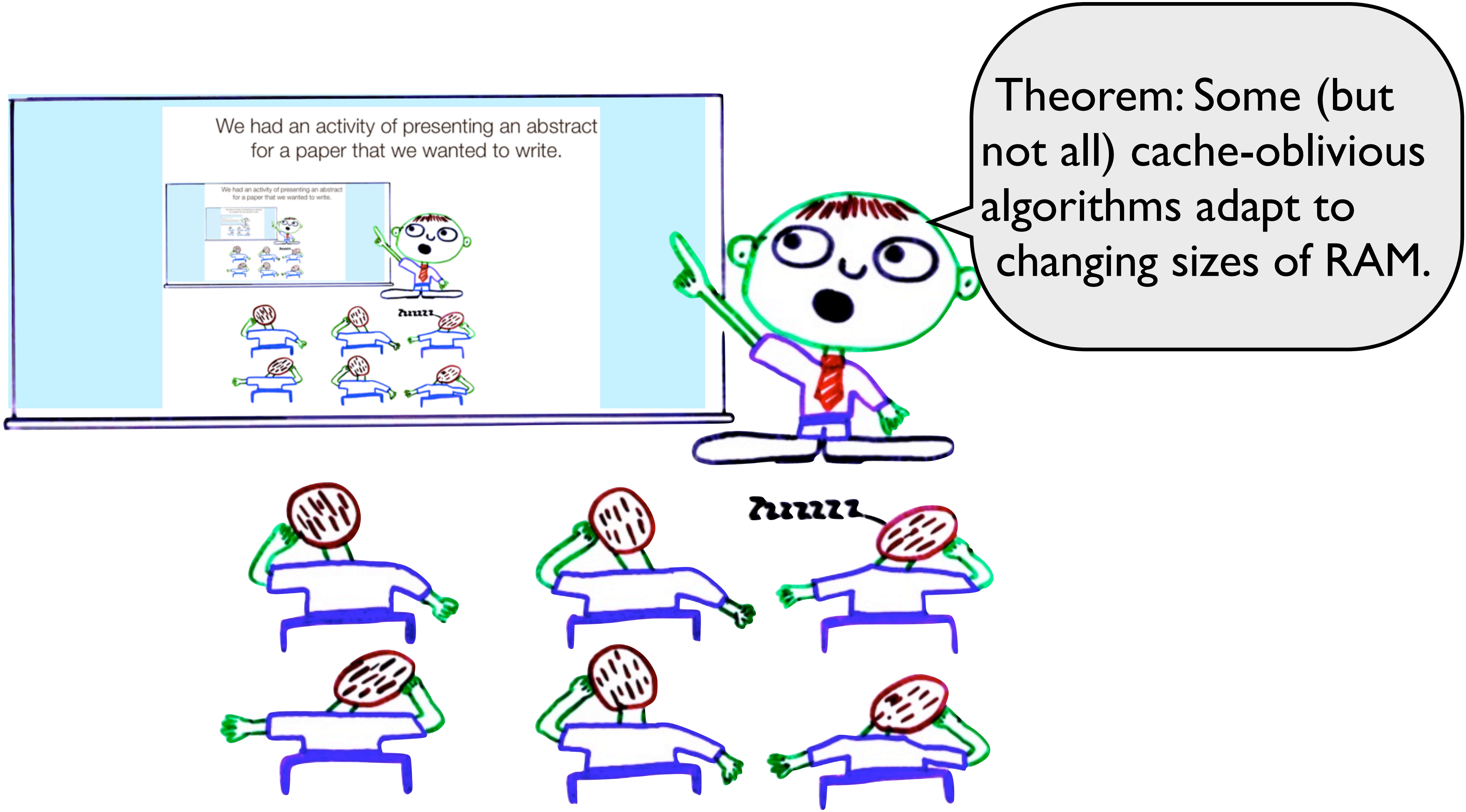
Michael at a Dagstuhl Workshop on Database Workload Management



We had an activity of presenting an abstract for a fictional paper that we wanted to write.



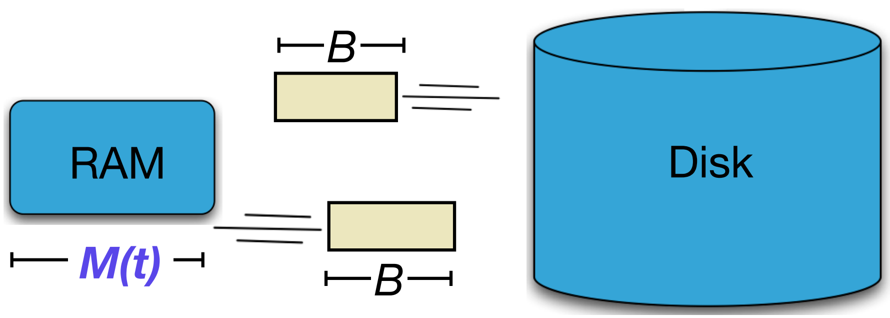
So I proved some theorems

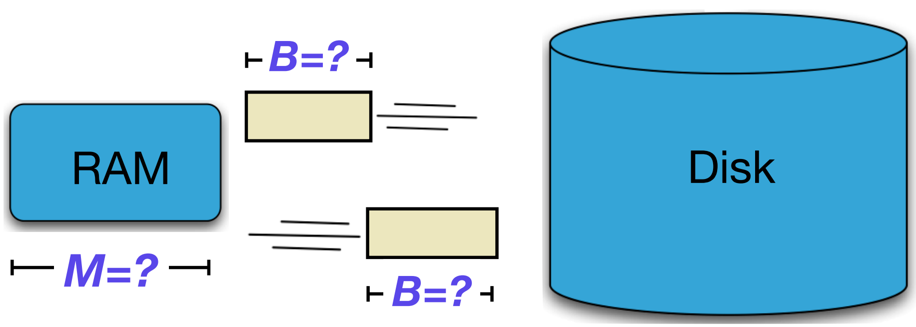


Some CO algorithms adapt when RAM changes

Some cache-oblivious algorithms run optimally even when the RAM changes arbitrarily over time.

- Sorting
- Problems with a special recursive structure (matrix multiplication, transpose, Gaussian elimination, all-pairs shortest paths)





Summary

In the cache-oblivious model, *B* and *M* are unknown to the coder.

(Of course, we still use *B* and *M* in proofs.)

It's remarkable how many common I/O-efficient data structures have cache-oblivious alternatives.

Sometimes it's better to optimize approximately for all *B* and *M* instead of picking the best *B* and *M*.

Data Structures and Algorithms for Big Data

Module 5: Log Structured Merge Trees

Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

Log Structured Merge Trees

[O'Neil, Cheng, Gawlick, O'Neil 96]

Log structured merge trees are write-optimized data structures developed in the 90s.

Over the past 5 years, LSM trees have become popular (for good reason).

Accumulo, Bigtable, bLSM, Cassandra, HBase, Hypertable, LevelDB are LSM trees (or borrow ideas).

http://nosql-database.org lists 122 NoSQL databases. Many of them are LSM trees.

Recall Optimal Search-Insert Tradeoff [Brodal, Fagerberg 03]

	insert	point query
Optimal tradeoff (function of ε=0...1)	$O\left(\frac{\log_{1+B^{\varepsilon}} N}{B^{1-\varepsilon}}\right)$	$O\left(\log_{1+B^{\varepsilon}} N\right)$

LSM trees don't lie on the optimal search-insert tradeoff curve.

But they're not far off.

We'll show how to move them back onto the optimal curve.

Log Structured Merge Tree

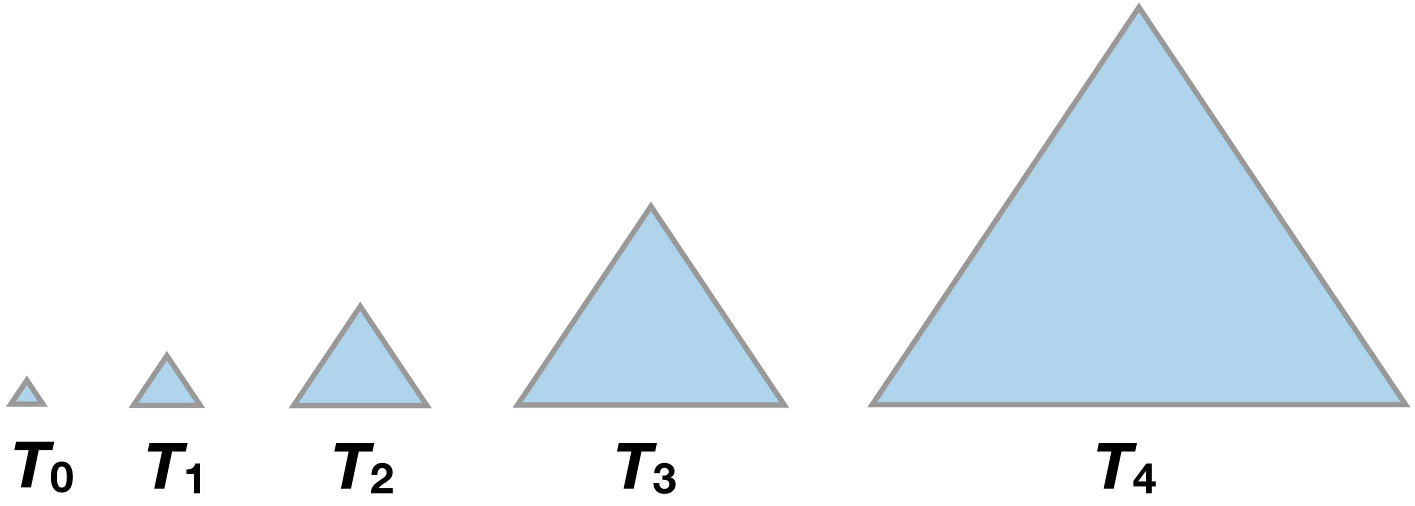
[O'Neil, Cheng, Gawlick, O'Neil 96]

An LSM tree is a cascade of B-trees.

Each tree T_j has a *target size* $|T_j|$.

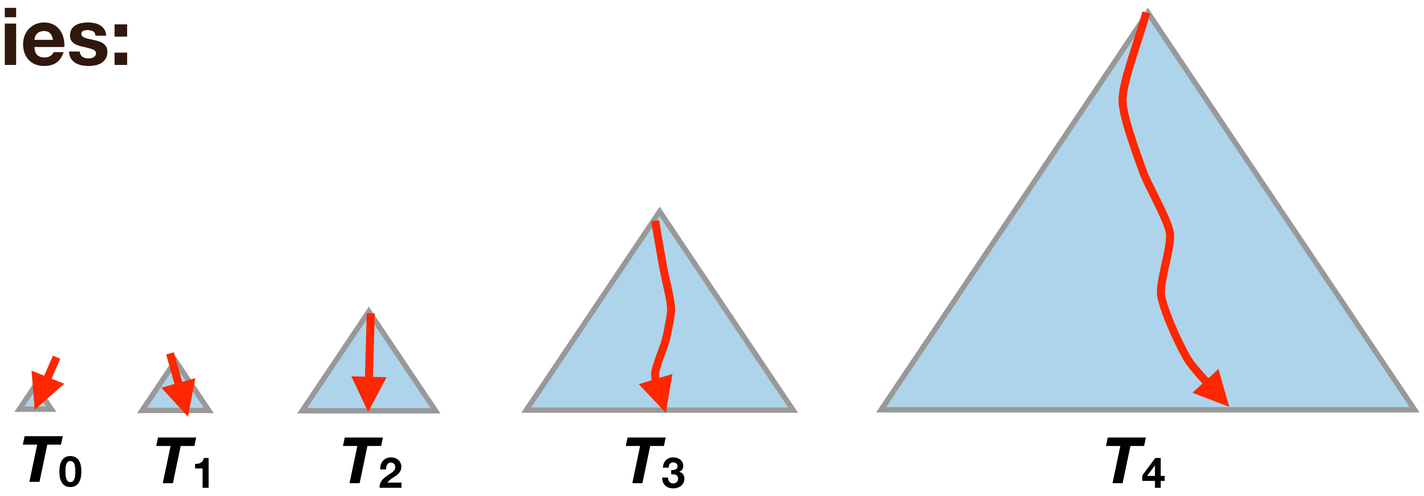
The target sizes are exponentially increasing.

Typically, target size $|T_{j+1}| = 10\,|T_j|$.



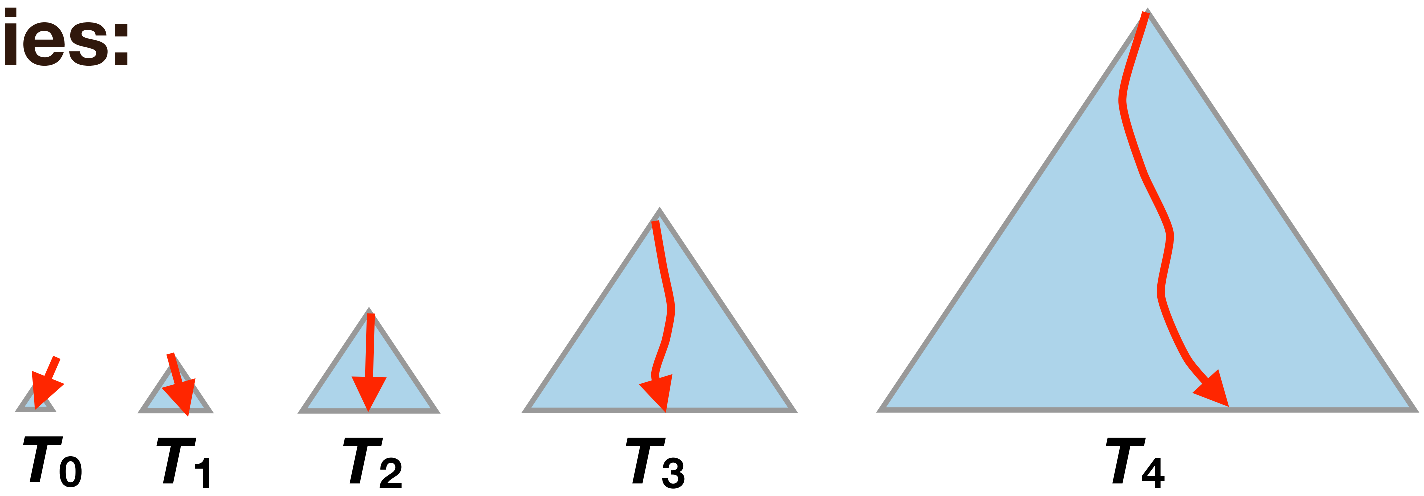
LSM Tree Operations

Point queries:

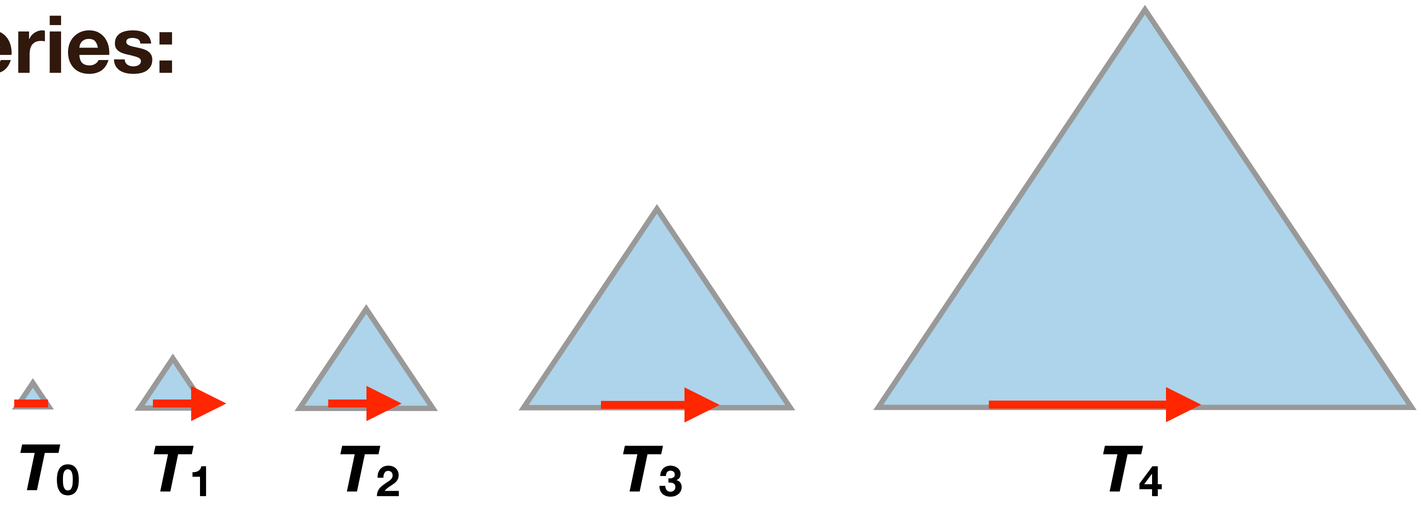


LSM Tree Operations

Point queries:



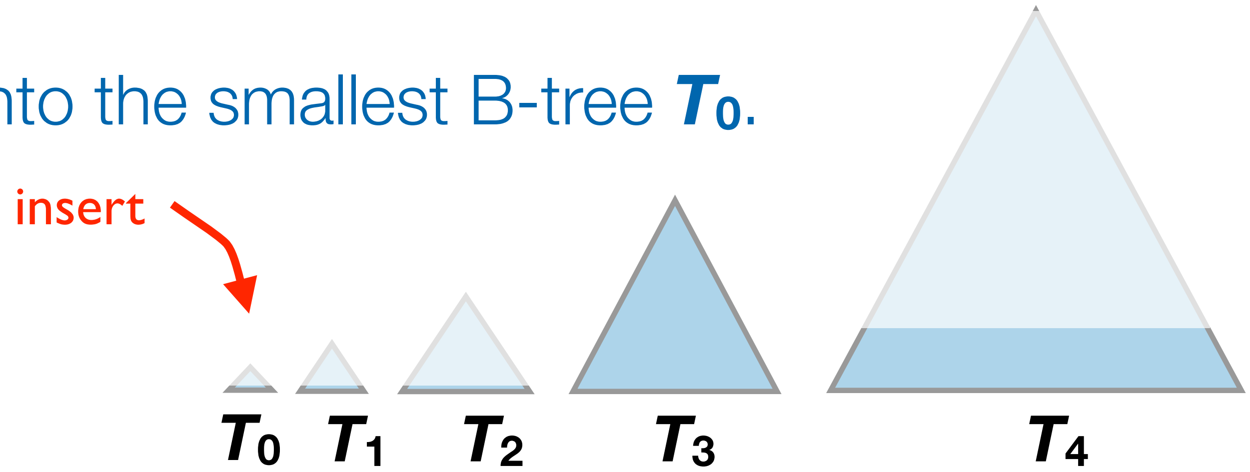
Range queries:



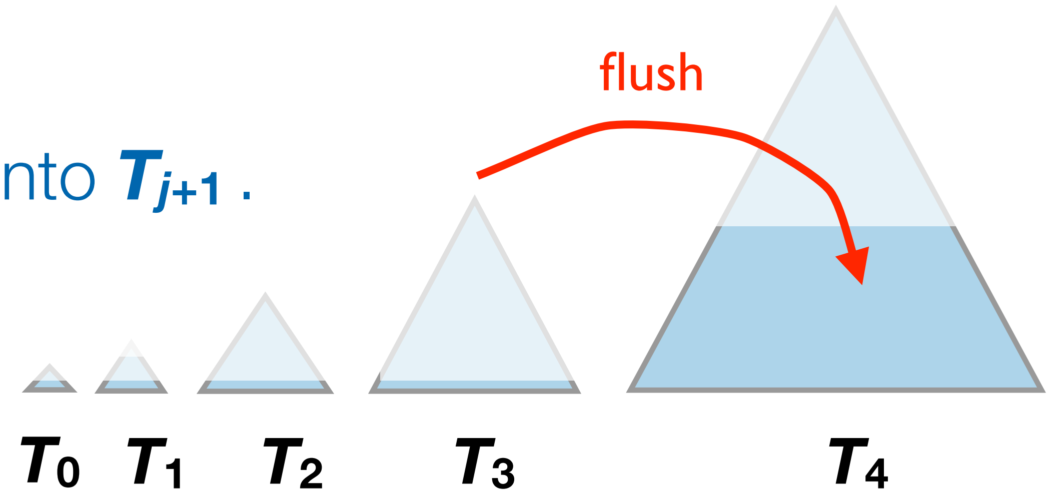
LSM Tree Operations

Insertions:

- Always insert element into the smallest B-tree $\boldsymbol{T_0}$.



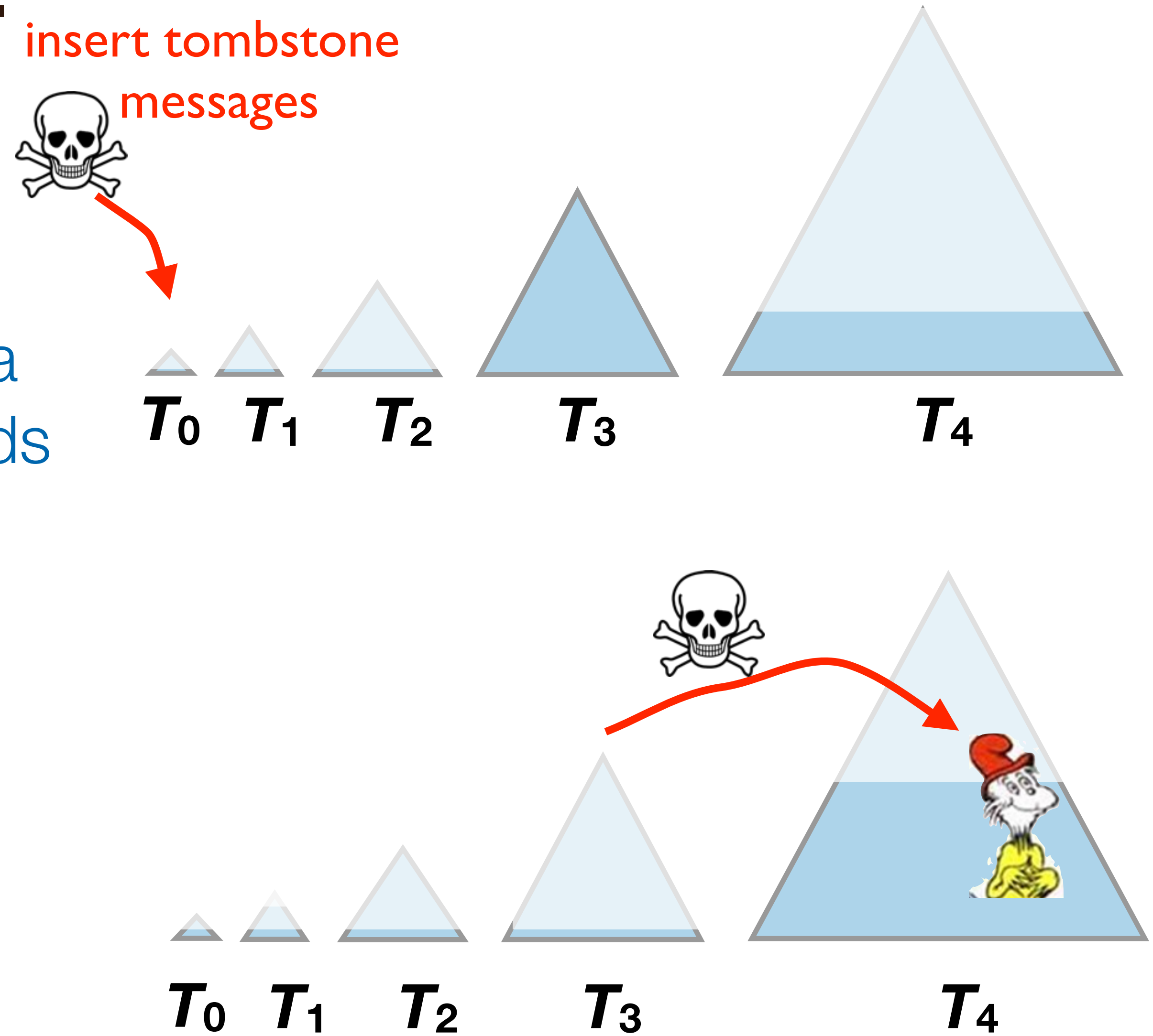
- When a B-tree $\boldsymbol{T_j}$ fills up, flush into $\boldsymbol{T_{j+1}}$.



LSM Tree Operations

Deletes are like inserts:

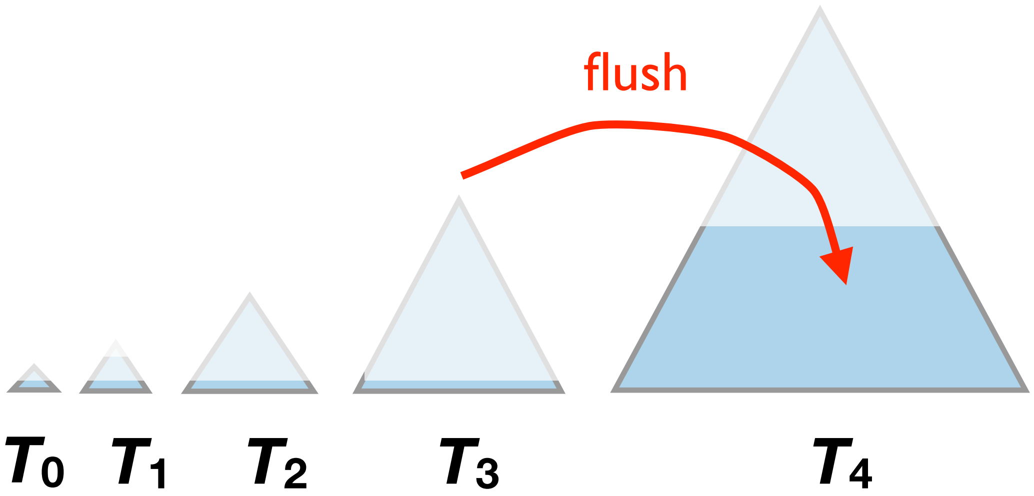
- Instead of deleting an element directly, insert tombstones.
- A tombstone knocks out a "real" element when it lands in the same tree.



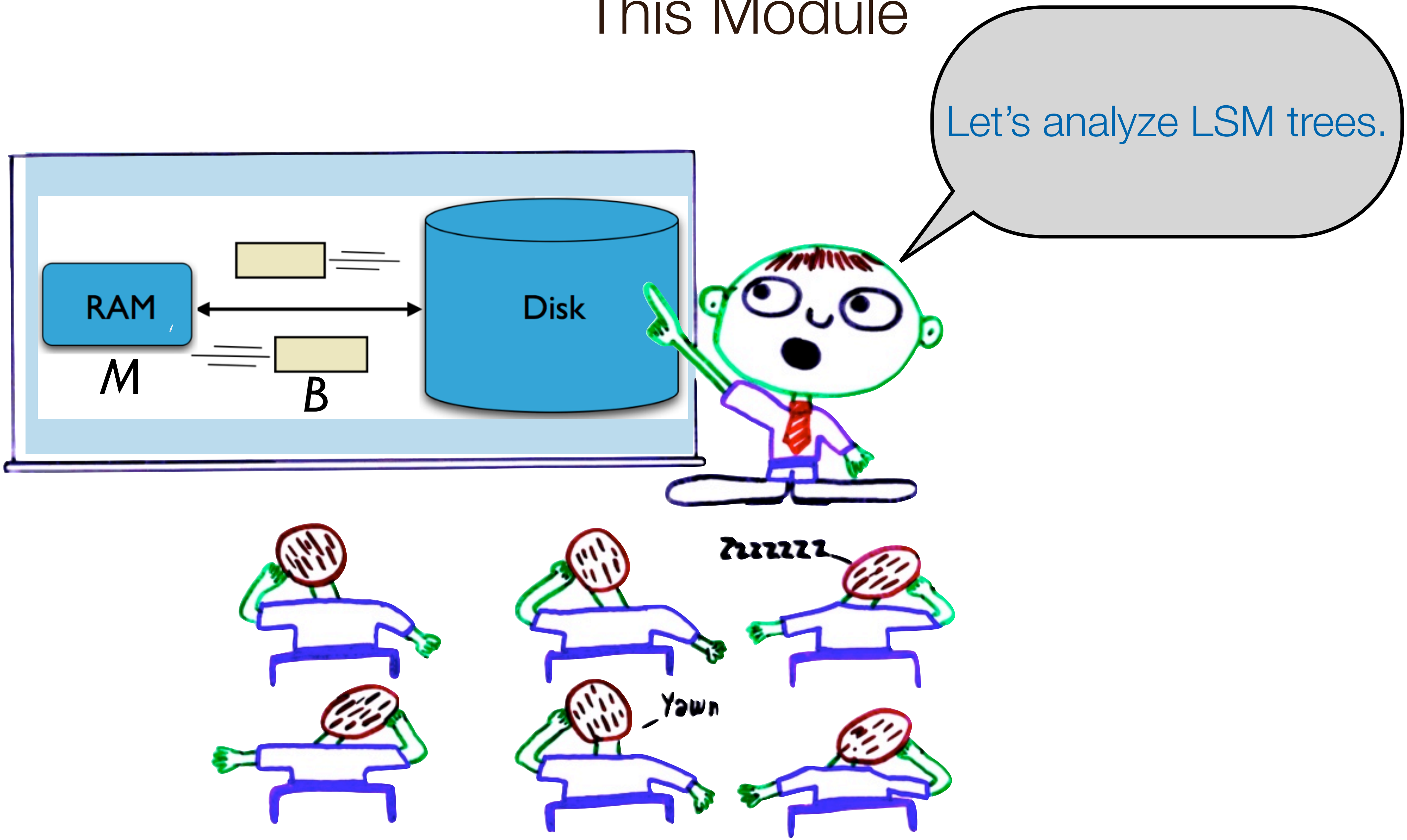
Static-to-Dynamic Transformation

An LSM Tree is an example of a "static-to-dynamic" transformation **[Bentley, Saxe '80]** **.**

- An LSM tree can be built out of ***static B-trees.***
- When $\boldsymbol{T}_3$ flushes into $\boldsymbol{T}_4$, $\boldsymbol{T}_4$ is rebuilt from scratch.

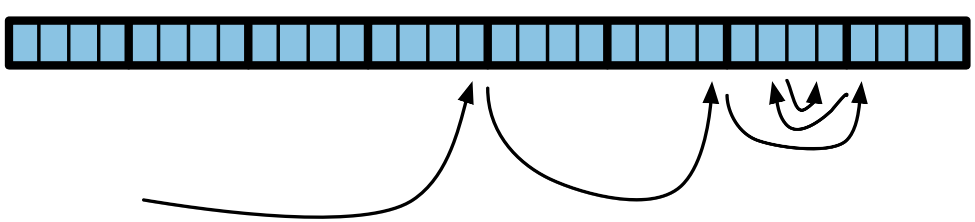


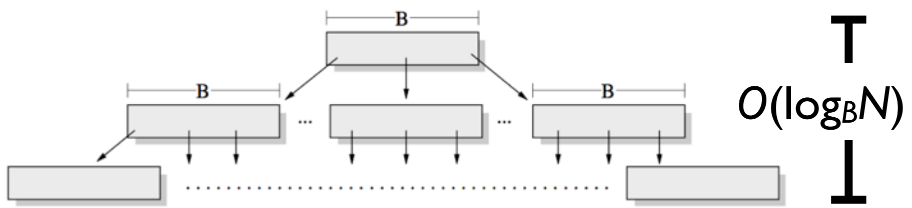
This Module



Recall: Searching in an Array Versus B-tree

Recall the cost of searching in an array versus a B-tree.

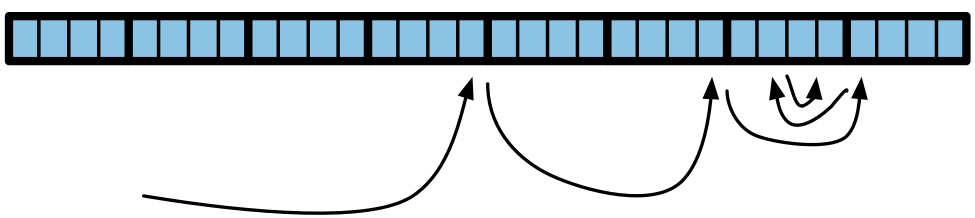




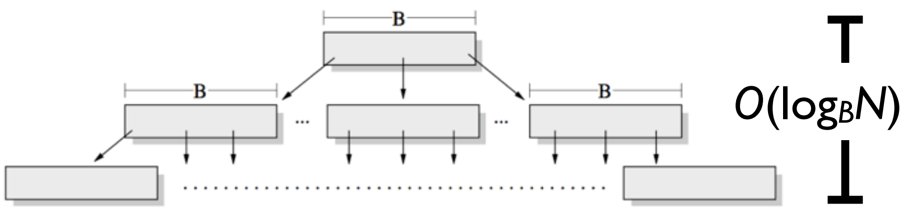
$$O(\log_B N) = O\left(\frac{\log_2 N}{\log_2 B}\right)$$

Recall: Searching in an Array Versus B-tree

Recall the cost of searching in an array versus a B-tree.



$$O\left(\log_2 \frac{N}{B}\right) \approx O(\log_2 N)$$



$$O(\log_B N) = O\left(\frac{\log_2 N}{\log_2 B}\right)$$

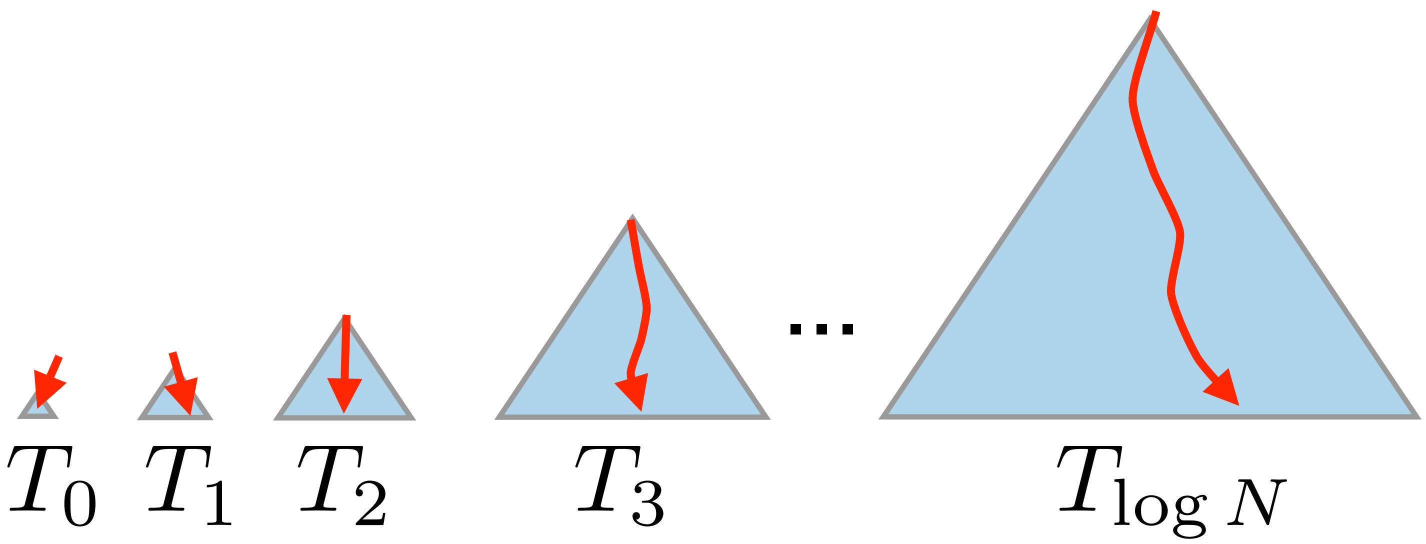
Analysis of point queries

Search cost:

$$\log_B N + \log_B N/2 + \log_B N/4 + \cdots + \log_B B$$

$$= \frac{1}{\log B} (\log N + \log N - 1 + \log N - 2 + \log N - 3 + \cdots + 1)$$

$$= O(\log N \log_B N)$$



Insert Analysis

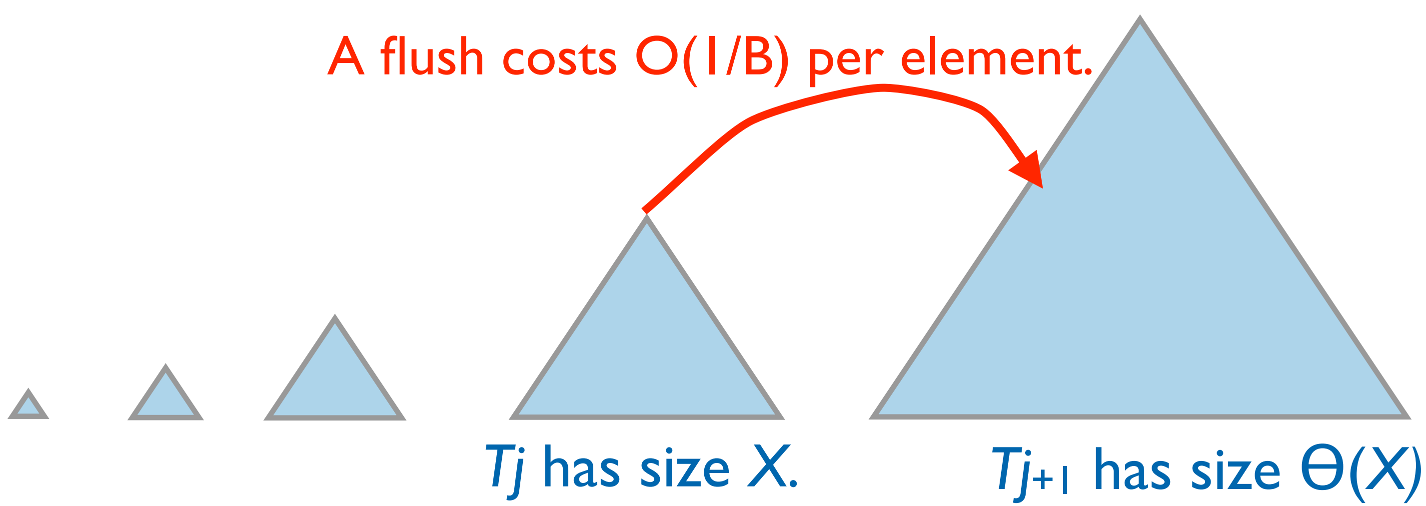
The cost to flush a tree *Tj* of size *X* is O(*X*/*B*).

- Flushing and rebuilding a tree is just a linear scan.

The cost per element to flush *Tj* is O(*1*/*B*).

The # times each element is moved is ≤ log *N*.

The insert cost is O((log *N*)/*B*) amortized memory transfers.



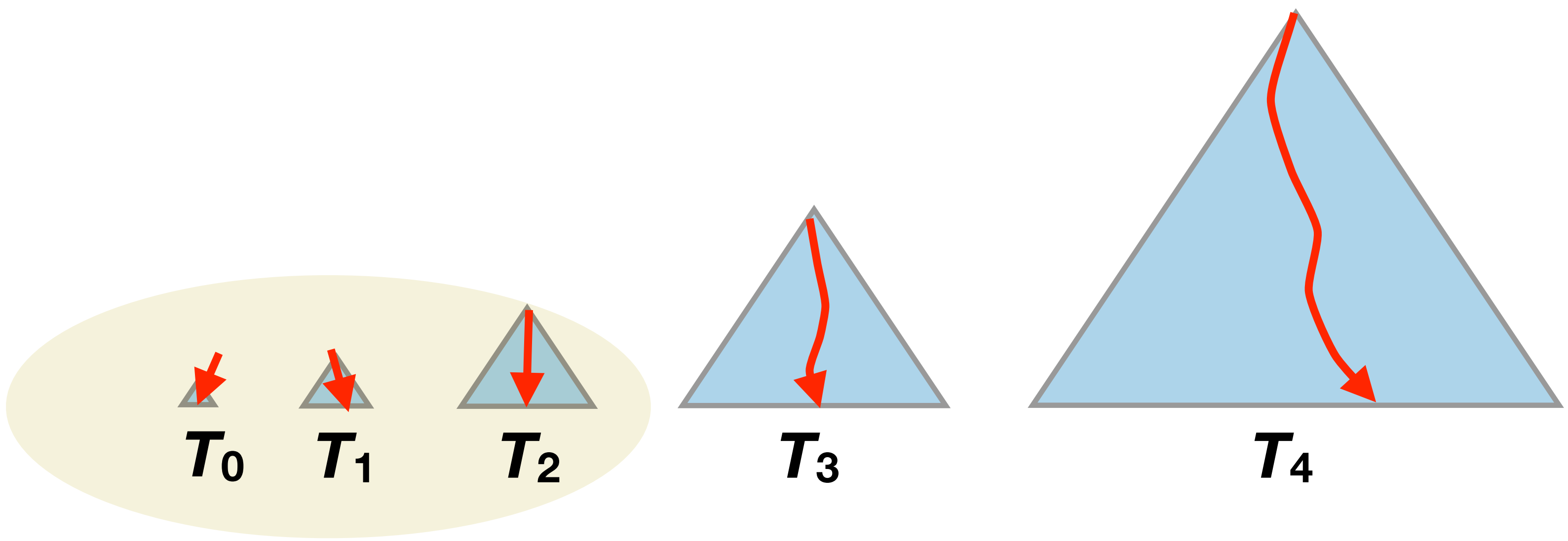
Samples from LSM Tradeoff Curve

	insert	point query
tradeoff (function of ε)	$O\left(\frac{\log_{1+B^\varepsilon} N}{B^{1-\varepsilon}}\right)$	$O\left((\log_B N)(\log_{1+B^\varepsilon} N)\right)$
sizes grow by *B* (ε=1)	$O\left(\log_B N\right)$	$O\left((\log_B N)(\log_B N)\right)$
sizes grow by *B*$^{1/2}$ (ε=1/2)	$O\left(\frac{\log_B N}{\sqrt{B}}\right)$	$O\left((\log_B N)(\log_B N)\right)$
sizes double (ε=0)	$O\left(\frac{\log N}{B}\right)$	$O\left((\log_B N)(\log N)\right)$

How to improve LSM-tree point queries?

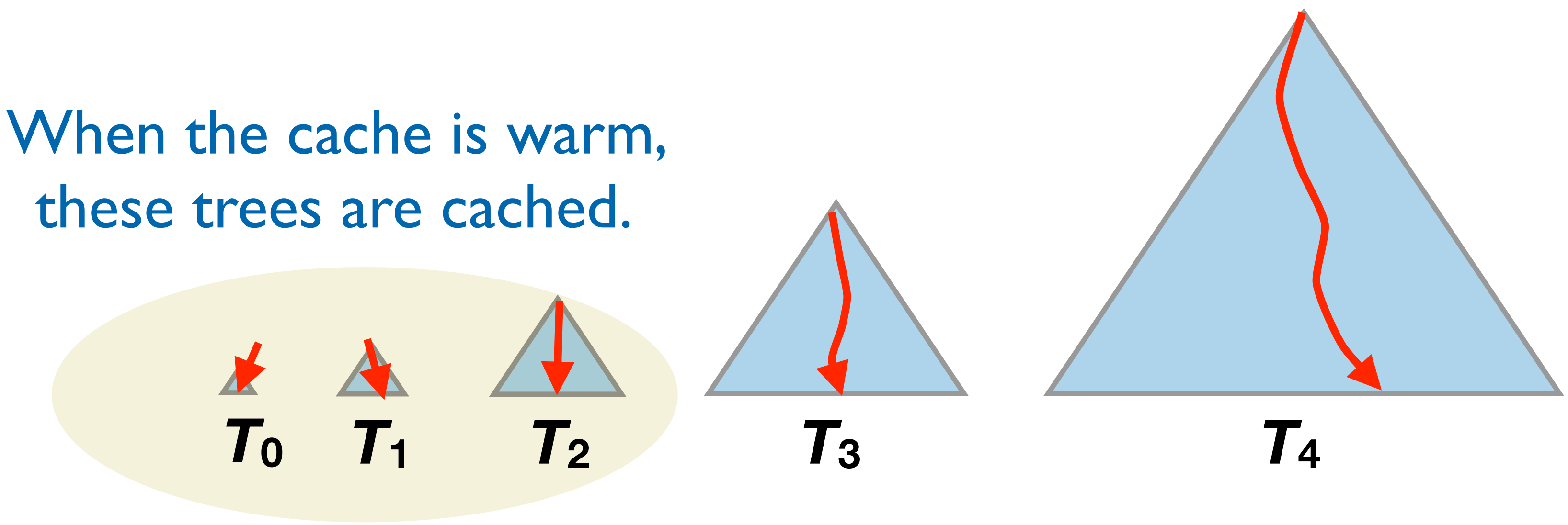
Looking in all those trees is expensive, but can be improved by

- caching,
- Bloom filters, and
- fractional cascading.



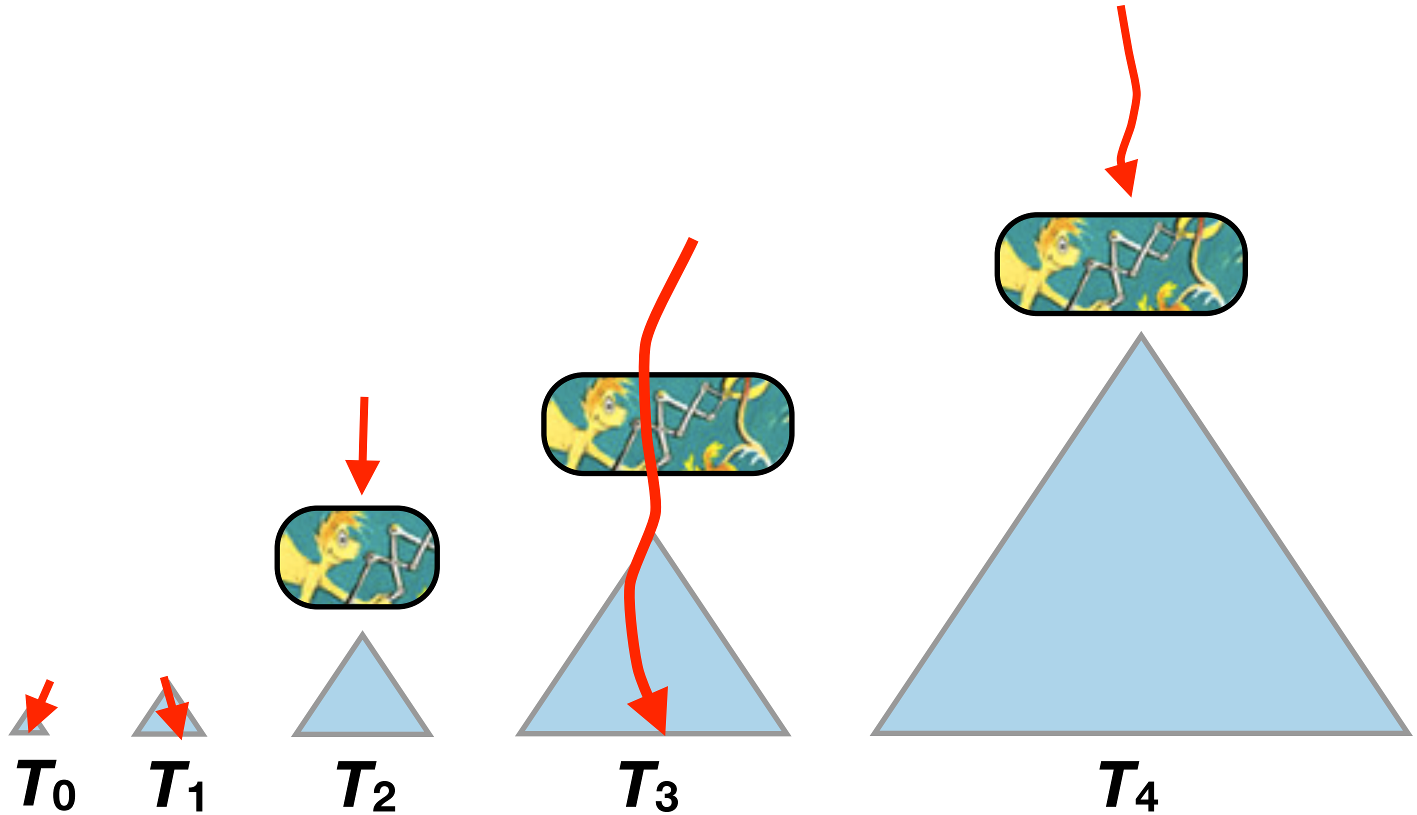
Caching in LSM trees

When the cache is warm, small trees are cached.



Bloom filters in LSM trees

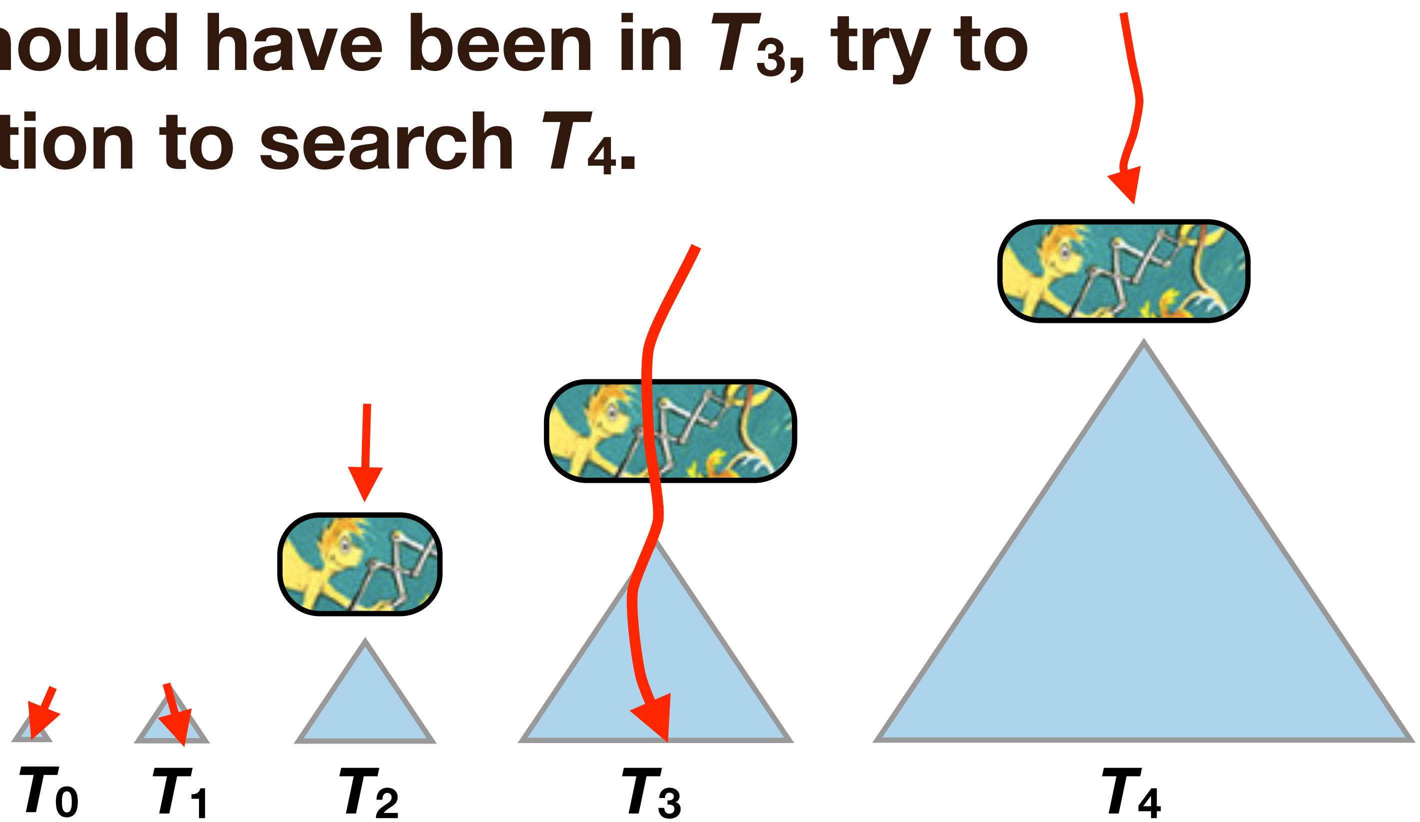
Bloom filters can avoid point queries for elements that are not in a particular B-tree.



Fractional cascading reduces the cost in each tree

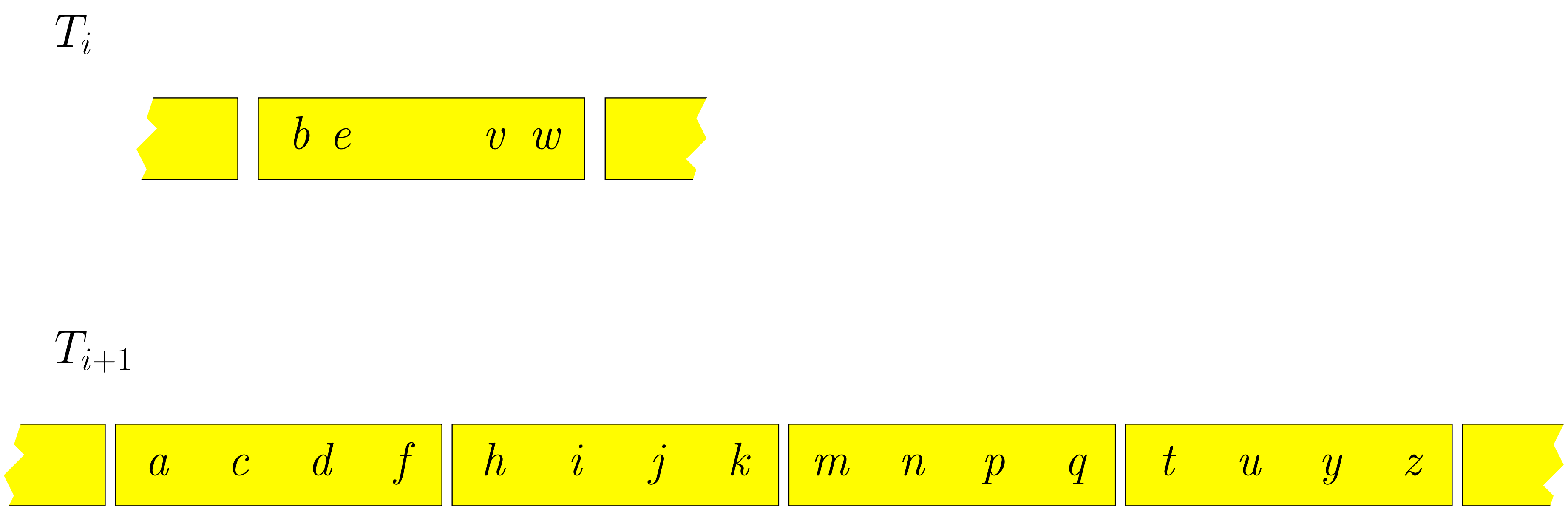
Instead of avoiding searches in trees, we can use a technique called *fractional cascading* to reduce the cost of searching each B-tree to $O(1)$.

Idea: We're looking for a key, and we already know where it should have been in T_3, try to use that information to search T_4.



Searching one tree helps in the next

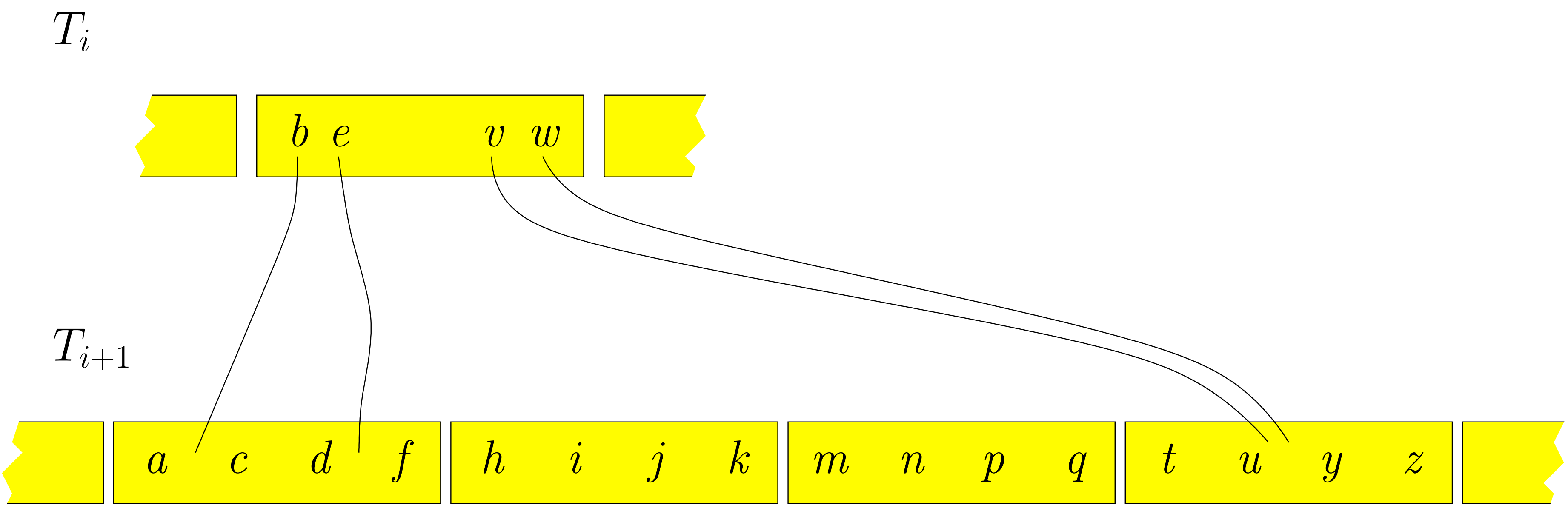
Looking up *c*, in *Ti* we know it's between *b*, and *e*.



Showing only the bottom level of each B-tree.

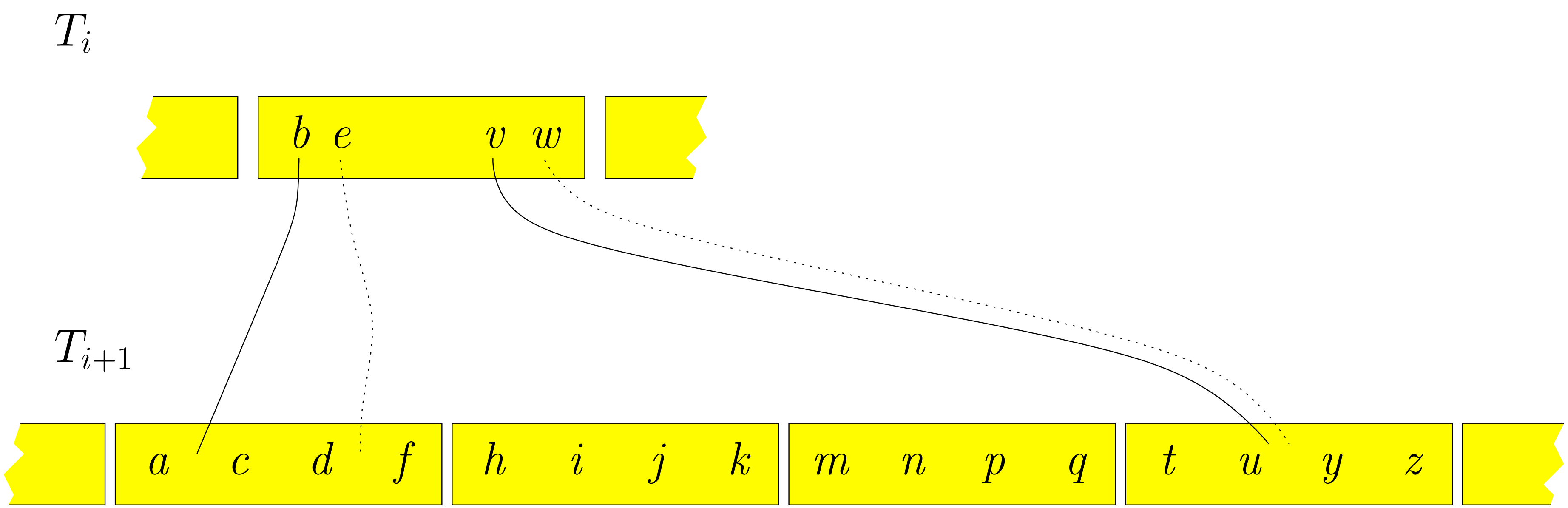
Forwarding pointers

If we add *forwarding pointers* to the first tree, we can jump straight to the node in the second tree, to find *c*.



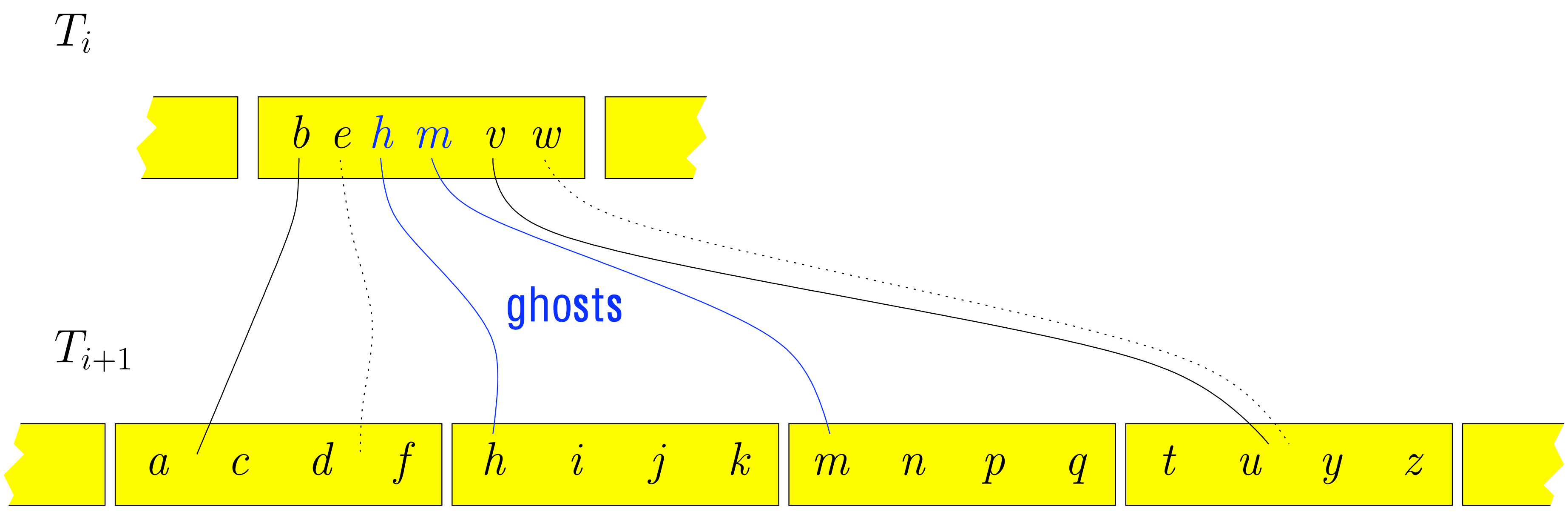
Remove redundant forwarding pointers

We need only one forwarding pointer for each block in the next tree. Remove the redundant ones.



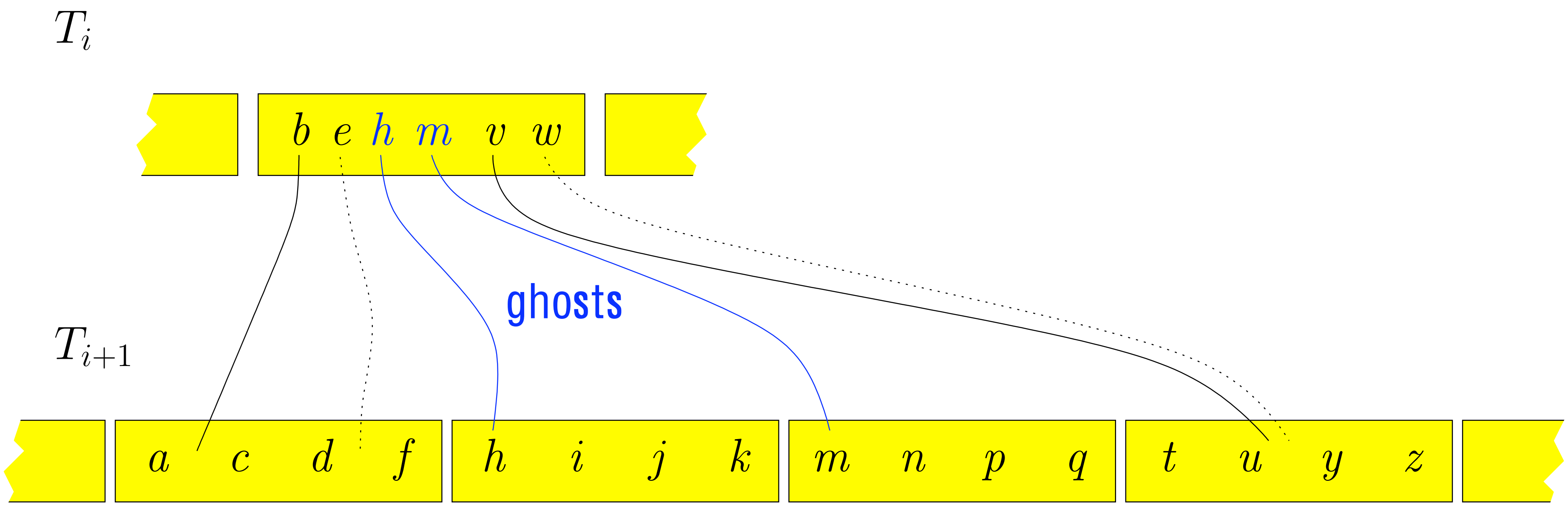
Ghost pointers

We need a forwarding pointer for every block in the next tree, even if there are no corresponding pointers in this tree. Add ghosts.



LSM tree + forward + ghost = fast queries

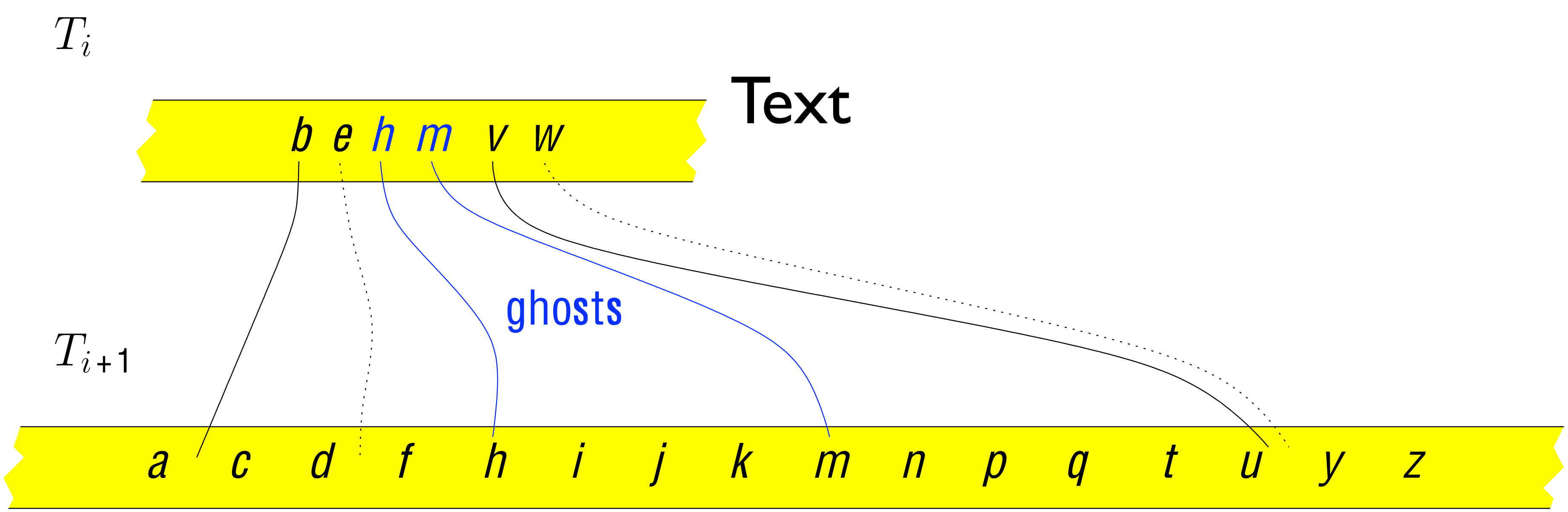
With forward pointers and ghosts, LSM trees require only one I/O per tree, and point queries cost only $O(\log_R N)$**.**



[Bender, Farach-Colton, Fineman, Fogel, Kuszmaul, Nelson 07]

LSM tree + forward + ghost = COLA

This data structure no longer uses the internal nodes of the B-trees, and each of the trees can be implemented by an array.



[Bender, Farach-Colton, Fineman, Fogel, Kuszmaul, Nelson 07]

Data Structures and Algorithms for Big Data

Module 6: What to Index

Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

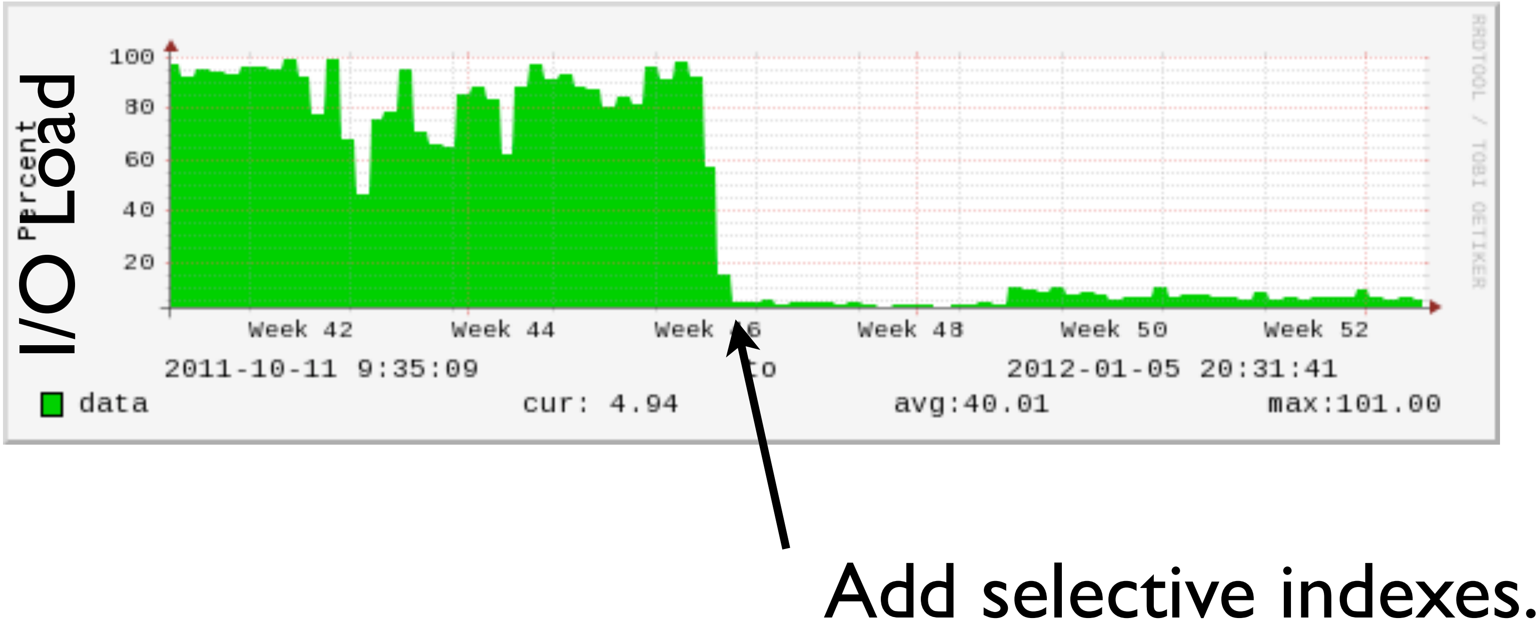
Story of this module

This module explores indexing.

Traditionally, (with B-trees), indexing improves queries, but cripples insertions.

But now we know that maintaining indexes is cheap. So what should we index?

An Indexing Testimonial



This is a graph from a real user, who added some indexes, and reduced the I/O load on their server. (They couldn't maintain the indexes with B-trees.)

What is an Index?

To understand what to index, we need to get on the same page for what an index is.

Row, Index, and Table

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

Row

- Key,value pair
- key = a, value = b,c

Index

- Ordering of rows by key (dictionary)
- Used to make queries fast

Table

- Set of indexes

```
create table foo (a int, b int, c int,
primary key(a));
```

An index is a dictionary

Dictionary API: maintain a set S subject to

- insert(x): $S \leftarrow S \cup \{x\}$
- delete(x): $S \leftarrow S - \{x\}$
- search(x): is $x \in S$?
- successor(x): return min $y > x$ s.t. $y \in S$
- predecessor(y): return max $y < x$ s.t. $y \in S$

We assume that these operations perform as well as a B-tree. For example, the successor operation usually doesn't require an I/O.

A table is a set of indexes

A table is a set of indexes with operations:

- Add index: `add key(f₁,f₂,...);`
- Drop index: `drop key(f₁,f₂,...);`
- Add column: adds a field to primary key value.
- Remove column: removes a field and drops all indexes where field is part of key.
- Change field type
- ...

Subject to index correctness constraints.

We want table operations to be fast too.

Next: how to use indexes to improve queries.

Indexes provide query performance

1. Indexes can reduce the amount of retrieved data.

- Less bandwidth, less processing, ...

2. Indexes can improve locality.

- Not all data access cost is the same
- Sequential access is MUCH faster than random access

3. Indexes can presort data.

- GROUP BY and ORDER BY queries do post-retrieval work
- Indexing can help get rid of this work

Indexes provide query performance

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3. Indexes can presort data.

- GROUP BY and ORDER BY queries do post-retrieval work
- Indexing can help get rid of this work

An index can select needed rows

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

```
count (*) where a<120;
```

An index can select needed rows

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

100	5	45
101	92	2

2

```
count (*) where a<120;
```

No good index means slow table scans

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

```
count (*) where b>50 and b<100;
```

No good index means slow table scans

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

3

```
count (*) where b>50 and b<100;
```

You can add an index

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

```
alter table foo add key(b);
```

A selective index speeds up queries

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

```
count (*) where b>50 and b<100;
```

A selective index speeds up queries

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	156
56	256
92	101

56	156
56	256
92	101

3

```
count (*) where b>50 and b<100;
```

Selective indexes can still be slow

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

```
sum(c) where b>50;
```

Selective indexes can still be slow

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

56	156
56	256
92	101
202	198

Selecting on b: fast

```
sum(c) where b>50;
```

Selective indexes can still be slow

56	156
56	256
92	101
202	198

Selecting on b: fast

Fetching info for summing c: slow

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

```
sum(c) where b>50;
```

Selective indexes can still be slow

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

56	156
56	256
92	101
202	198

Selecting on b: fast

156	56	45

Fetching info for summing c: slow

```
sum(c) where b>50;
```

Selective indexes can still be slow

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b	a
5	100
6	165
23	206
43	412
56	156
56	256
92	101
202	198

56	156
56	256
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5	100
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23	206
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56	156
56	256
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↓

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Selective indexes can still be slow

Poor data locality

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Selecting on b: fast

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56	256
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56	156
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92	101
202	198

Selecting on b: fast

↓

156	56	45
256	56	2
101	92	2
198	202	56

Fetching info for summing c: slow

↓

105

```
sum(c) where b>50;
```

Indexes provide query performance

1. Indexes can reduce the amount of retrieved data.

- Less bandwidth, less processing, ...

2. Indexes can improve locality.

- Not all data access cost is the same
- Sequential access is MUCH faster than random access

3. Indexes can presort data.

- GROUP BY and ORDER BY queries do post-retrieval work
- Indexing can help get rid of this work

Covering indexes speed up queries

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b,c	a
5,45	100
6,2	165
23,252	206
43,45	412
56,2	256
56,45	156
92,2	101
202,56	198

```
alter table foo add key(b,c);
sum(c) where b>50;
```

Covering indexes speed up queries

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b,c	a
5,45	100
6,2	165
23,252	206
43,45	412
56,2	256
56,45	156
92,2	101
202,56	198

56,2	256
56,45	156
92,2	101
202,56	198

↓

105

```
alter table foo add key(b,c);
sum(c) where b>50;
```

Indexes provide query performance

1. Indexes can reduce the amount of retrieved data.

- Less bandwidth, less processing, ...

2. Indexes can improve locality.

- Not all data access cost is the same
- Sequential access is MUCH faster than random access

3. Indexes can presort data.

- GROUP BY and ORDER BY queries do post-retrieval work
- Indexing can help get rid of this work

Indexes can avoid post-selection sorts

a	b	c
100	5	45
101	92	2
156	56	45
165	6	2
198	202	56
206	23	252
256	56	2
412	43	45

b,c	a
5,45	100
6,2	165
23,252	206
43,45	412
56,2	256
56,45	156
92,2	101
202,56	198

b	sum(c)
5	45
6	2
23	252
43	45
56	47
92	2
202	56

```
select b, sum(c) group by b;
```

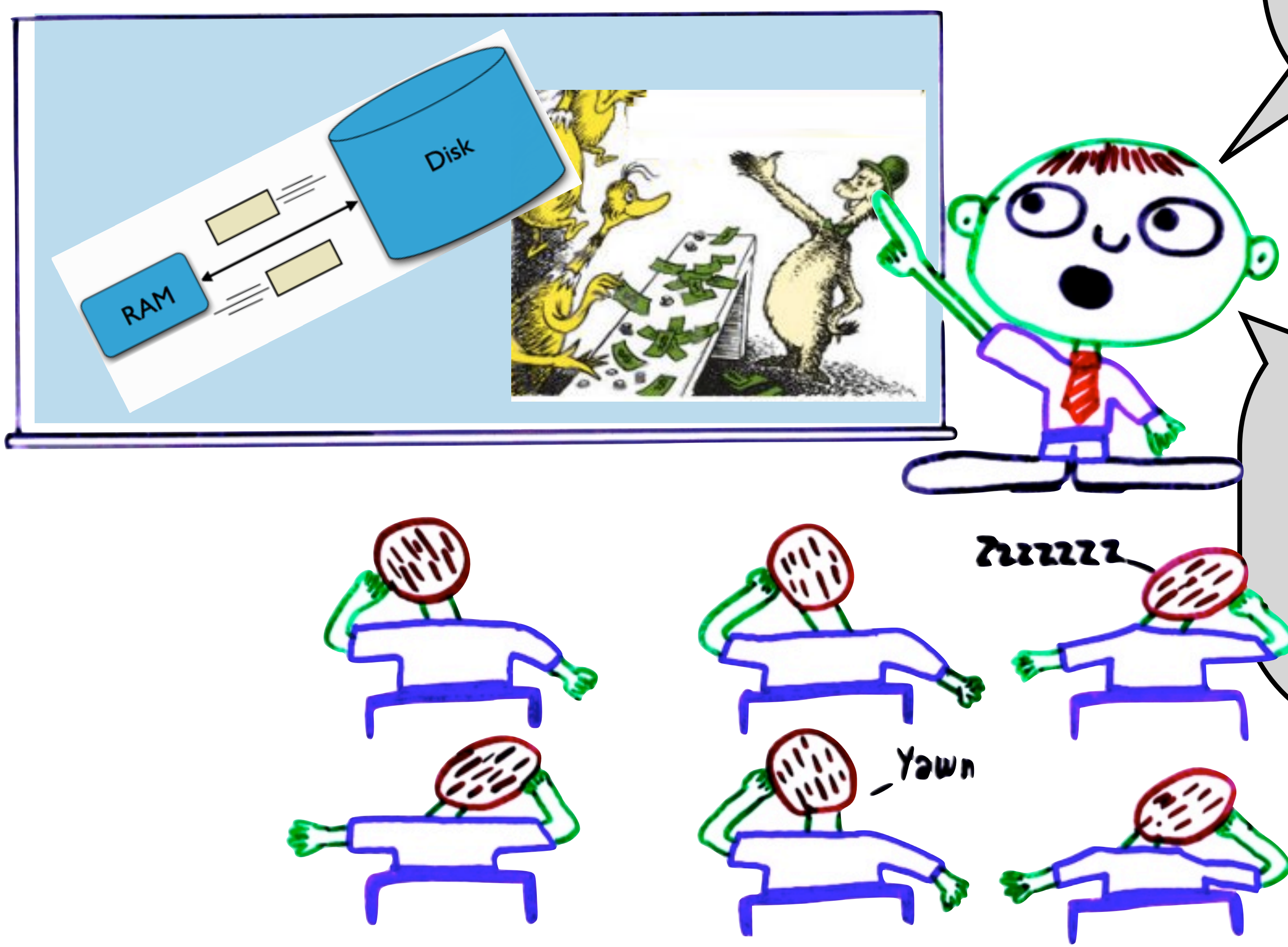
Data Structures and Algorithms for Big Data
Module 7: Paging

Michael A. Bender
Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

This Module

The algorithmics of cache-management.



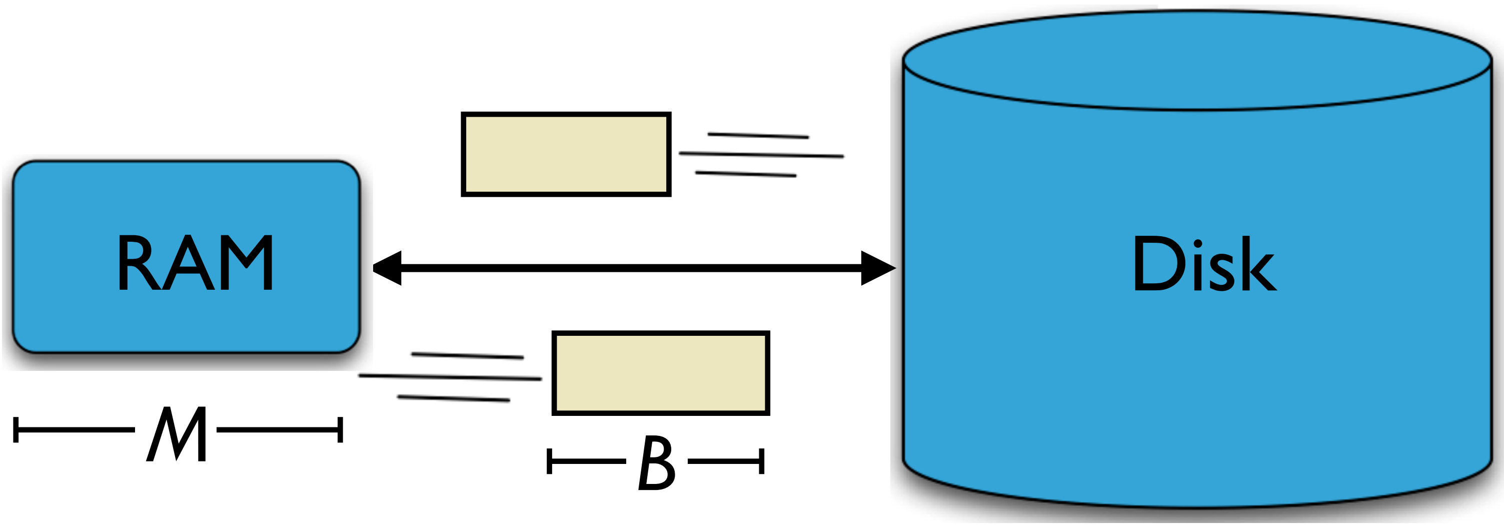
This will help us understand I/O- and cache-efficient algorithms.

Recall Disk Access Model

Goal: minimize # block transfers.

- Data is transferred in blocks between RAM and disk.
- Performance bounds are parameterized by B, M, N.

When a block is cached, the access cost is 0. Otherwise it's 1.



[Aggarwal+Vitter '88]

Recall Cache-Oblivious Analysis

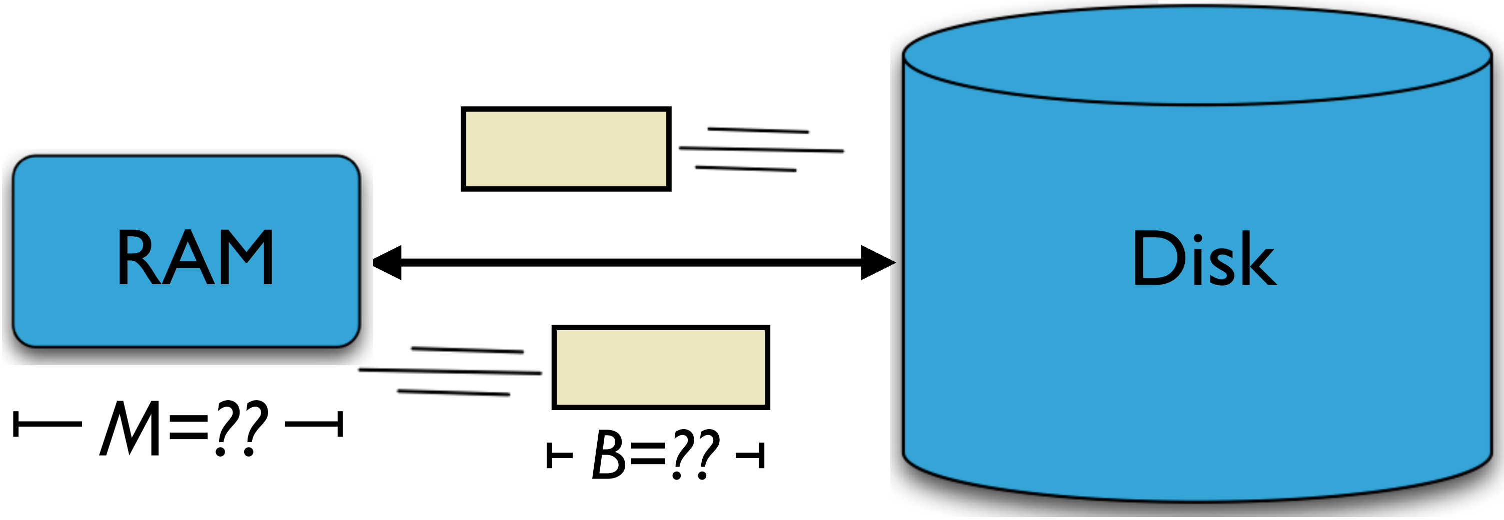
Disk Access Model (DAM Model):

- Performance bounds are parameterized by *B*, *M*, *N*.

Goal: Minimize # of block transfers.

Beautiful restriction:

- Parameters *B*, *M* are unknown to the algorithm or coder.



[Frigo, Leiserson, Prokop, Ramachandran '99]

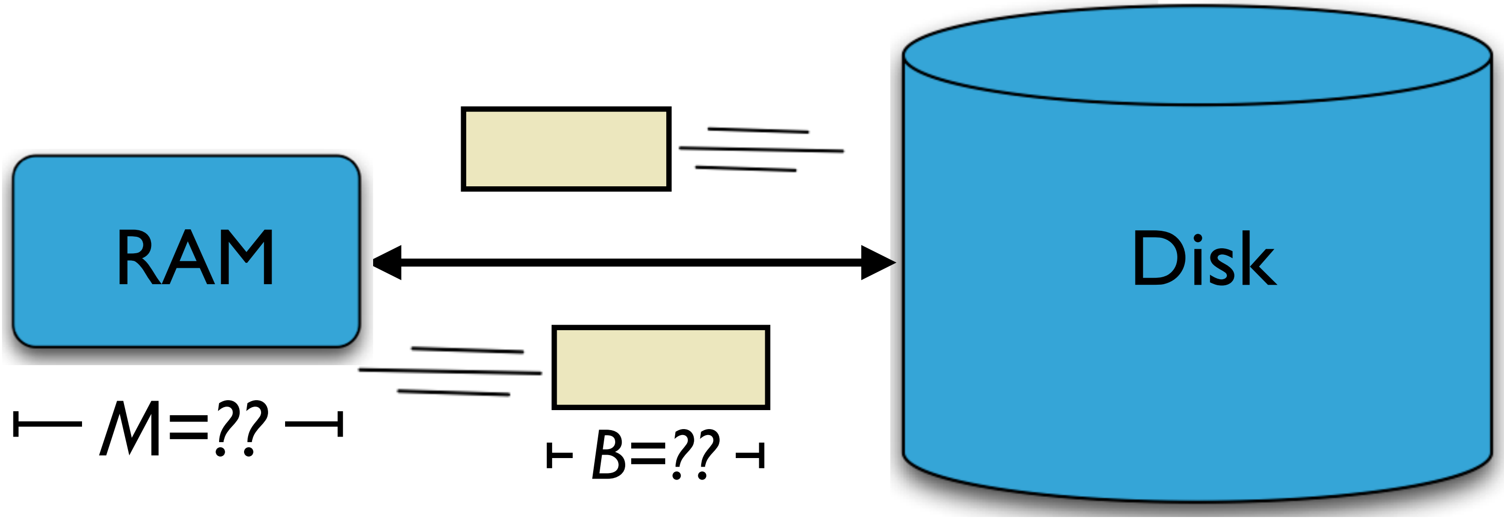
Recall Cache-Oblivious Analysis

CO analysis applies to unknown multilevel hierarchies:

- Cache-oblivious algorithms work for all *B* and *M*...
- ... and all levels of a multi-level hierarchy.

Moral:

- It's better to optimize approximately for all *B*, *M* rather than to try to pick the best *B* and *M*.

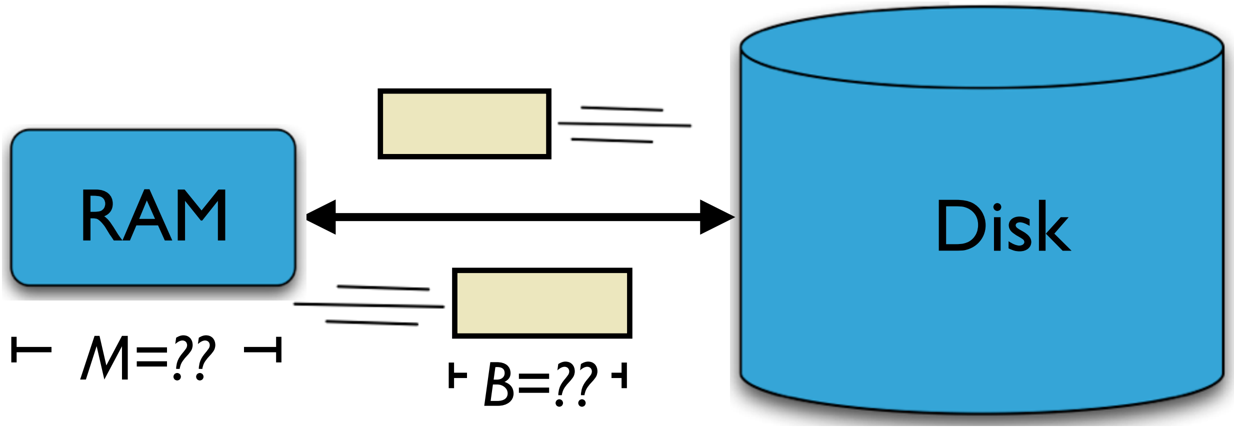


[Frigo, Leiserson, Prokop, Ramachandran '99]

Cache-Replacement in Cache-Oblivious Algorithms

Which blocks are currently cached in RAM?

- The system performs its own caching/paging.
- If we knew *B* and *M* we could explicitly manage I/O. (But even then, what should we do?)

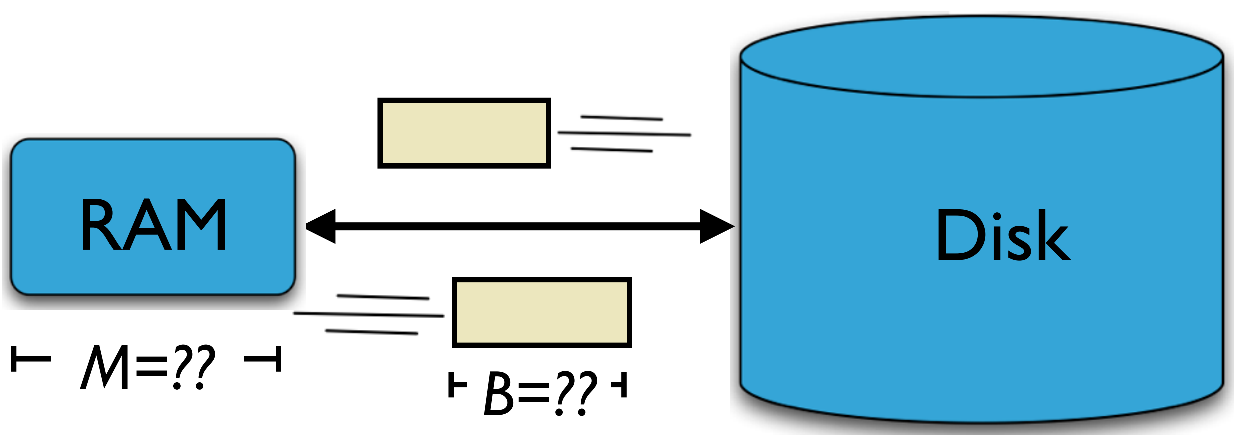


Cache-Replacement in Cache-Oblivious Algorithms

Which blocks are currently cached in RAM?

- The system performs its own caching/paging.
- If we knew B and M we could explicitly manage I/O. (But even then, what should we do?)

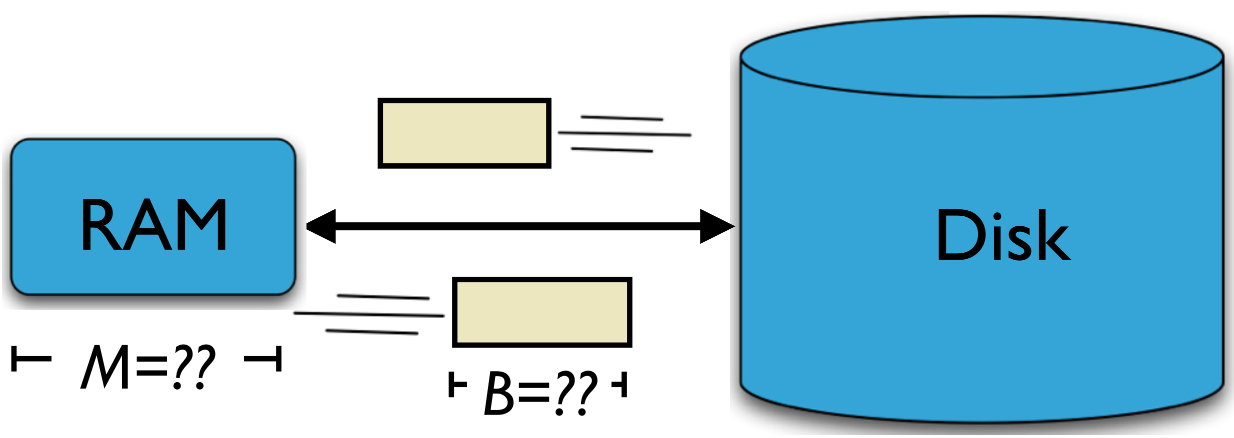
But systems may use different mechanisms, so what can we actually assume?



This Module: Cache-Management Strategies

With cache-oblivious analysis, we can assume a memory system with optimal replacement.

Even though the system manages memory, we can assume all the advantages of explicit memory management.



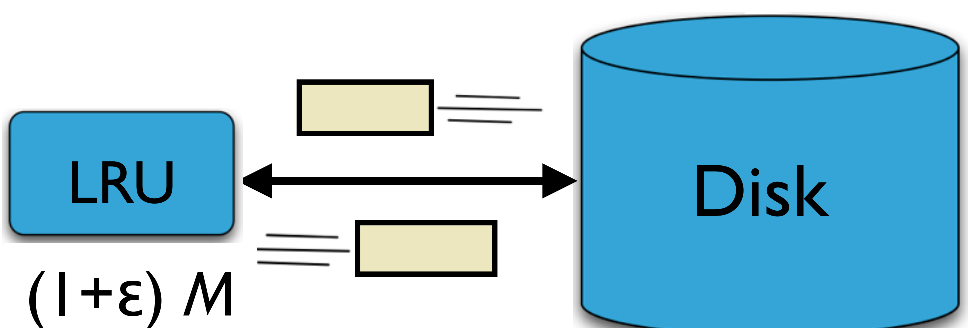
This Module: Cache-Management Strategies

An LRU-based system with memory *M* performs cache-management < 2x worse than the optimal, prescient policy with memory *M*/2.

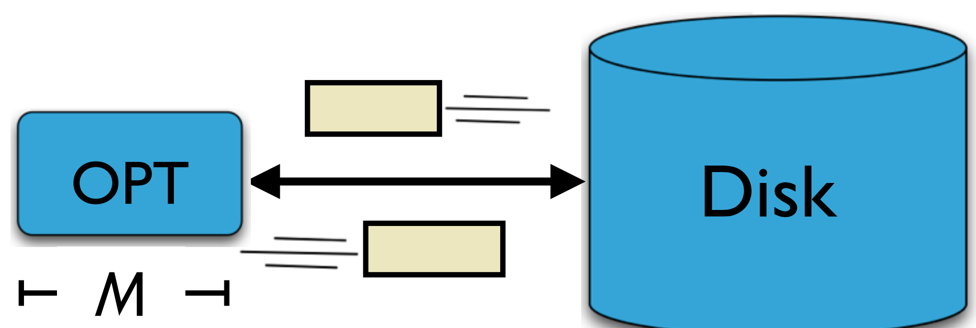
Achieving optimal cache-management is hard because predicting the future is hard.

But LRU with (1+ε)*M* memory is almost as good (or better), than the optimal strategy with *M* memory.

[Sleator, Tarjan 85]



LRU with (1+ε) more memory is nearly as good or better...



... than OPT.

The paging/caching problem

A *program* is just sequence of block requests:

$$r_1, r_2, r_3, \ldots$$

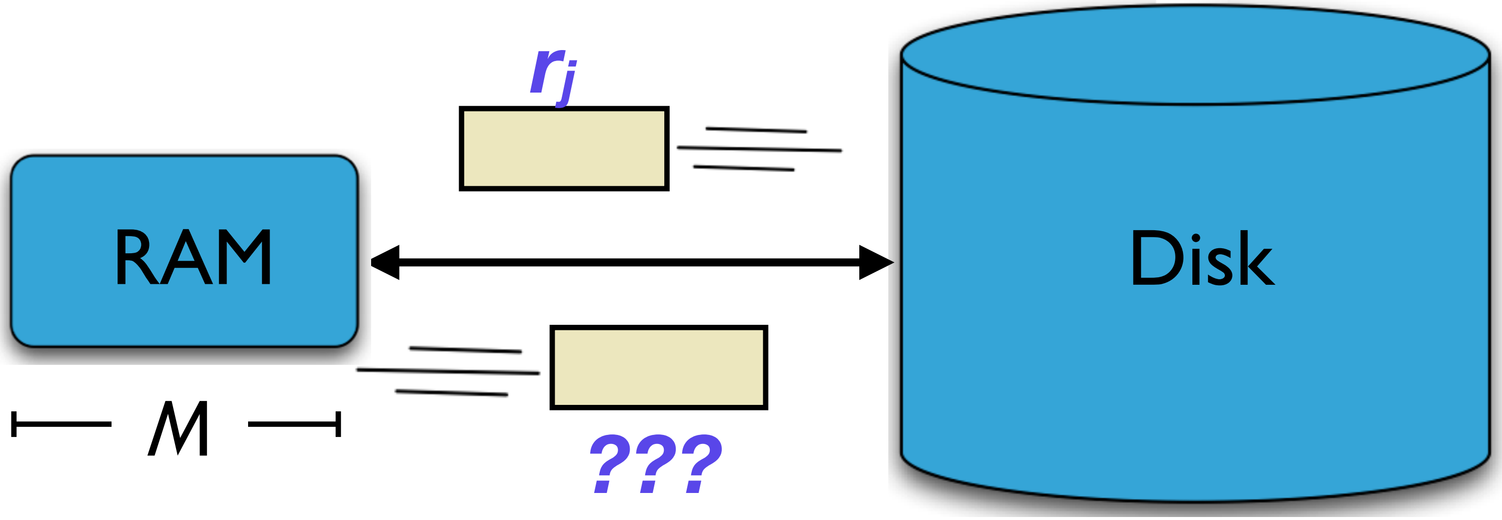
Cost of request *r_j*

$$\text{cost}(r_j) = \begin{cases} 0 & \text{block } r_j \text{ is already cached,} \\ 1 & \text{block } r_j \text{ is brought into cache.} \end{cases}$$

The paging/caching problem

RAM holds only *k=M/B* blocks.

Which block should be ejected when block r_j is brought into cache?



Paging Algorithms

LRU (least recently used)

- Discard block whose most recent access is earliest.

FIFO (first in, first out)

- Discard the block brought in longest ago.

LFU (least frequently used)

- Discard the least popular block.

Random

- Discard a random block.

LFD (longest forward distance)=OPT [Belady 69]

- Discard block whose next access is farthest in the future.

Optimal Page Replacement

LFD (Longest Forward Distance) [Belady '69]**:**

- Discard the block requested farthest in the future.

Optimal Page Replacement

LFD (Longest Forward Distance) [Belady ’69]**:**

- Discard the block requested farthest in the future.

Cons: Who knows the Future?!

Optimal Page Replacement

LFD (Longest Forward Distance) [Belady '69]**:**

- Discard the block requested farthest in the future.

Cons: Who knows the Future?!

Pros: LFD can be viewed as a point of comparison with online strategies.

Competitive Analysis

An online algorithm *A* is *k-competitive*, if for every request sequence *R*:

$$\mathrm{cost}_A(R) \leq k\,\mathrm{cost}_{\mathrm{opt}}(R)$$

Idea of competitive analysis:

- The optimal (prescient) algorithm is a yardstick we use to compare online algorithms.

LRU is no better than *k*-competitive

Memory holds 3 blocks

$$M/B = k = 3$$

The program accesses 4 different blocks

$$r_j \in \{1, 2, 3, 4\}$$

The request stream is

$$1, 2, 3, 4, 1, 2, 3, 4, \cdots$$

LRU is no better than k-competitive

requests	1	2	3	4	1	2	3	4	1	2
blocks in memory of size 3	1	1	1		1	1	1		1	1
		2	2	2		2	2	2		2
			3	3	3		3	3	3	
				4	4	4		4	4	4

There's a block transfer at every step because LRU ejects the block that's requested in the next step.

LRU is no better than k-competitive

requests	1	2	3	4	1	2	3	4	1	2
blocks in memory of size 3	1	1	1	1	1	1	1	1	1	1
		2	2	2	2	2				2
			3				3	3	3	3
				4	4	4	4	4	4	4

LFD (longest forward distance) has a block transfer every k=3 steps.

LRU is k-competitive [Sleator, Tarjan 85]

In fact, LRU is *k=M/B*-competitive.

- I.e., LRU has $k=M/B$ times more transfers than OPT.
- A depressing result because k is huge so $k \cdot$ OPT is nothing to write home about.

On the other hand, the LRU bad example is fragile

requests	1	2	3	4	1	2	3	4	1	2
blocks in memory of size 3	1	1	1		1	1	1		1	1
		2	2	2		2	2	2		2
			3	3	3		3	3	3	
				4	4	4		4	4	4

If $k=M/B=4$, not 3, then both LRU and OPT do well.
If $k=M/B=2$, not 3, then neither LRU nor OPT does well.

LRU is 2-competitive with more memory [Sleator, Tarjan 85]

LRU is at most twice as bad as OPT, when LRU has twice the memory.

$$\mathrm{LRU}_{|\mathrm{cache}|=k}(R) \leq 2\,\mathrm{OPT}_{|\mathrm{cache}|=k/2}(R)$$

In general, LRU is nearly as good as OPT when LRU has a little more memory than OPT.

LRU is 2-competitive with more memory [Sleator, Tarjan 85]

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LRU has more memory, but OPT=LFD can see the future.

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LRU has more memory, but OPT=LFD can see the future.

In general, LRU is nearly as good as OPT when LRU has a little more memory than OPT.

LRU Performance Proof

Divide LRU into phases, each with *k* faults.

$$r_1, r_2, \ldots, r_i, r_{i+1}, \ldots, r_j, r_{j+1}, \ldots, r_\ell, r_{\ell+1}, \ldots$$

LRU Performance Proof

Divide LRU into phases, each with *k* faults.

$$\underbrace{r_1, r_2, \ldots, r_i}, \underbrace{r_{i+1}, \ldots, r_j}, \underbrace{r_{j+1}, \ldots, r_\ell}, \underbrace{r_{\ell+1}, \ldots}$$

OPT[*k*] must have ≥ 1 fault in each phase.

- Case analysis proof.
- LRU is *k*-competitive.

LRU Performance Proof

Divide LRU into phases, each with *k* faults.

$$r_1, r_2, \ldots, r_i, r_{i+1}, \ldots, r_j, r_{j+1}, \ldots, r_\ell, r_{\ell+1}, \ldots$$

OPT[*k*] must have ≥ 1 fault in each phase.

- Case analysis proof.
- LRU is *k*-competitive.

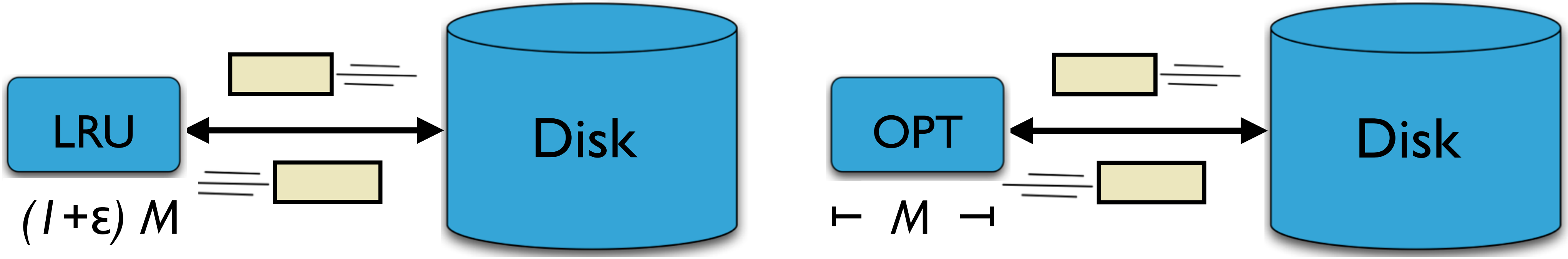
OPT[*k*/2] must have ≥ *k*/2 faults in each phase.

- Main idea: each phase must touch *k* different pages.
- LRU is 2-competitive.

Under the hood of cache-oblivious analysis

Moral: with cache-oblivious analysis, we can analyze based on a memory system with optimal, omniscient replacement.

- Technically, an optimal cache-oblivious algorithm is asymptotically optimal versus any algorithm on a memory system that is slightly smaller.
- Empirically, this is just a technicality.



This is almost as good or better... ... than this.

Ramifications for New Cache-Replacement Policies

Moral: There's not much performance on the table for new cache-replacement policies.

- Bad instances for LRU versus LFD are fragile and depend on a particular cache size.

There are still research questions:

- What if blocks have different sizes [Irani 02][Young 02]?
- There's a write-back cost? (Complexity unknown.)
- LRU may be too costly to implement (clock algorithm).
- The cache-size changes over time.
  [Bender, Ebrahimi, Fineman, Ghasemiesfeh, Johnson, McCauley 13]

Data Structures and Algorithms for Big Data

Module 8: Sorting Big Data

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Stony Brook & Tokutek

Bradley C. Kuszmaul
MIT & Tokutek

Story for Module

Another way to create an index is to sort

- Sorting creates an index all-at-once.
- Sorting does not incrementally maintain an index.
- Sorting is faster than the best algorithms to incrementally maintain an index.

I/O-efficient mergesort

Parallel sort

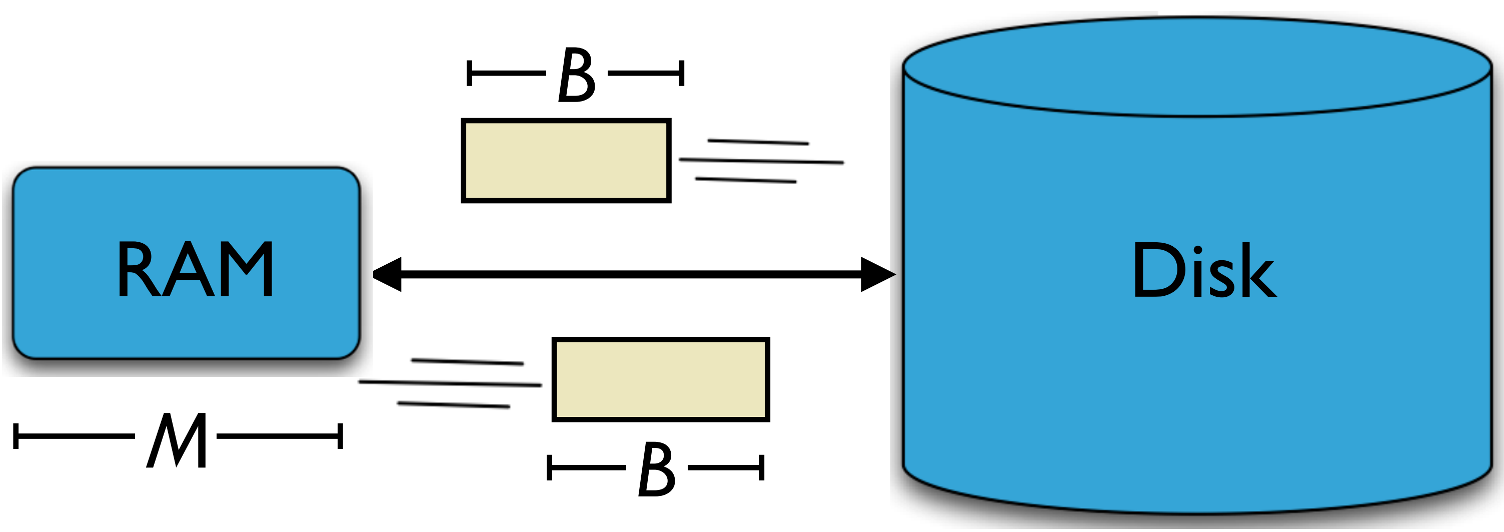
Modeling I/O Using the Disk Access Model

How computation works:

- Data is transferred in blocks between RAM and disk.
- The # of block transfers dominates the running time.

Goal: Minimize # of block transfers

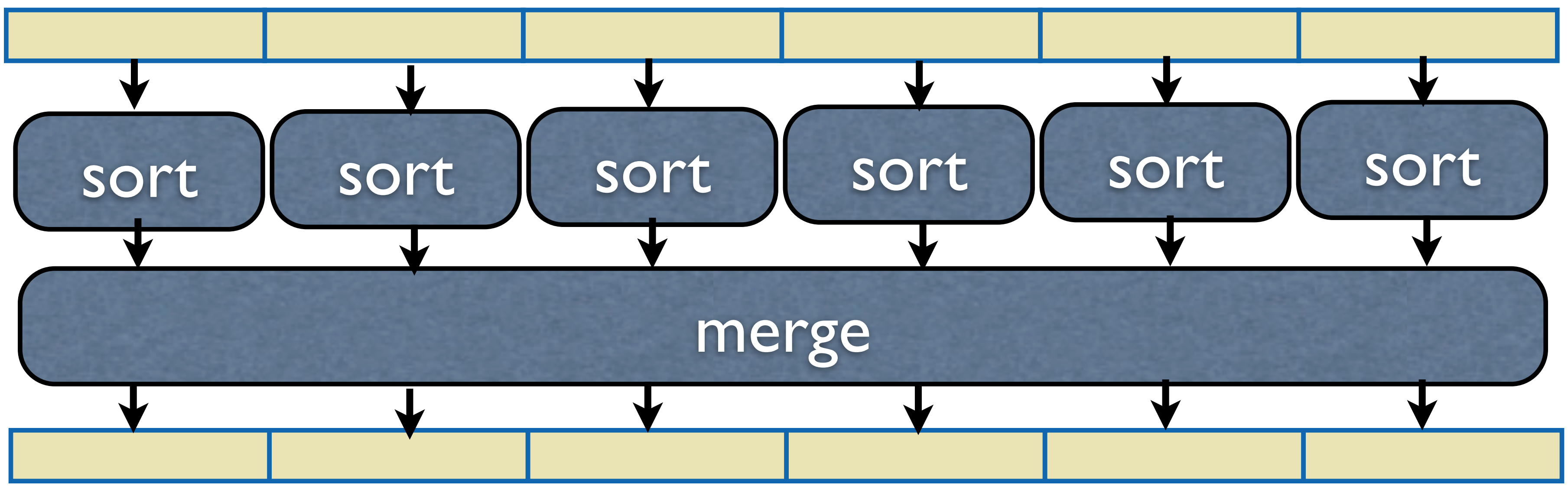
- Performance bounds are parameterized by block size ***B***, memory size ***M***, data size ***N***.



[Aggarwal+Vitter '88]

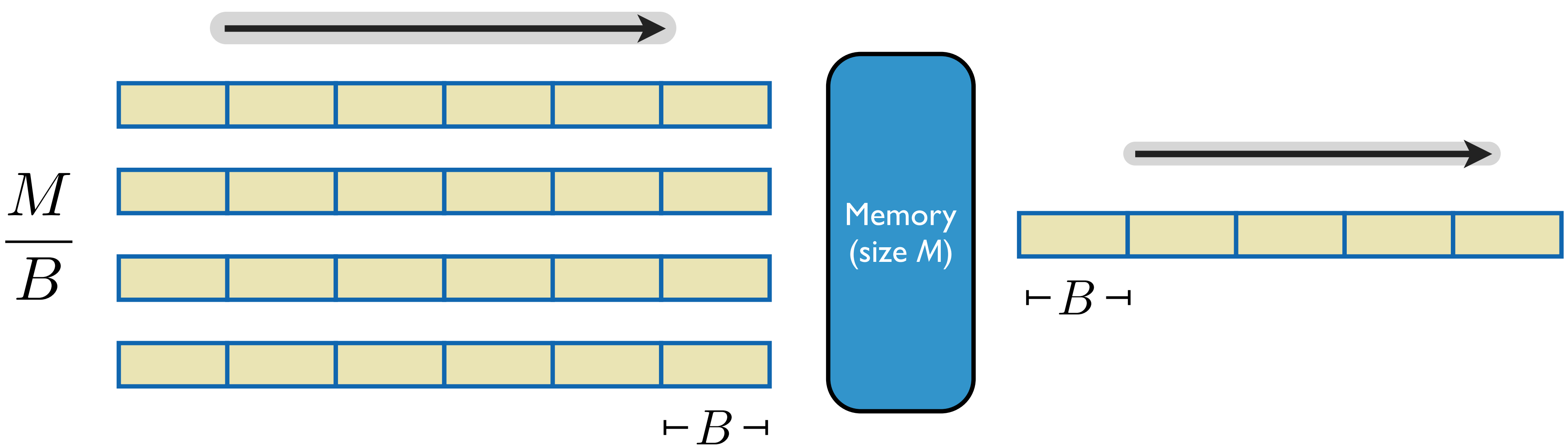
To sort an array of *N* objects

- If *N* fits in main memory, then just sort elements.
- Otherwise,
  - divide the array into *M*/*B* pieces;
  - sort each piece (recursively); and
  - merge the *M*/*B* pieces.

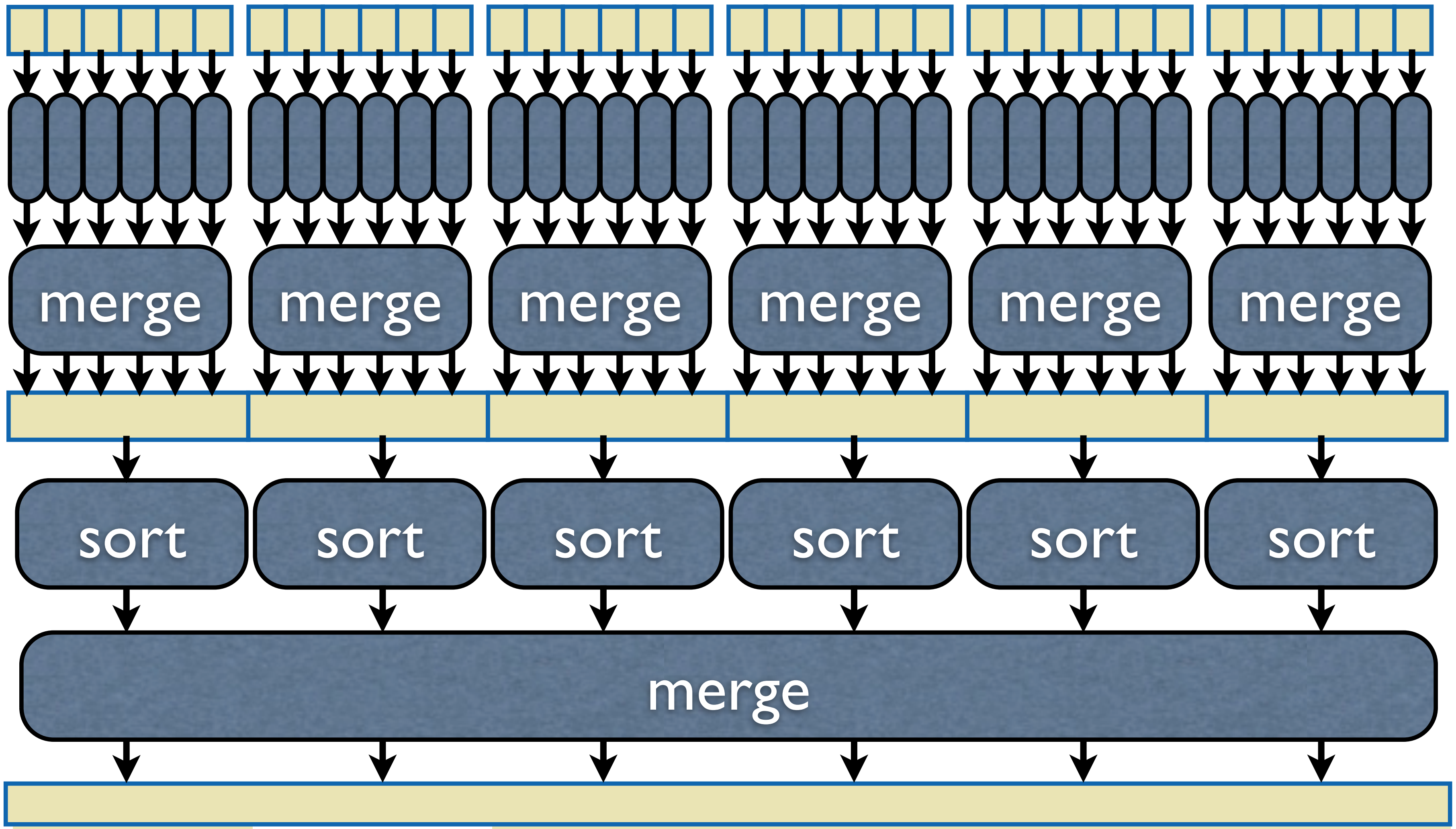


Why Divide into *M/B* pieces?

- **We want as much fan-in as possible.**
- **The merge needs to cache one block for each sorted subinput.**
- **Plus one block for the output.**
- **There are *M/B* blocks in memory.**
- **So the fan-in can be at most O(*M/B*)**



Merge Sort


merge
merge
merge
merge
merge
merge
sort
sort
sort
sort
sort
sort
merge

Intuition for Merge Sort Analysis

Question: How many I/Os to sort *N* elements?

- First run takes *N*/*B* I/Os.
- Each level of the merge tree takes *N*/*B* I/Os.
- How deep is the merge tree?

$$O\left(\underbrace{\frac{N}{B}}_{\text{Cost to scan data}}\underbrace{\log_{M/B}\frac{N}{B}}_{\text{\# of scans of data}}\right)$$

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- How deep is the merge tree?

$$O\left(\underbrace{\frac{N}{B}}_{\text{Cost to scan data}}\underbrace{\log_{M/B}\frac{N}{B}}_{\text{\# of scans of data}}\right)$$

This bound is the best possible.

Merge Sort Analysis

T(N), the number of I/Os to sort *N* items, satisfies this recurrence:

$$T(N) = \underbrace{\frac{M}{B}}_{\text{\# of pieces}} \cdot \underbrace{T\left(\frac{N}{M/B}\right)}_{\text{cost to sort each piece recursively}} + \underbrace{\frac{N}{B}}_{\text{cost to merge}}$$

$$T(N) = \frac{N}{B} \text{ when } N < M$$

cost to sort something that fits in memory

Solution:

$$O\left(\underbrace{\frac{N}{B}}_{\text{Cost to scan data}} \underbrace{\log_{M/B} \frac{N}{B}}_{\text{\# of scans of data}}\right)$$

Sorting is Faster Than Index Maintenance

I/Os to sort *N* objects:

$$O\left(\frac{N}{B}\log_{M/B}\frac{N}{B}\right)$$

I/Os to insert *N* objects into a COLA:

$$O\left(\frac{N}{B}\lg(N/M)\right)$$

I/Os to insert *N* objects into a B-tree:

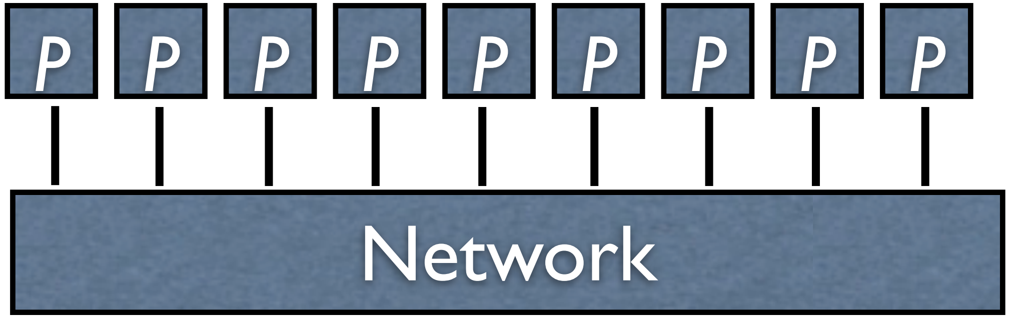
$$O\left(N\log_B(N/M)\right)$$

Sorting can usually be done in 2 passes since *M/B* is large.

Parallel Sort

Big data might not fit on one machine.

So use many machines and parallelize.

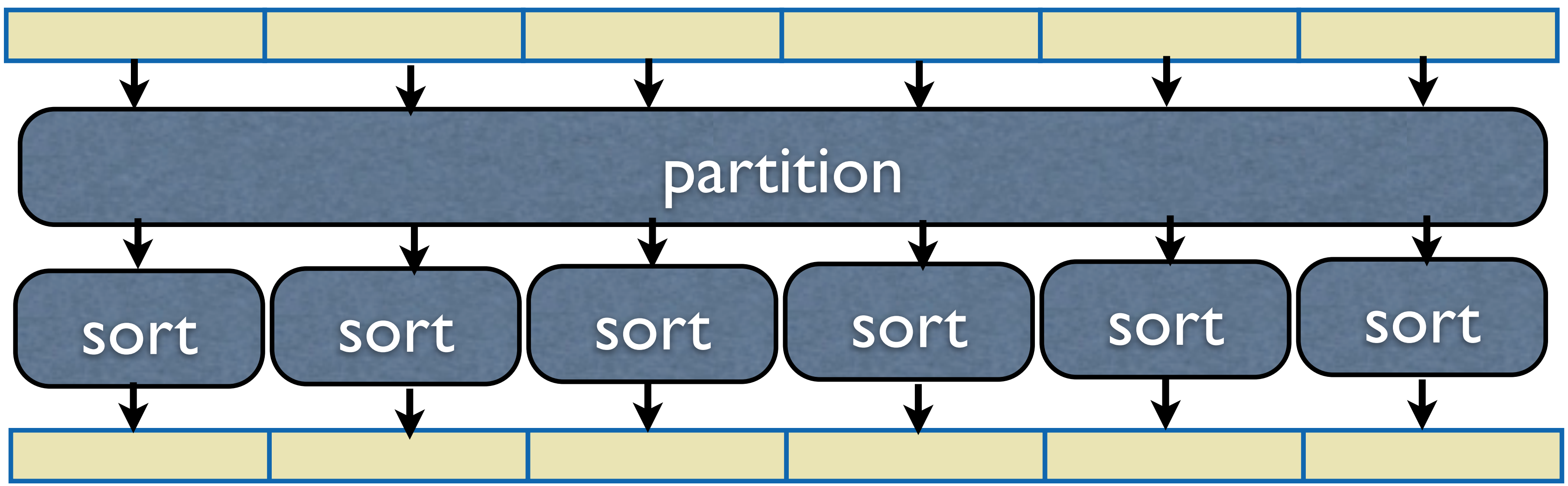


Parallelizing merge sort is tricky, however.

Distribution Sort

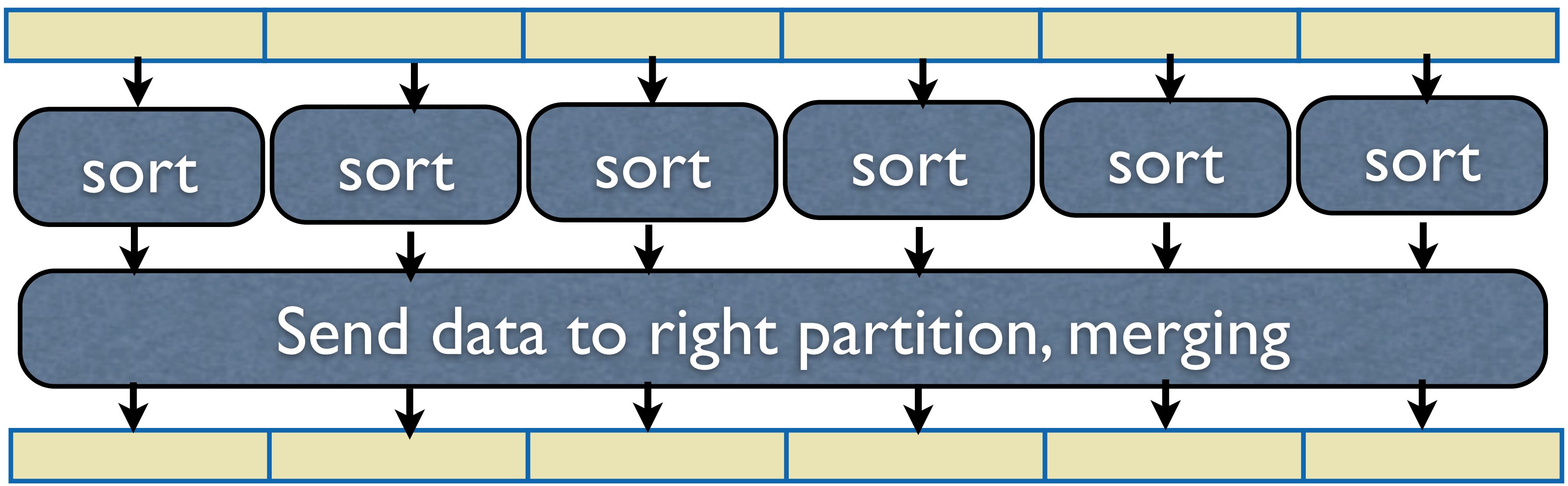
To sort an array of *N* objects

- If *N* fits in main memory, then just sort elements.
- Otherwise
  - pick *M*/*B* pivot keys;
  - partition data according to pivot keys; and
  - sort each partition (recursively).



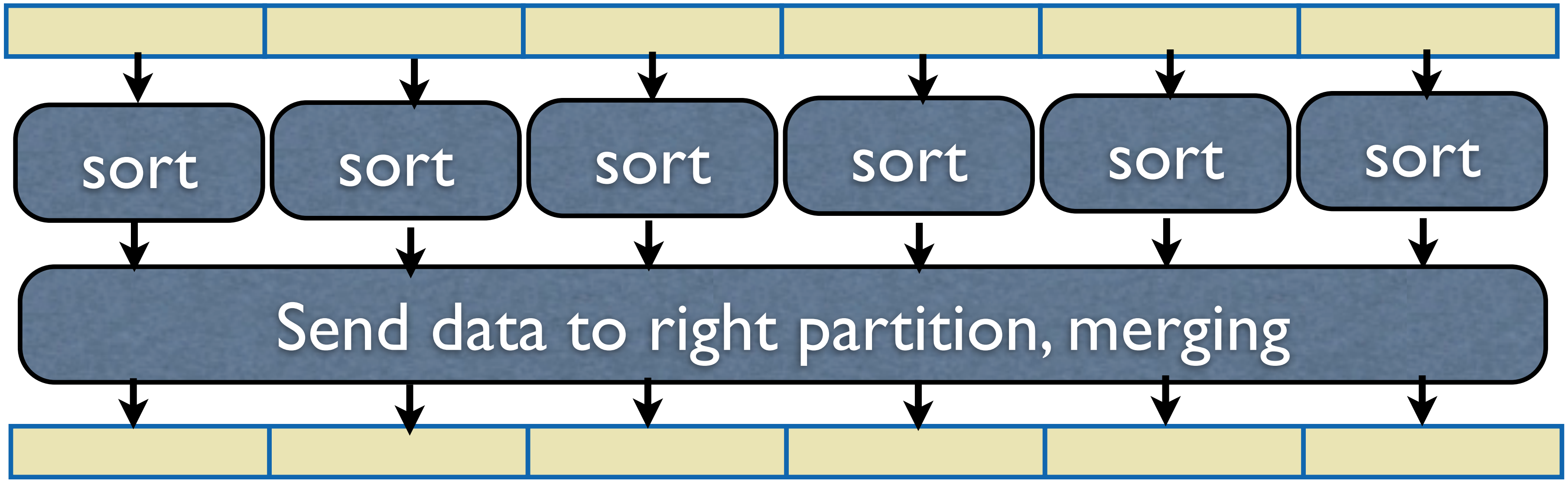
Parallelizing Partitioning

- **Broadcast the pivot keys to every processor.**
- **Compute the local rank of each pivot on each processor.**
  - **Sort local data to make this fast.**
- **Sum the local ranks to get global ranks.**
- **Send each datum to the right processor.**
- **The final step is a merge, since the local data was sorted.**



Engineering Parallel Sort

- **Scheduling:**
  - Overlap I/O with computation and network communcation.
  - Schedule network communication carefully to avoid network contention.
- **Hardware:**
  - Use a serious network.
  - Get rid of slow disks. Some disks are 10x slower than average. Probably failing.
- **In memory:**
  - Must compute local pivot ranks efficiently.
  - Employ a heap data structure to perform merge efficiently.



Sorting Contest

Bradley holds the world record for sorting a Terabyte: sortbenchmark.org

- 400 dual core machines with 2400 disks in 2007.
- Ran in 3.28 minutes.
- Used a distribution sort.
- Terabyte sort now deprecated, since it's the same as minute sort (how much can you sort in a minute).
- Today to compete, you must sort 100TB in less than two hours.

Summary

Fast sorting is an important tool for big data.

Sorting provides many opportunities for cleverness.

No one can take my Terabyte sorting trophy!

Closing Words

We want to feel your pain.

We are interested in hearing about other scaling problems.

Come talk to us.

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bradley@mit.edu

bradley@tokutek.com

Big Data Epigrams

The problem with big data is microdata.

Sometimes the right read optimization is a write-optimization.

It's often better to optimize approximately for all B, M than to pick the best B and M.

As data becomes bigger, the asymptotics become more important.