# CSE 304 Compiler Design

## Lexical Analysis

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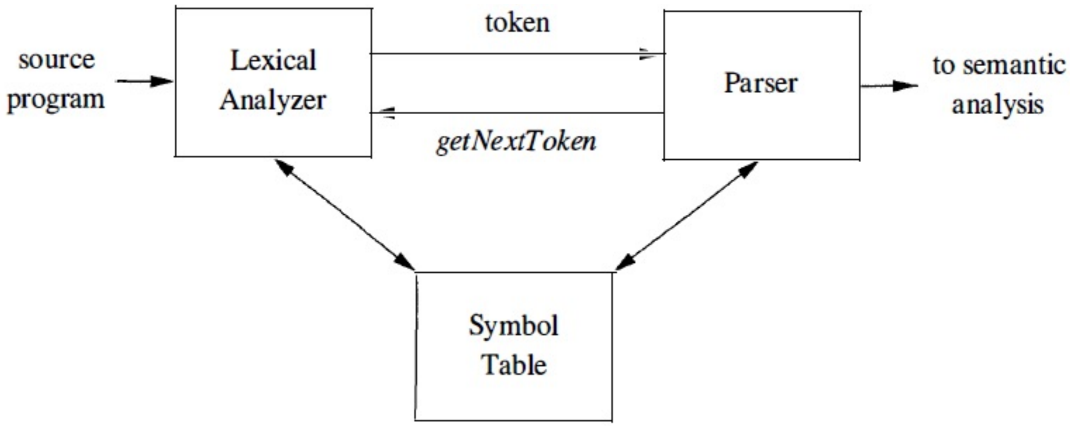
# The Role of the Lexical Analyzer




Why separating lexical analysis and parsing

- Simplify design (comments, white spaces…)
- Improve compiler efficiency (simpler algorithm)
- Improve compiler portability

# Specification of Tokens

String and Language

- Alphabet (character class): any finite set of symbols.
- A string of some alphabet: a finite sequence of symbols drawn from the alphabet.
- Language: any set of strings over some fixed alphabet.

Operations on Language

| OPERATION | DEFINITION AND NOTATION |
|---|---|
| *Union* of $L$ and $M$ | $L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$ |
| *Concatenation* of $L$ and $M$ | $LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$ |
| *Kleene closure* of $L$ | $L^* = \cup_{i=0}^{\infty} L^i$ |
| *Positive closure* of $L$ | $L^+ = \cup_{i=1}^{\infty} L^i$ |

# Regular Expressions

Rules that define the regular expression over alphabet Σ

- $\epsilon$ is a regular expression denoting $\{\epsilon\}$
- If $a \in \Sigma$, $a$ is a regular expression denoting $\{a\}$
- $(r)|(s)$ is a regular expression denoting $L(r) \cup L(s)$
- $(r)(s)$ is a regular expression denoting $L(r)L(s)$
- $(r)*$ is a regular expression denoting $(L(r))*$
- $(r)$ is a regular expression denoting $L(r)$,
  where $r$ and $s$ are regular expressions denoting $L(r)$ and $L(s)$

Exercise: Find the language `L( a(b|c)* )`

# Nonregular Sets

Balanced or nested structure

- e -> ( e )

Repeating strings

- {wcw | w is a string of a's and b's}

Arbitrary number of repetitions

- n H $a_1$ $a_2$ … $a_n$

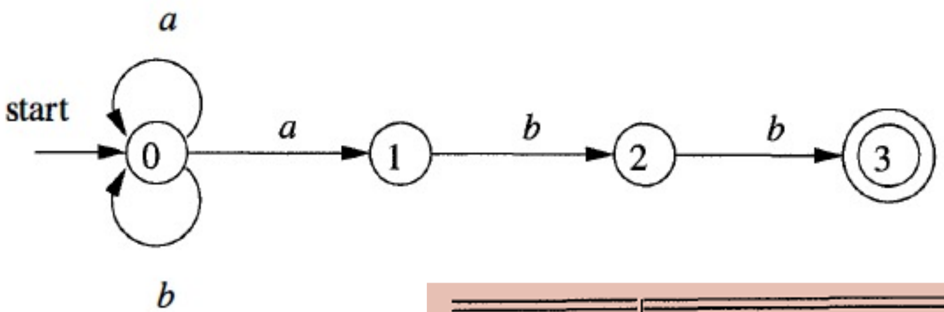
# Finite Automata

A recognizer for a language L is a program that takes a string x as an input and answers "yes" if x $\in$ L and "no" otherwise.

Nondeterministic Automata (NFA) consist of

1. a set of status $S$
2. a set of input symbol $\Sigma$
3. a transition function *move*: maps $(S, \Sigma)$ to $S$
4. an initial state $s_0 \in S$
5. a set of final states $F \subseteq S$

# NFA example




| STATE | $a$ | $b$ | $\epsilon$ |
|---|---|---|---|
| 0 | $\{0,1\}$ | $\{0\}$ | $\emptyset$ |
| 1 | $\emptyset$ | $\{2\}$ | $\emptyset$ |
| 2 | $\emptyset$ | $\{3\}$ | $\emptyset$ |
| 3 | $\emptyset$ | $\emptyset$ | $\emptyset$ |

In NFA, the same input string can result in  different states.

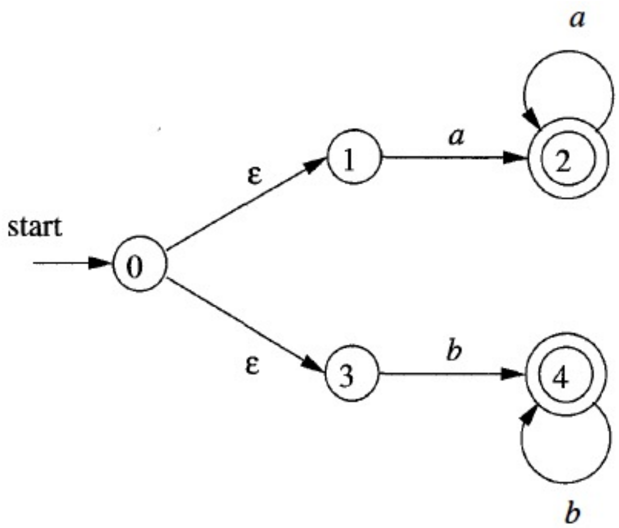
$$0 \xrightarrow{a} 0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3$$

$$0 \xrightarrow{a} 0 \xrightarrow{a} 0 \xrightarrow{b} 0 \xrightarrow{b} 0$$

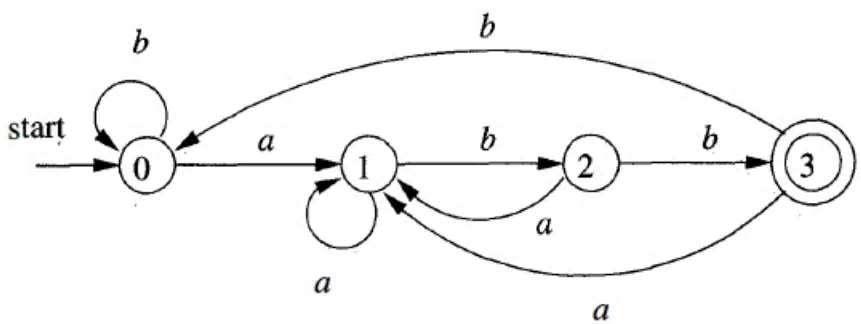
# Finite Automata

Deterministic Finite Automata (DFA)

- DFA is a special case of NFA with
  - No state has an ϵ-transition
  - Each state has at most 1 edge for each input symbol.



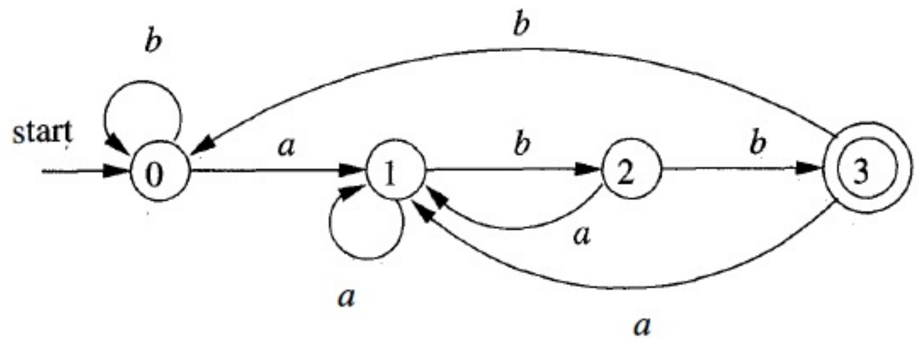

NFA accepting $\mathbf{aa}^*|\mathbf{bb}^*$




DFA accepting $(\mathbf{a}|\mathbf{b})^*\mathbf{abb}$

# Simulating DFA

```
s = s0;
c = nextChar();
while ( c != eof ) {
        s = move(s, c);
        c = nextChar();
}
if ( s is in F ) return "yes";
else return "no";
```




DFA accepting $(\mathbf{a}|\mathbf{b})^*\mathbf{abb}$

Exercise:

1. Check if aabbabb is in the language of (a|b)*abb
2. Check if aabbaa is in the language of (a|b)*abb

# NFA to DFA

Goal: Starting with an NFA, produce *Dtrans,* a transition table for an equivalent DFA along with a set of states that comprise the DFA, *Dstates*.

Idea: Create 'sets of states', T, where each NFA state in T can be reached after a specific input string. The creation of these sets handle ε-transitions.

# NFA to DFA

| OPERATION | DESCRIPTION |
|---|---|
| $\epsilon\text{-}closure(s)$ | Set of NFA states reachable from NFA state $s$ on $\epsilon$-transitions alone. |
| $\epsilon\text{-}closure(T)$ | Set of NFA states reachable from some NFA state $s$ in set $T$ on $\epsilon$-transitions alone; $= \cup_{s \text{ in } T}\ \epsilon\text{-}closure(s)$. |
| $move(T, a)$ | Set of NFA states to which there is a transition on input symbol $a$ from some state $s$ in $T$. |

```
initially, ϵ-closure(s0) is the only state in Dstates, and it is unmarked;
while ( there is an unmarked state T in Dstates ) {
        mark T;
        for ( each input symbol a ) {
                U = ϵ-closure(move(T, a));
                if ( U is not in Dstates )
                        add U as an unmarked state to Dstates;
                Dtran[T, a] = U;
        }
}
```

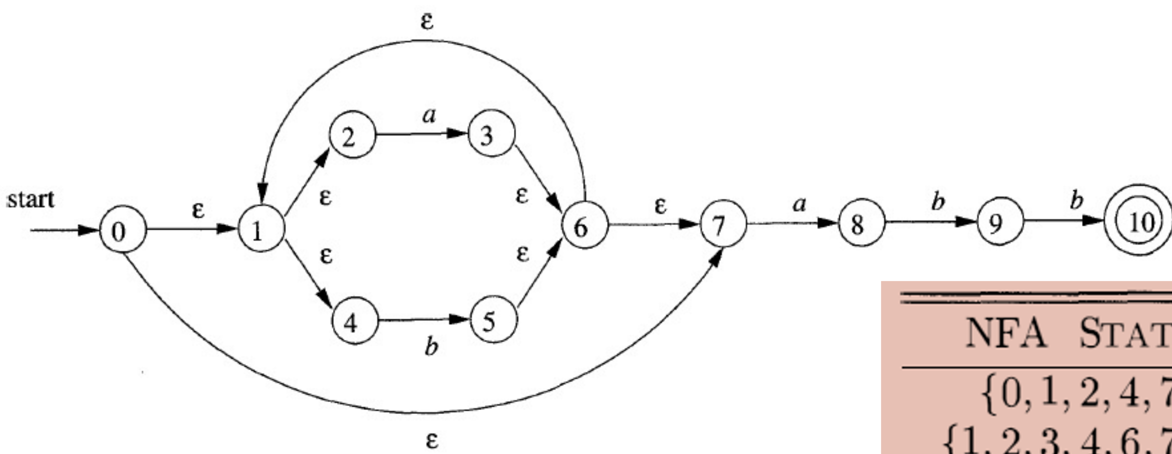
The subset construction

# NFA to DFA Conversion

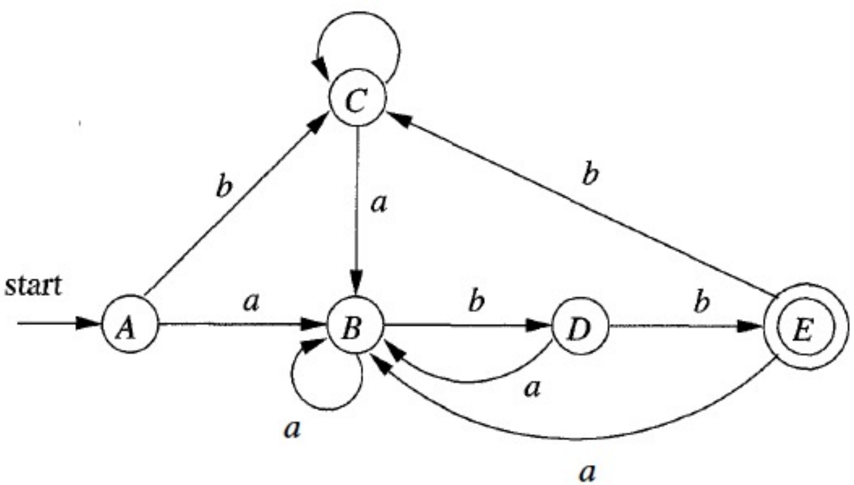
```
push all states of T onto stack;
initialize ε-closure(T) to T;
while ( stack is not empty ) {
        pop t, the top element, off stack;
        for ( each state u with an edge from t to u labeled ε )
                if ( u is not in ε-closure(T) ) {
                        add u to ε-closure(T);
                        push u onto stack;
                }
}
```

Computing $\epsilon$-*closure*$(T)$

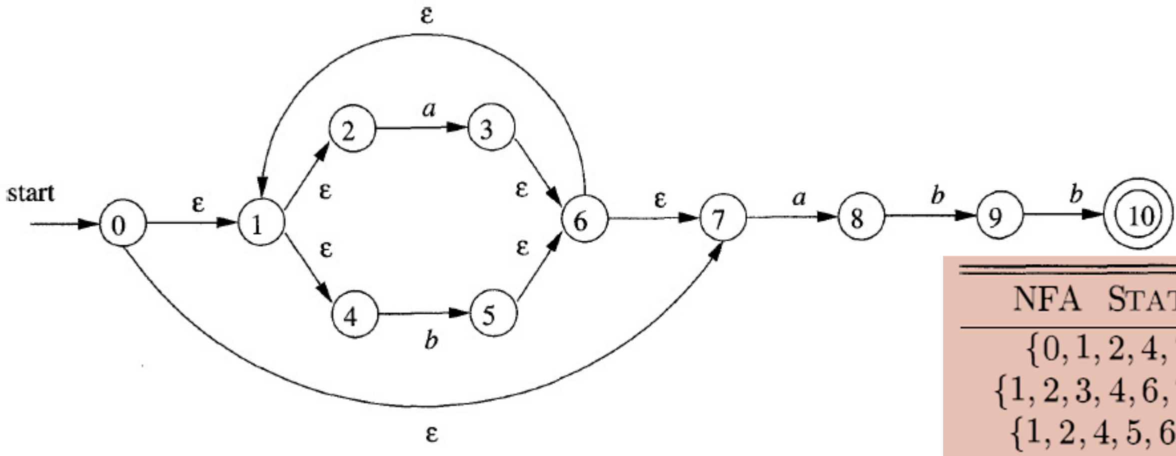
# NFA to DFA Conversion example




| NFA STATE | DFA STATE | $a$ | $b$ |
|---|---|---|---|
| $\{0,1,2,4,7\}$ | $A$ | $B$ | $C$ |
| $\{1,2,3,4,6,7,8\}$ | $B$ | $B$ | $D$ |
| $\{1,2,4,5,6,7\}$ | $C$ | $B$ | $C$ |
| $\{1,2,4,5,6,7,9\}$ | $D$ | $B$ | $E$ |
| $\{1,2,3,5,6,7,10\}$ | $E$ | $B$ | $C$ |

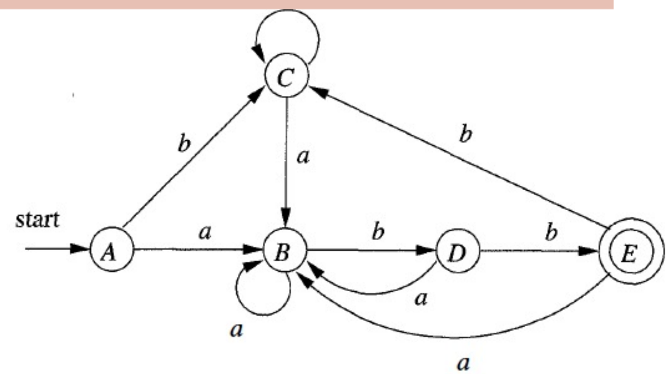


# NFA to DFA Conversion example



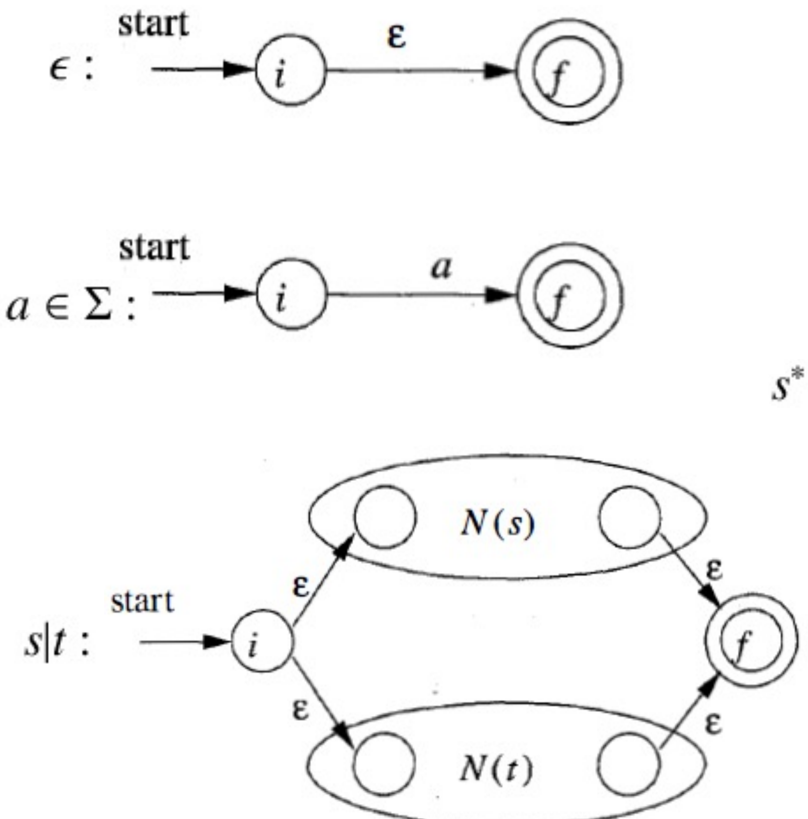

| NFA STATE | DFA STATE | $a$ | $b$ |
|---|---|---|---|
| $\{0,1,2,4,7\}$ | $A$ | $B$ | $C$ |
| $\{1,2,3,4,6,7,8\}$ | $B$ | $B$ | $D$ |
| $\{1,2,4,5,6,7\}$ | $C$ | $B$ | $C$ |
| $\{1,2,4,5,6,7,9\}$ | $D$ | $B$ | $E$ |
| $\{1,2,3,5,6,7,10\}$ | $E$ | $B$ | $C$ |

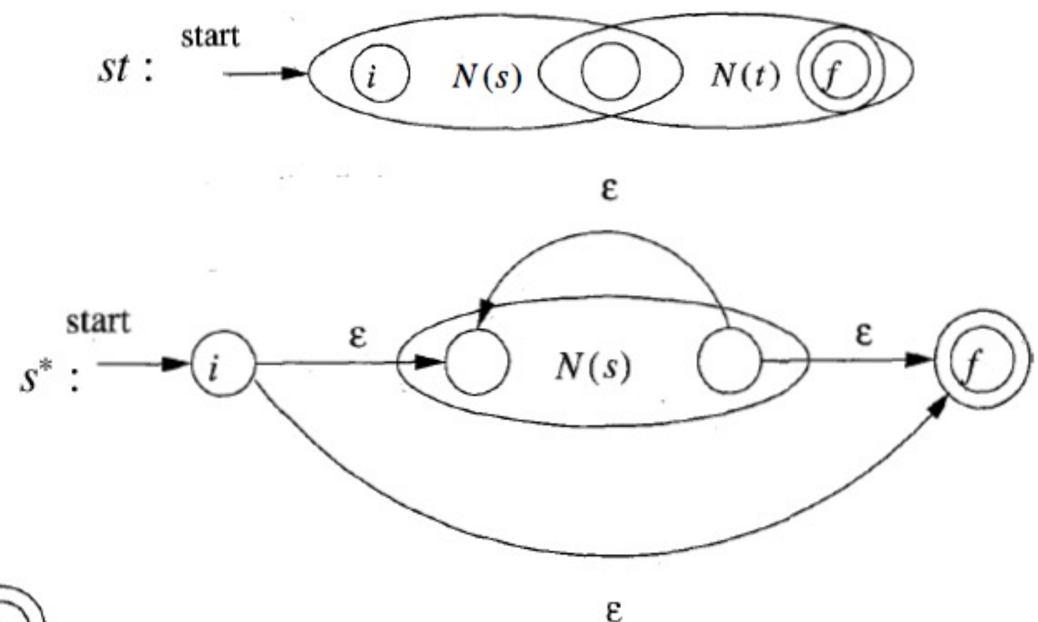
1. ε -closure(O), or A = {O, 1, 2,4, 7}
2. mark A and compute
   a. Dtran[A, a] = ε -closure(move(A,a)) (DFA state B)
   b. Dtran[A, b] = ε -closure(move(A,b)) (DFA state C)



# Regular Expression to NFA
## (Thompson's Construction Algorithm)

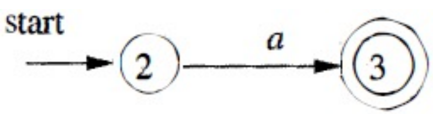
Let N(s) and N(t) be NFAs for s and t

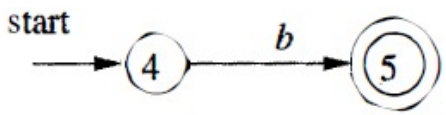


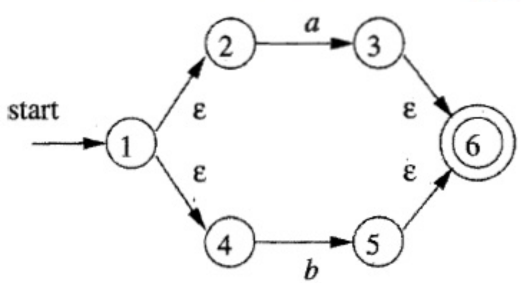



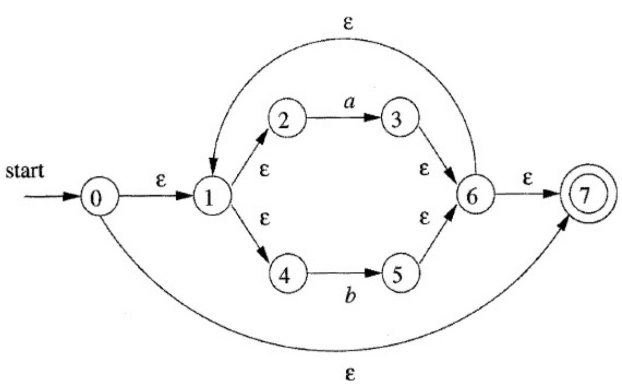

$$(s) : N((s)) = N(s)$$

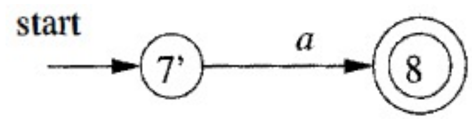
# Regular Expression to NFA: (a|b)*abb

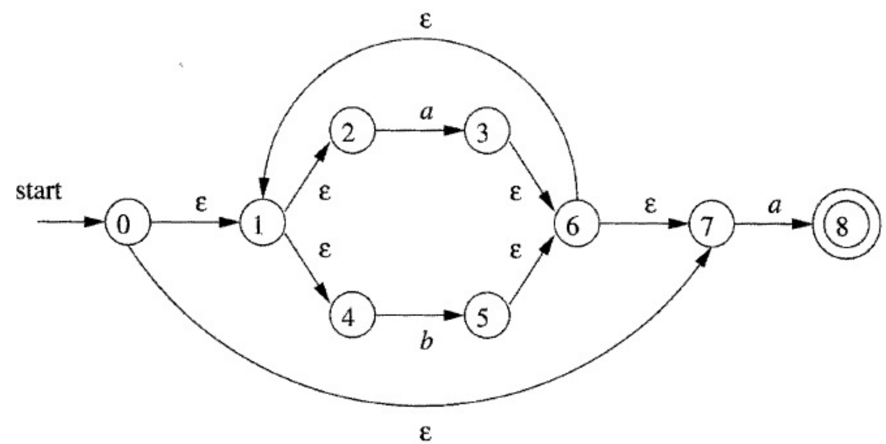


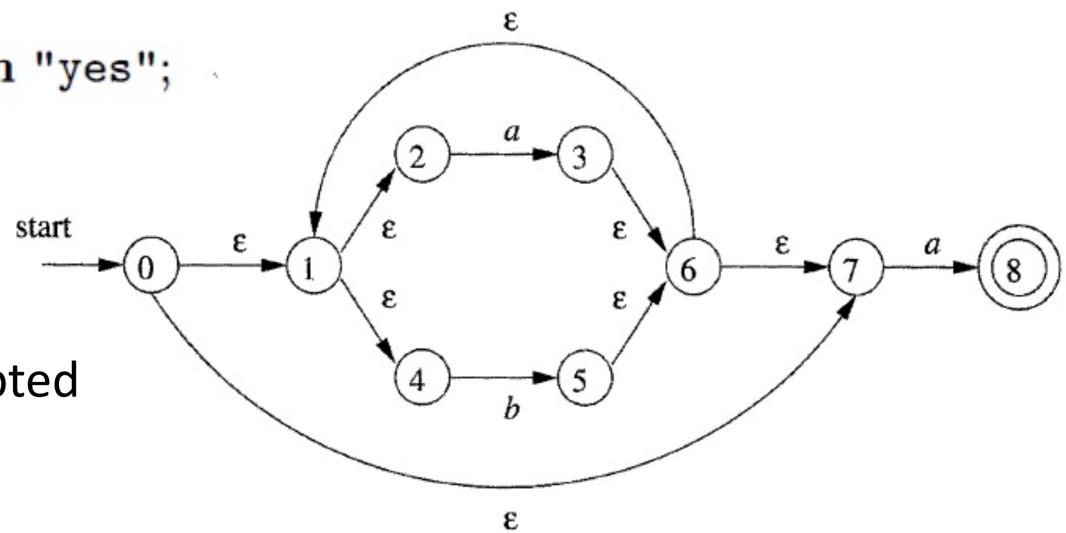




Exercise:

- Build an NFA for a(a|b)*b
- Convert the NFA to a DFA

# Simulating NFA

$$
\begin{array}{l}
S = \epsilon\text{-}closure(s_0); \\
c = nextChar(); \\
\textbf{while} \ ( \ c \ != \textbf{eof} \ ) \ \{ \\
\qquad S = \epsilon\text{-}closure(move(S, c)); \\
\qquad c = nextChar(); \\
\} \\
\textbf{if} \ ( \ S \cap F \ != \emptyset \ ) \ \textbf{return "yes";} \\
\textbf{else return "no";}
\end{array}
$$



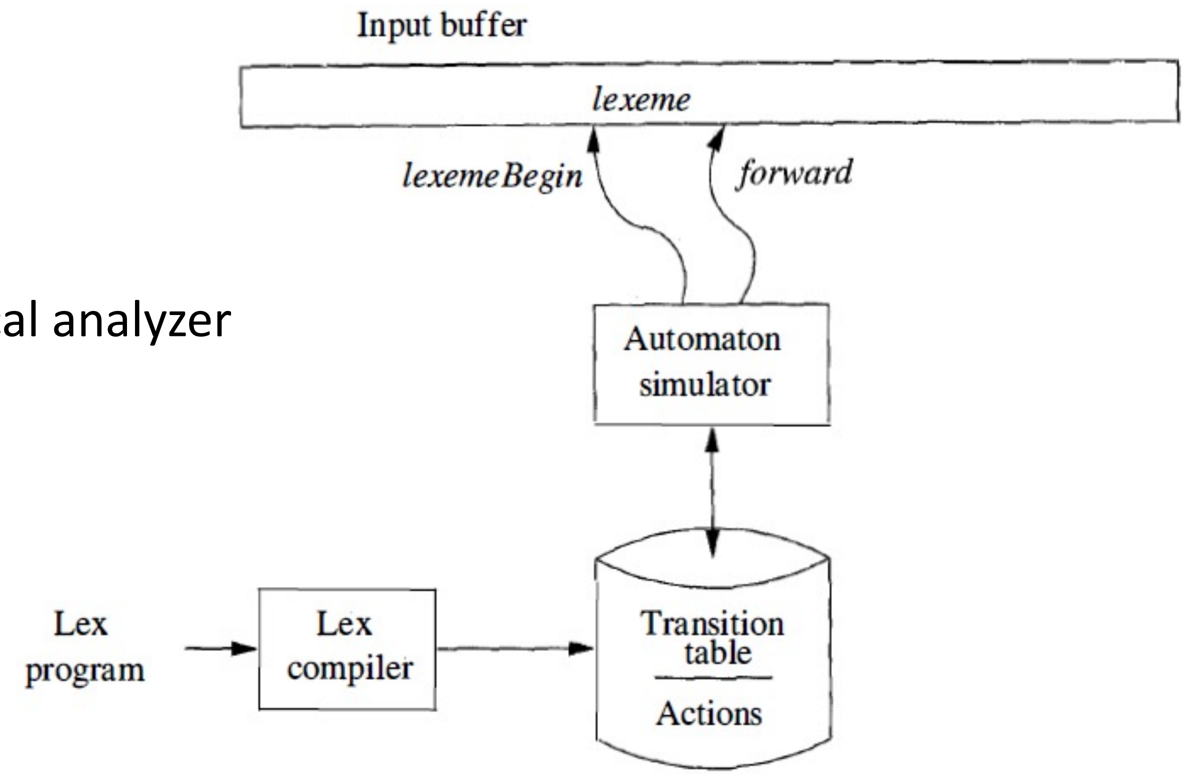

Quiz:

- Check if ababa will be accepted by the NFA on the right

# Lexical Analyzer

$p_1$ { $action_1$ }
$p_2$ { $action_2$ }
... ...
$p_n$ { $action_n$ }

Specification of a lexical analyzer

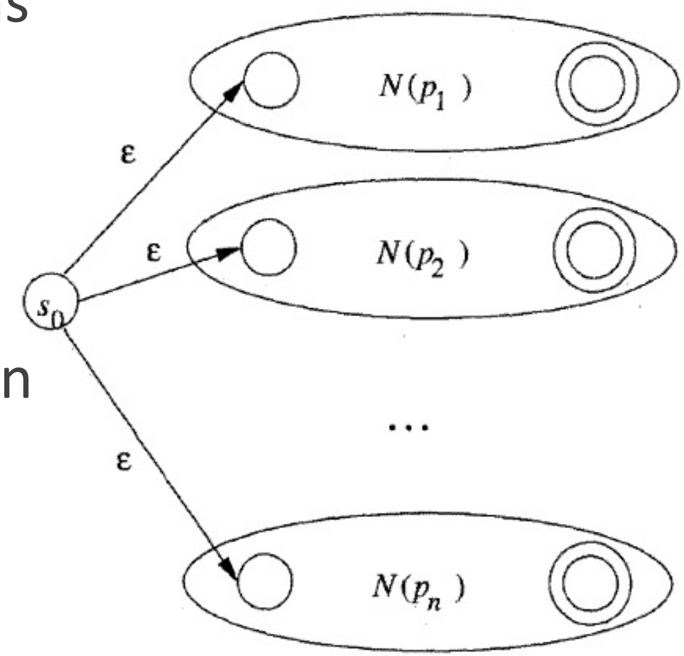



Model of Lex compiler

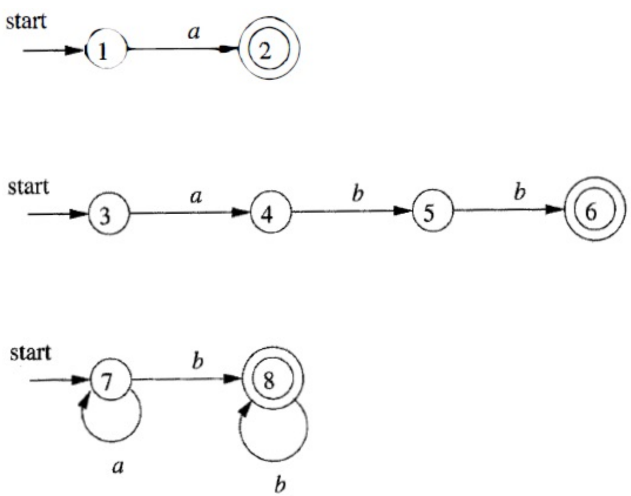
# Pattern Matching with NFAs

For patterns $p_1$, … , $p_n$,

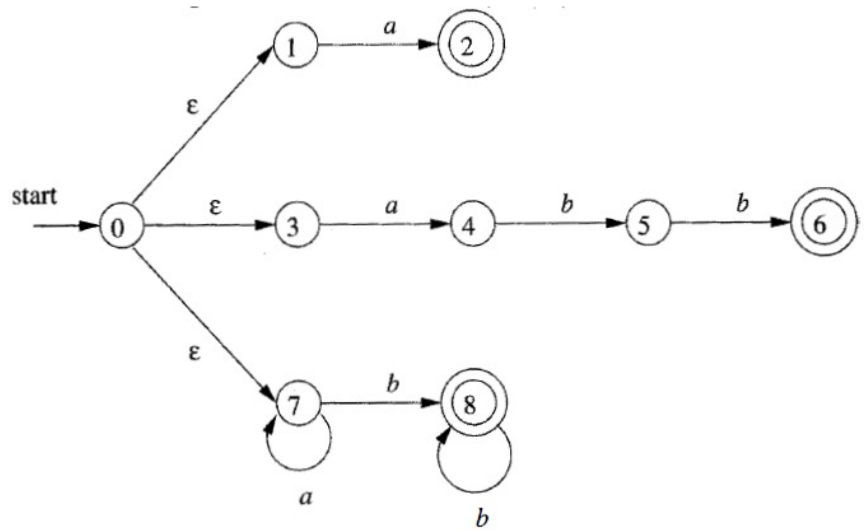
- Construct NFAs $N(p_1)$, … ,$N(p_n)$
- Add a start state $s_0$ and add ϵ transitions from $s_0$ to each $N(p_i)$.
- To match the longest pattern, keep simulating the NFA until there are no more transitions.
- Move backward to the last state with an accepting state.



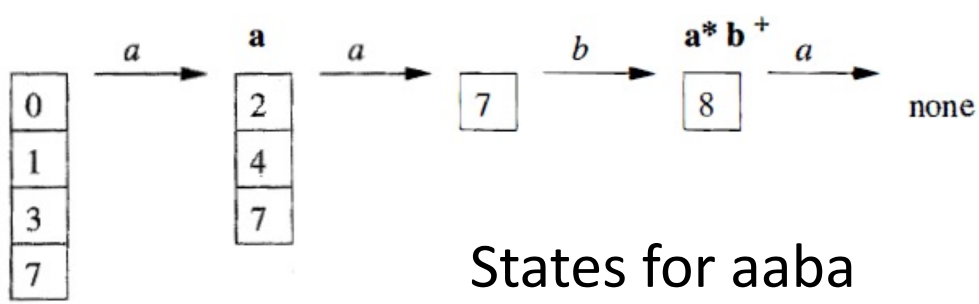
# Pattern Matching Example




NFAs for a, abb, a*b+




Combined NFA




States for aaba

# The lookahead operator

`r1/r2`: match a string in `r1` only if followed by a string in `r2`

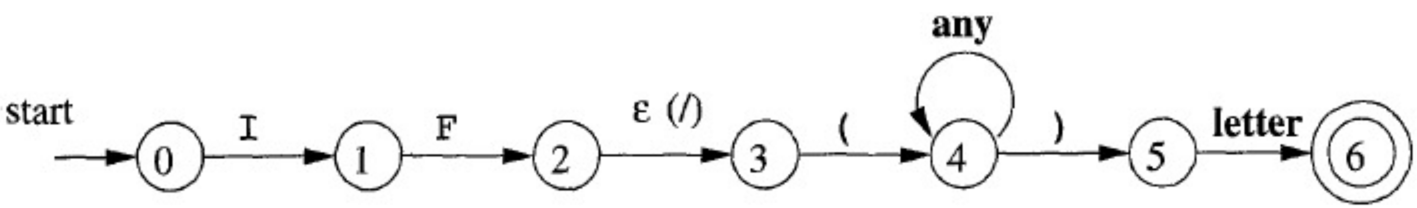
- E.g. in Fortran: `DO5I=1.25 vs DO5I=1,25`
  `DO/{letter_or_digit}*={letter_or_digit}*,`

Implementing lookahead operator

- When converting to NFA, treat / as ϵ
- When a string is recognized, truncate the lexeme at the position where the last transition on the (imaginary) / occurred.

E.g. `IF / \( .* \) {letter}`

- `IF ( 123 ) a`



# Regular Expressions to DFA

Important States of NFA

- An NFA state is important if it has a non-ϵ transition
- Subset construction algorithm uses only important states ( ϵ-closure(move(T,a)) )
- Two subsets can be identified if
  1. They have the same important states and
  2. They both have an accepting state or neither have one.
- Thompson's construction builds an important state exactly when a symbol in the alphabet appears.

Augmented regular expression

- Append a unique marker # to a regular expression r: (r)#
- Any DFA state with a transition on # is an accepting state.

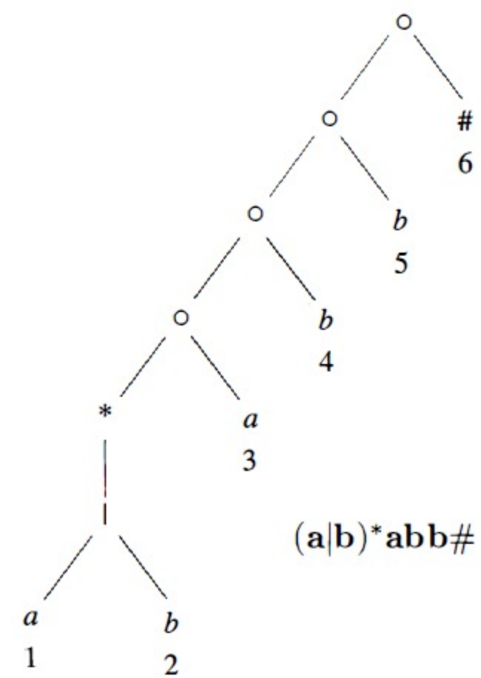
# Regular Expressions to DFA

Position: label non-ϵ leaves of a syntax tree for a regular expression with a unique number.

For a node n in a syntax tree, let r be the subexpression corresponding to n.

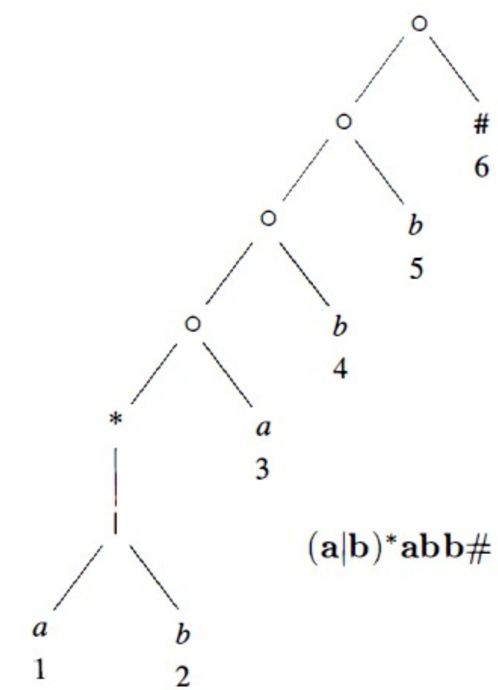
- nullable(n): if r can generate an empty string.
- firstpos(n): the set of positions that can match the first symbols of the strings generated by r.
- lastpos(n): the set of positions that can match the last symbols of the strings generated by r.

For a position i, followpos(i): the set of positions j such that there is some input string …cd… such that i corresponds to c and j to d.



# Regular Expressions to DFA

| NODE $n$ | $nullable(n)$ | $firstpos(n)$ |
|---|---|---|
| A leaf labeled $\epsilon$ | **true** | $\emptyset$ |
| A leaf with position $i$ | **false** | $\{i\}$ |
| An or-node $n = c_1 \mid c_2$ | $nullable(c_1)$ **or** $nullable(c_2)$ | $firstpos(c_1) \cup firstpos(c_2)$ |
| A cat-node $n = c_1 c_2$ | $nullable(c_1)$ **and** $nullable(c_2)$ | **if** ( $nullable(c_1)$ ) $firstpos(c_1) \cup firstpos(c_2)$ **else** $firstpos(c_1)$ |
| A star-node $n = c_1{}^*$ | **true** | $firstpos(c_1)$ |



$(\mathbf{a}|\mathbf{b})^*\mathbf{abb}\#$

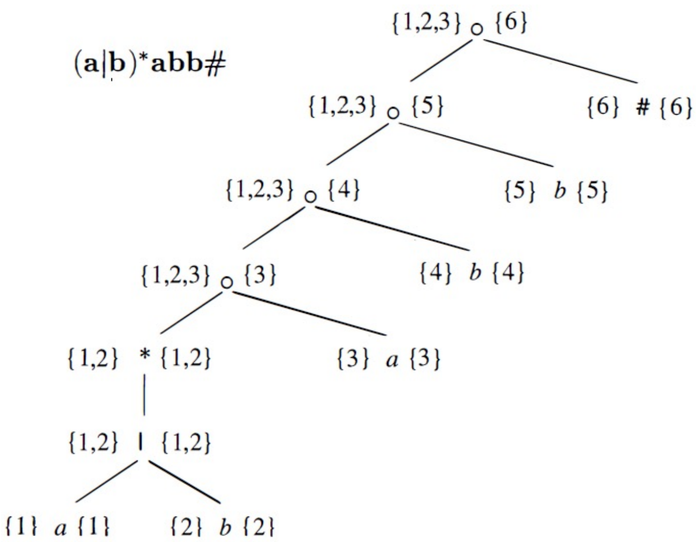
## followpos(i)

- If n is a cat-node with left c1 and right c2, and i ∈ lastpos(c1), then all positions in firstpos(c2) are in followpos(i).
- If n is a star-node and i ∈ lastpos(n), then all positions in firstpos(n) are in followpos(i).

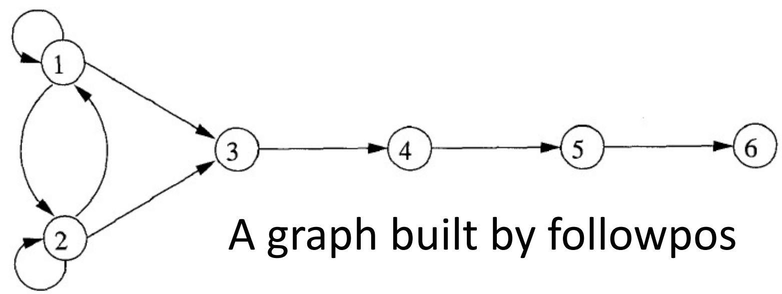
# Regular Expressions to DFA

Example

(a|b)*abb#




| NODE $n$ | $followpos(n)$ |
|---|---|
| 1 | $\{1, 2, 3\}$ |
| 2 | $\{1, 2, 3\}$ |
| 3 | $\{4\}$ |
| 4 | $\{5\}$ |
| 5 | $\{6\}$ |
| 6 | $\emptyset$ |




A graph built by followpos

Construct NFA without ϵ-transition

1. Make all positions in the firstpos of the root initial states
2. Label each edge (i,j) with the symbol at position i.
3. Make the position for # the only accepting state.

# Regular Expressions to DFA

Apply the subset construction algorithm directly to the implicit NFA.

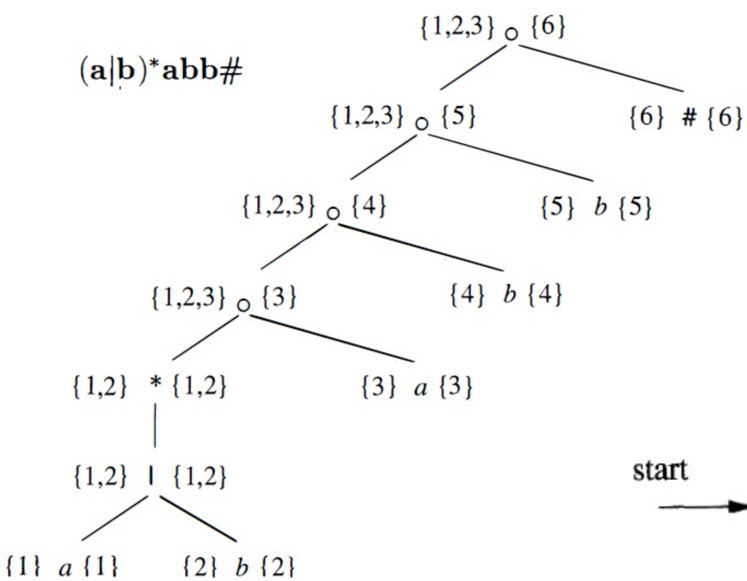
1. Construct a syntax tree for (r)#
2. Construct, nullable, firstpos, lastpos, and followpos
3. Construct Dstates and Dtran using the algorithm below. The start state is firstpos(root), the accepting states are the ones with the position for #.

```
initialize Dstates to contain only the unmarked state firstpos(n0),
        where n0 is the root of syntax tree T for (r)#;
while ( there is an unmarked state S in Dstates ) {
        mark S;
        for ( each input symbol a ) {
                let U be the union of followpos(p) for all p
                        in S that correspond to a;
                if ( U is not in Dstates )
                        add U as an unmarked state to Dstates;
                Dtran[S, a] = U;
        }
}
```

# Regular Expressions to DFA

Example

(a|b)*abb#




| NODE $n$ | $followpos(n)$ |
|---|---|
| 1 | $\{1,2,3\}$ |
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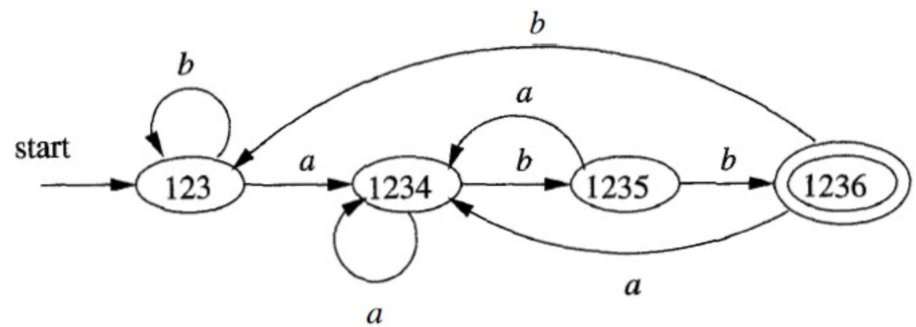


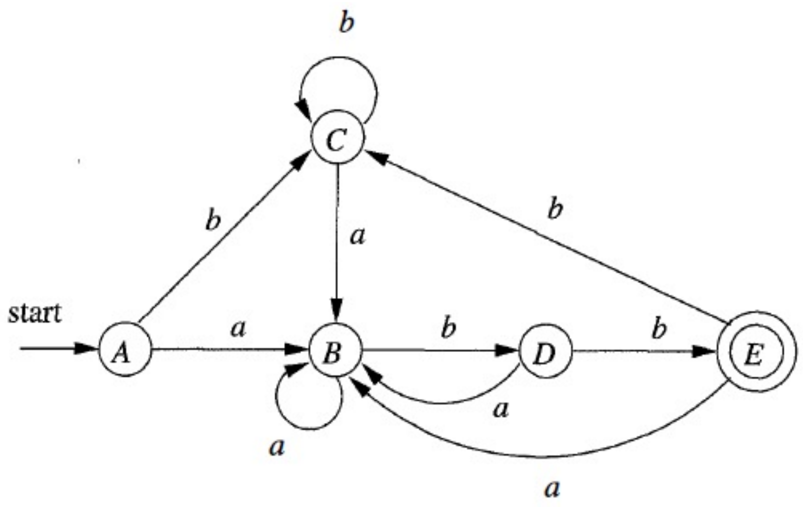

Exercise: Build a DFA for a(a|b)*b

# Minimizing the number of DFA states

Make every state has a transition on every input symbol. (add a dead state d if necessary)

String w distinguishes states s and t if feeding w from the states ended up with an accepting state in one case and a non-accepting state in the other.

- Starting from F and S-F, keep partitioning the states until they are not distinguishable.



# Minimizing the number of DFA states

1. Start with an initial partition $\Pi$ with two groups, $F$ and $S - F$, the accepting and nonaccepting states of $D$.

2. Apply the procedure of Fig. 3.64 to construct a new partition $\Pi_{\text{new}}$.

```
initially, let Π_new = Π;
for ( each group G of Π ) {
        partition G into subgroups such that two states s and t
                are in the same subgroup if and only if for all
                input symbols a, states s and t have transitions on a
                to states in the same group of Π;
        /* at worst, a state will be in a subgroup by itself */
        replace G in Π_new by the set of all subgroups formed;
}
```

Figure 3.64: Construction of $\Pi_{\text{new}}$

3. If $\Pi_{\text{new}} = \Pi$, let $\Pi_{\text{final}} = \Pi$ and continue with step (4). Otherwise, repeat step (2) with $\Pi_{\text{new}}$ in place of $\Pi$.

4. Choose one state in each group of $\Pi_{\text{final}}$ as the *representative* for that group. The representatives will be the states of the minimum-state DFA $D'$. The other components of $D'$ are constructed as follows:

# Questions?