CSE 304/504 Compiler Design Syntax Analysis (Top-Down Parsing)

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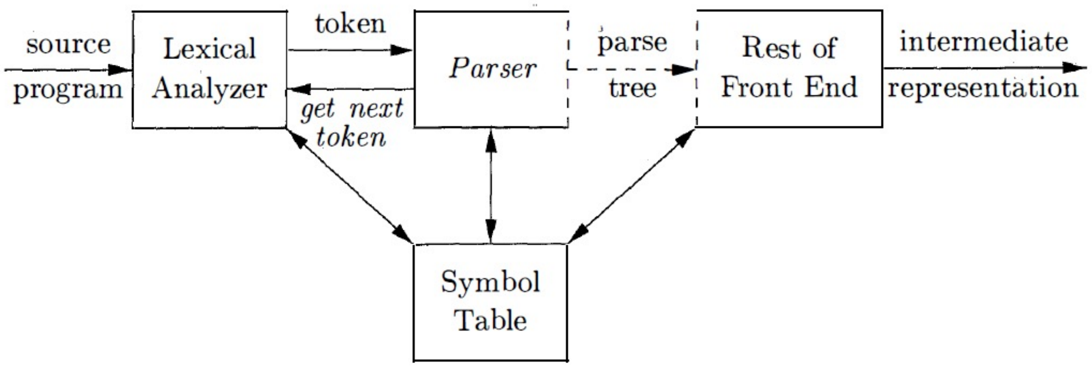
The Role of the Parser



Obtains strings of tokens from the lexical analyzer and verifies that the string can be generated by the grammar.

Efficient parsing methods

- Top-down Parsers:
  - Build parse trees from the root to the leaves
  - Handmade parsers (e.g. LL grammars)
- Bottom-up Parsers
  - Build parse trees from the leaves to the top
  - Generated by automated tools (e.g. LR grammars)

Context-Free Grammars

Terminals (tokens)

- Basic symbols from which strings are formed

Nonterminals

- Syntactic variables that denote sets of strings

Start symbol

- A nonterminal that denotes the language defined by the grammar

Productions

- The manner in which the terminals and nonterminals can be combined to from strings.

Notational Conventions

a, b, c (small earlier part of the alphabet): a single terminal symbol.

A, B, C (large earlier part of the alphabet): a single nonterminal symbol.

x, y, z (small later part of the alphabet): a string of terminals.

X, Y, Z (large later part of the alphabet): a single grammar symbol (a terminal or a nonterminal symbol).

α, β, γ (small Greek letters): a string of grammar symbols.

S: the start symbol.

Derivations

A production is treated as a rewriting rule

- The nonterminal on the LHS is replaced by the string on the RHS of the production.
- Example E -> E + E | E * E | (E) | - E | ID,
  E => - E : "E derives - E"
  E => - E => - (E) => -(ID) : derivation of -(ID) from E
  $E \Rightarrow^* -(ID)$
- => : derives in one step,
  $\Rightarrow^*$: derives in zero or more steps,
  $\Rightarrow^+$: derives in one or more steps.
- $\alpha \Rightarrow^* \alpha$
  $\alpha \Rightarrow^* \beta$ and $\beta \Rightarrow \gamma$, then $\alpha \Rightarrow^* \gamma$

Derivations

Let S be the start symbol of G, then a string of terminals w is in L(G) iff $S \Rightarrow^{+} w$.

- The string w is called a **sentence** of G

A language generated by a grammar is called a **context-free language**

Two grammars are called equivalent if they generate the same language.

If $S \Rightarrow^{+} \alpha$, where α may contain nonterminals, then α is a **sentential form**.

- A sentence is a sentential form with no nonterminals.
- Leftmost derivation: derivations in which only the leftmost nonterminal in any sentential form is replaced.
- Rightmost derivation: derivations in which only the rightmost nonterminal in any sentential form is replaced.

Elimination of Left Recursion

A grammar is **left recursive** if there is a derivation $A \Rightarrow^{+} A\alpha$ for a nonterminal A and some string α.

- ***Top-down parsing mechanism cannot handle left-recursive grammars.***

In section 2.4, $A \rightarrow A\ \alpha\ |\ \beta$ is converted to

$A \rightarrow \beta\ R$
$R \rightarrow \alpha\ R\ |\ \epsilon$.

- It does not eliminate left recursions involving two or more steps of derivations.

  S -> A a | b
  A -> A c | S d | ϵ

- Solution: give orders to nonterminals and if there is a production whose first RHS nonterminal is higher than the LHS, replace the RHS nonterminal with its productions.

Eliminating Left Recursion

arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
for (each i from 1 to n) {
  for (each j from 1 to $i-1$) {
    replace each production of the form $A_i \to A_j\gamma$ by the
      productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \cdots \mid \delta_k\gamma$, where
      $A_j \to \delta_1 \mid \delta_2 \mid \cdots \mid \delta_k$ are all current A_j-productions
  }
  eliminate the immediate left recursion among the A_i-productions
}

Example

- S -> A a | b,
  A -> A c | S d | ϵ.
- Order nonterminals as S, A
- When i = 2, A -> S d is converted to
  A -> A c | A ad | bd | ϵ

S -> A a | b
A -> bd A' | A'
A' -> c A' | ad A' | ϵ

Left Factoring

In predictive parsing, when we cannot select the production rule immediately, modify the grammar to defer the decision.

```
stmt -> IF expr THEN stmt ELSE stmt
      |  IF expr THEN stmt
```

If $A \rightarrow \alpha\ \beta_1 \mid \alpha\ \beta_2$, then modify the grammar as

$A \rightarrow \alpha\ A'$

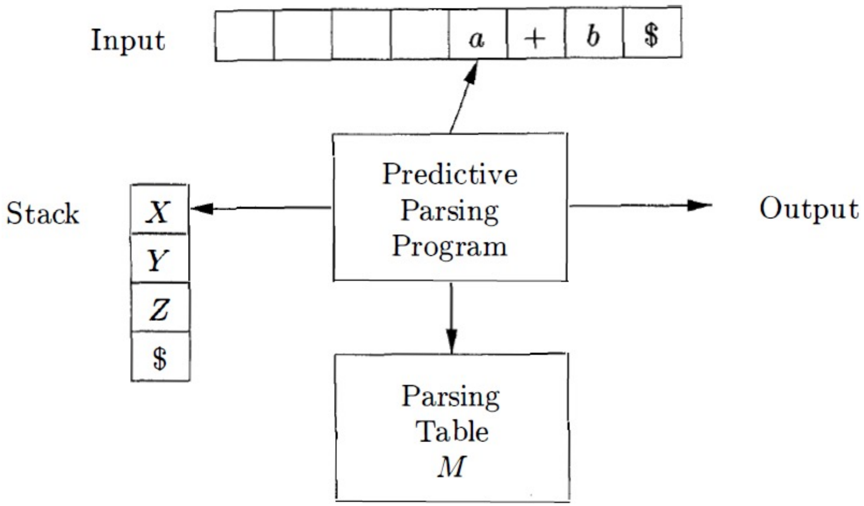
$A' \rightarrow \beta_1 \mid \beta_2$

```
stmt -> IF expr THEN stmt stmt'
stmt' -> ELSE stmt | ϵ
```

Top-Down Parsing

In many cases, left-recursion removal and left factoring results in a grammar that can be parsed by a recursive-decent parser without backtracking (i.e. a predictive parser).

Nonrecursive Predictive Parsing



Input: string of terminals followed by $

Stack: sequence of grammar symbols with $ on the bottom.

Parsing table: M[A,a], where A is a nonterminal, a is a terminal or $.

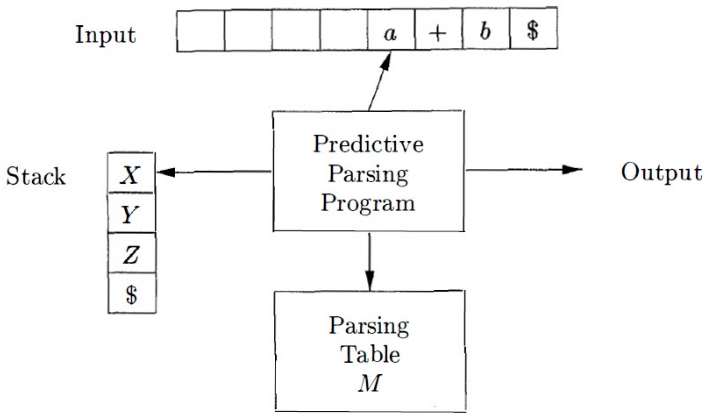
- Let X be the symbol on top of the stack, and a be the current input
- If X = a = $, announce the success.
- If X = a ≠ $, pops X and advance the input pointer
- If X is nonterminal
  - If M[X,a] = {X->UVW}, replace X on top of the stack with WVU (with U on top)
  - If M[X,a] = error, declare an error

Nonrecursive Predictive Parsing

```
set ip to point to the first symbol of w;
set X to the top stack symbol;
while ( X ≠ $ ) { /* stack is not empty */
        if ( X is a ) pop the stack and advance ip;
        else if ( X is a terminal ) error();
        else if ( M[X, a] is an error entry ) error();
        else if ( M[X, a] = X → Y1 Y2 ··· Yk ) {
                output the production X → Y1 Y2 ··· Yk;
                pop the stack;
                push Yk, Yk−1, ... , Y1 onto the stack, with Y1 on top;
        }
        set X to the top stack symbol;
}
```

Example

id + id * id



NON-TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \textbf{id}$			$F \to (E)$		

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $E \to TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $T \to FT'$
	$\textbf{id}\ T'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $F \to \textbf{id}$
$\textbf{id}$	$T'E'\$$	$+ \textbf{id} * \textbf{id}\$$	match **id**
$\textbf{id}$	$E'\$$	$+ \textbf{id} * \textbf{id}\$$	output $T' \to \epsilon$
$\textbf{id}$	$+\ TE'\$$	$+ \textbf{id} * \textbf{id}\$$	output $E' \to +\ TE'$
$\textbf{id}\ +$	$TE'\$$	$\textbf{id} * \textbf{id}\$$	match +
$\textbf{id}\ +$	$FT'E'\$$	$\textbf{id} * \textbf{id}\$$	output $T \to FT'$
$\textbf{id}\ +$	$\textbf{id}\ T'E'\$$	$\textbf{id} * \textbf{id}\$$	output $F \to \textbf{id}$
$\textbf{id} + \textbf{id}$	$T'E'\$$	$* \textbf{id}\$$	match **id**
$\textbf{id} + \textbf{id}$	$*\ FT'E'\$$	$* \textbf{id}\$$	output $T' \to *\ FT'$
$\textbf{id} + \textbf{id}\ *$	$FT'E'\$$	$\textbf{id}\$$	match *
$\textbf{id} + \textbf{id}\ *$	$\textbf{id}\ T'E'\$$	$\textbf{id}\$$	output $F \to \textbf{id}$
$\textbf{id} + \textbf{id} * \textbf{id}$	$T'E'\$$	$\$$	match **id**
$\textbf{id} + \textbf{id} * \textbf{id}$	$E'\$$	$\$$	output $T' \to \epsilon$
$\textbf{id} + \textbf{id} * \textbf{id}$	$\$$	$\$$	output $E' \to \epsilon$

- Quiz: id + (id)

FIRST and FOLLOW

FIRST(α)

- The set of terminals that begin the strings derived from α
- If $\alpha \Rightarrow^* a\ \beta$ then a is in FIRST(α)
- If $\alpha \Rightarrow^* \epsilon$, then ϵ is in FIRST(α)

FOLLOW(A)

- The set of terminals a that can appear immediately to the right of A in some sentential form.
- If $S \Rightarrow^* \alpha\ A\ a\ \beta$, then a is in FOLLOW(A)

Compute FIRST(X)

- If X is terminal, then FIRST(X) is {X}.
- If $X \rightarrow \epsilon$ is a production, then add ϵ to FIRST(X).
- If X is nonterminal and $X \rightarrow Y_1\ Y_2 \ldots Y_k$ is a production,
  - Add a to FIRST(X) if $a \in$ FIRST(Y_i) and $\epsilon \in$ FIRST(Y_j) for 1 <= j < i.
  - Add ϵ to FIRST(X) if $\epsilon \in$ FIRST(Y_j) for 1 <= j <= k.

FIRST and FOLLOW

Compute FIRST($X_1 \ldots X_n$)

- Add a to FIRST($X_1 \ldots X_n$) if $a \in$ FIRST(X_i) and $\epsilon \in$ FIRST(X_j) for 1 <= j < i.
- Add ϵ to FIRST($X_1 \ldots X_n$) if $\epsilon \in$ FIRST(X_j) for 1 <= j < n.

Compute FOLLOW(A)

- Add \$ to FOLLOW(S) if S is the start symbol.
- If there is a production $A \to \alpha\ B\ \beta$, then add FIRST(β)-{ϵ} to FOLLOW(B).
- If there is a production $A \to \alpha\ B$ or a production $A \to \alpha\ B\ \beta$ where $\epsilon \in$ FIRST(β), then add FOLLOW(A) to FOLLOW(B).

FIRST and FOLLOW

Example

```
E   -> T E'
E'  -> + T E' | ϵ
T   -> F T'
T'  -> * F T' | ϵ
F   -> ( E ) | ID
```

FIRST(E) = FIRST(T) = FIRST(F) = { (, ID }

FIRST(E') = { +, ϵ }

FIRST(T') = { *, ϵ }

FOLLOW(E) = FOLLOW(E') = {), $ }

FOLLOW(T) = FOLLOW(T') = { +,), $ }

FOLLOW(F) = { +, *,), $ }

Building a Predictive Parsing Table

For each production $A \rightarrow \alpha$ do

- For each terminal a in FIRST(α), add $A \rightarrow \alpha$ to $M[A,a]$.
- If $\epsilon \in$ FIRST(α), add $A \rightarrow \alpha$ to $M[A,b]$ for each $b \in$ FOLLOW(A). (b is a terminal or $)
- Make each undefined entry of M be error.

Building a Predictive Parsing Table

Example

```
E -> T E'
E' -> + T E' | ϵ
T -> F T'
T' -> * F T' | ϵ
F -> ( E ) | ID
```

FIRST(E) = FIRST(T) = FIRST(F) = { (, ID }
FIRST(E') = { +, ϵ }
FIRST(T') = { *, ϵ }
FOLLOW(E) = FOLLOW(E') = {), $ }
FOLLOW(T) = FOLLOW(T') = { +,), $ }
FOLLOW(F) = { +, *,), $ }

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \textbf{id}$			$F \to (E)$		

LL(1) Grammars

LL(1): a grammar whose predictive parsing table has no multiply-defined entries.

- First L: scanning input from left to right
- Second L: producing a leftmost derivation.
- 1: using 1 input symbol of lookahead

Grammar G is LL(1) iff whenever $A \rightarrow \alpha \mid \beta$ are two distinct productions of G, then the following holds

- For no terminal a do both α and β derive strings beginning with a.
- At most one of α and β can derive the empty string
- If $\beta \Rightarrow^* \epsilon$, then α does not derive any string beginning with a terminal in FOLLOW(A).

Questions?