Computer Science Principles

CHAPTER 7 – RANDOM NUMBERS AND OBJECT-ORIENTED PROGRAMMING

Announcements

Read Chapter 7 in the Conery textbook (Explorations in Computing)

Acknowledgement: These slides are revised versions of slides prepared by Prof. Arthur Lee, Tony Mione, and Pravin Pawar for earlier CSE 101 classes. Some slides are based on Prof. Kevin McDonald at SBU CSE 101 lecture notes and the textbook by John Conery.

Games Involving Chance

Many games involve chance of some kind:

- Card games with drawing cards from a shuffled deck
- Rolling dice to determine how many places we move a piece on a game board
- Spinning a wheel to randomly determine an outcome

Video games use chance and randomization all the time

- Determining random events or damage
- Choosing among potential moves the computer will make

We expect these outcomes to be **random** or unbiased – in other words, unpredictable

- Computers can be programmed to generate *apparently* "random" sequences of numbers and other quantities for such games and other applications

Lecture Overview

In this lecture we will explore algorithms for generating values that are apparently random and unpredictable

- We say "apparently" because we need to use mathematical formulas to generate sequences of numbers that at the very least appear to be random

Since we will use an algorithm to generate "random" values, we really can't say the sequence of values is truly random

We say instead that a computer generates **pseudorandom** numbers

Pseudorandom Numbers

Randomness is a difficult property to quantify

- Is the list **[3, 7, 1, 4]** more or less random than **[4, 1, 7, 3]**?

The algorithm that generates pseudorandom numbers is called **pseudorandom number generator**, or PRNG

- The goal is for the algorithm to generate numbers without any kind of apparent predictability
- Python has a built-in capability to generate random values through its **random** module

To generate a random integer in the range 1-100:

```
import random
num = random.randint(1,100)    # up to 100, not 101!
```

Modular Arithmetic

The **modulo** operator, **%** in Python, will be a key part of generating pseudorandom numbers

Suppose we want to generate a seemingly random sequence of numbers, all in the range 0 through 11

Let's start with the number 0 and store it in a new list named **t**:

```
t = [0]
```

One basic formula for generating numbers involves:

(1) Adding a value to the previously-generated number and then

(2) Performing a modulo operation

Modular Arithmetic

For our particular example, we could generate a new number by adding 7 to the prior value then mod by 12

- Conveniently, the Python language lets us write **`t[-1]`** to mean "retrieve the last element of list **t**"

So in general we can write **`t.append((t[-1]+7)%12)`** to generate and store the "next" pseudorandom number

If we put this code inside a loop, we can generate a series of random values and store them in the list

Modular Arithmetic

```
t = [0]
for i in range(15):
   t.append((t[-1] + 7) % 12)
```

The above code will generate the list:

```
[0,7,2,9,4,11,6,1,8,3,10,5,0,7,2,9]
```

How "random" are these numbers?

- They look pretty random, but we notice that eventually they start to repeat

Can we improve things?

- Part of the issue is the divisor of 12, but the formula itself is a little too simplistic

Modular Arithmetic

A more general formula for generating pseudorandom numbers is:

x_{i+1}=(a*x_i+c) mod *m*

- x_{i+1} is the "next" random number
- x_i is the most recently generated random number
- *i* is the position of the number in the list
- *a*, *c* and *m* are constants called the *multiplier*, *increment*, and *modulus*, respectively

If the values *a*, *c* and *m* are chosen carefully, then every value from 0 through *m-1* will appear in the list exactly once before the sequence repeats

The number of items in the repetitive part of the list is called the **period** of the list

- We want the period to be as long as possible to make the numbers as unpredictable as possible

We will implement the above formula, but first we need to explore some new programming concepts

Numbers on Demand

One possibility for working with random numbers is to generate as many as we need and store them in a list

- Often we don't know exactly how many random numbers we will ultimately need
- Also, in practice we might not want to generate a very long list of random numbers and store them

Typically, we need only one or just a few random numbers at a time, so generating thousands or even millions of them at once is a waste of time and memory

Rather than building such a list, we can instead generate the numbers one at a time, on demand

Numbers on Demand

We will define a function **rand()** and a **global variable x** to store the most recently generated random number

- A *global variable* is a variable defined outside functions and is available for use by any function in a .py file

The value of a global variable is preserved between function calls, unlike local variables, which disappear when a function returns

If we want a function to change the value of a global variable, we need to indicate this by using the **global** keyword in the function

- If we are only reading the global variable, we do not need to use the **global** keyword

The rand() Function (v1)

Let's consider a function for generating random numbers that uses the formula we saw earlier:

```
x = 0      # global variable
def rand(a, c, m):
    global x
    x = (a * x + c) % m
    return x
```

Call the function several times with *a*=1, *c*=7, *m*=12:

```
rand(1, 7, 12)    # returns 7 and updates x to 7
rand(1, 7, 12)    # returns 2 and updates x to 2
rand(1, 7, 12)    # returns 9 and updates x to 9
```

Let's see why x is updated in this way

The rand() Function (v1)

The key line of code is `x = (a * x + c) % m`

Initially, `x = 0`

1. `rand(1,7,12)`: `x = (1 * 0 + 7) % 12 = 7`

   So, `x` becomes `7`
2. `rand(1,7,12)`: `x = (1 * 7 + 7) % 12 = 2`

   So, `x` becomes `2`
3. `rand(1,7,12)`: `x = (1 * 2 + 7) % 12 = 9`

   So, `x` becomes `9`

The only reason this series of computations works correctly is because the value of **x** is preserved between function calls

Modules and Encapsulation

Suppose we wanted to use our new **`rand()`** function in several files. We have two options:

- Copy and paste the function into each file (bad idea)
- Place the function in a Python program file to create a **module** that can be imported using an **`import`** statement (the right way)

We should place our function in a separate file (module) along with the global variable **`x`**

This global variable will be "hidden" inside the module so that there is no danger of a "name clash", meaning that other modules could have their own global variables named **`x`** if they want to

Modules and Encapsulation

This idea of gathering functions and their related data values (variables) into a single package is called ***encapsulation***

- It's an extension of the concept called **abstraction** we studied earlier in the course

We know that the **math** module has some useful functions and constants, like **`sqrt()`** and **`pi`**

A module like **math** is an example of a ***namespace***, a collection of names that could be names of functions, objects or anything else in Python that has a name

- A module/namespace is one way of implementing the concept of encapsulation in Python

Modules and Encapsulation

To create a new module, all we need to do is save the functions and variables of the module in a file ending in .py

- For example, if we were to save the **rand()** function in the file **prng.py**, we could then import the **rand()** function in a new Python program by typing **import prng** at the top of the new program

Next slide shows a revised version of our **rand()** function that encapsulates the function in a module and stores the values of *x*, *a*, *c* and *m* as global variables

- This means the user no longer needs to pass *a*, *c,* or *m* as arguments anymore
- We will also add a new function **reset()** to reset the PRNG to its starting state

The rand() Function (v2)

```
x = 0
a = 81
c = 337
m = 1000

def reset(mult, inc, mod):
    global x, a, c, m
    x = 0
    a = mult
    c = inc
    m = mod

def rand():
    global x
    x = (a * x + c) % m
    return x
```

The rand() Function (v2)

x = 0

a = 81

c = 337

m = 1000

Examples:

1. **rand(): (81 * 0 + 337) % 1000 = 337**
2. **rand(): (81 * 337 + 337) % 1000 = 634**
3. **rand(): (81 * 634 + 337) % 1000 = 691**

The rand() Function (v2)

We can change the values of *a*, *c*, and *m* by calling the **reset()** function.

- For example: **reset(19, 4, 999)**, which also sets **x = 0**.

Now we will generate a different sequence of random numbers:

1. **rand()**: **(19 * 0 + 4) % 999 = 4**
2. **rand()**: **(19 * 4 + 4) % 999 = 80**
3. **rand()**: **(19 * 80 + 4) % 999 = 525**

Games with Random Numbers

Suppose we wanted to simulate the rolling of a six-sided die in a board game

- We would want to generate integers in the range 1 through 6, inclusive
- However, our function **rand()** generates values outside this range

We can solve this problem using an expression like **rand() % 6 + 1**

- The expression **rand() % 6** gives us a value in the range 0 through 5, which we can then “shift up” by adding 1

Why not do **rand() % 7** instead?

Games with Random Numbers

If we always initialize *x*, *a*, *c,* and *m* to the same values, then every program that uses the **rand()** function will get the same exactly sequence of pseudorandom values

We want these numbers to be different. Some options:

- We could allow someone using our code to set the starting value of *x*, which we call the **seed** of the pseudorandom number generator
- Another option is to have the computer pick the seed by using the system clock

The time module has a function called **time()** which returns the number of seconds since January 1, 1970

- Fractions of a second are also included in the returned value

Games with Random Numbers

Our revised module shown below uses **time.time()** to pick a random seed

```
import time
a = 81
c = 337
m = 1000
x = int(time.time()) % m
def rand():
   global x
   x = (a * x + c) % m
   return x
```

See random_numbers.py

Random Numbers in a Range

In general, how can we generate random integers from an arbitrary range?

The formula for this is:

```
    rand() % (high – low + 1) + low
```

For example, suppose we wanted to generate a value in the range -5 through 10, inclusive

The formula indicates we should use this code:

```
    rand() % (10 - (-5) + 1) + (-5)
```

Simplifying gives us:

```
    rand() % 16 – 5
```

See random_numbers.py

List Comprehensions

Python features a very compact syntax for generating a list called a **list comprehension**

- Write a pair of square brackets and inside the brackets put an expression that describes each list item

For example, to make a list of numbers from 1 to 10 write

```
[i for i in range(1,11)]
```

To make a list of the first 10 perfect squares we can write

```
[i**2 for i in range(1,11)]
```

In general, we write an expression that describes each new item in the new list and a loop that describes a set of existing values to work from

A list of 10 random numbers:

```
[rand() for i in range(10)]
```

List Comprehensions

Suppose we wanted to take a list of words and capitalize them all:

```
names = ['bob', 'DANE', 'mikey', 'ToMmY']
names = [s.capitalize() for s in names]
```

names would become **['Bob', 'Dane', 'Mikey', 'Tommy']**

Or perhaps we wanted to extract the first initial of each person and capitalize it:

```
initials = [s[0].upper() for s in names]
```

initials would be **['B', 'D', 'M', 'T']**

Random Shuffles

Suppose we needed the ability to randomly *permute* (shuffle) a list of items, such as a deck of 52 playing cards

- Let's explore how we might write a function that does exactly this

The **RandomLab** module defines a **class** called **Card**

A ***class*** defines a type of object in an object-oriented programming language like Python

- Remember that int, float, and str (string) are all classes

Random Shuffles –

We use a special function called the **constructor** to create (construct) new objects of the class

```
from PythonLabs.RandomLab import Card
card = Card()
```

We have been using constructors for a while, for example:

```
some_float = 5.3
now_a_string = str(some_float)     # this constructs a string with the value '5.3'
```

The Card Class

A **Card** object has a separate **rank** and **suit**, which we can query using the **rank()** and **suit()** methods, respectively

- The 2 through Ace are ranked 0 through 12

The suits are mapped to integers as follows:

- Clubs: 0
- Diamonds: 1
- Hearts: 2
- Spades: 3

For example, for a **Card** object representing the 9 of Spades:

- **rank()** would return 7
- **suit()** would return 3

The Card Class

The ranks and suits are numbered so that we can uniquely identify each card of a standard 52-card deck

- When calling the constructor to create a **Card** object, we provide a number in the range 0 through 51 to identify which card we want

Examples:

- **Card(0)** and **Card(1)** are the 2 and 3 of Clubs, respectively
- **Card(50)** and **Card(51)** are the King and Ace of Spades, respectively
- **Card(46)** is 9 of Spades

print(Card(51)) would output **A**   (yes, including that Spade symbol!)

The Card Class

We can use a list comprehension to generate all 52 cards and store them in a list:

```
deck = [Card(i) for i in range(0,52)]
```

With slicing we can take a look at the first 5 cards by appending **[:5]** to the name of the variable

- Remember this notation means “slice out all the elements of the list up to (but not including) the element at index 5”

```
print(deck[:5])
```

Output: **[2   , 3   , 4   , 5   , 6   ]**

Shuffling Card Objects

The order of the cards generated by the list comprehension (i.e., sequential order) is only one particular ordering or **permutation** of the cards

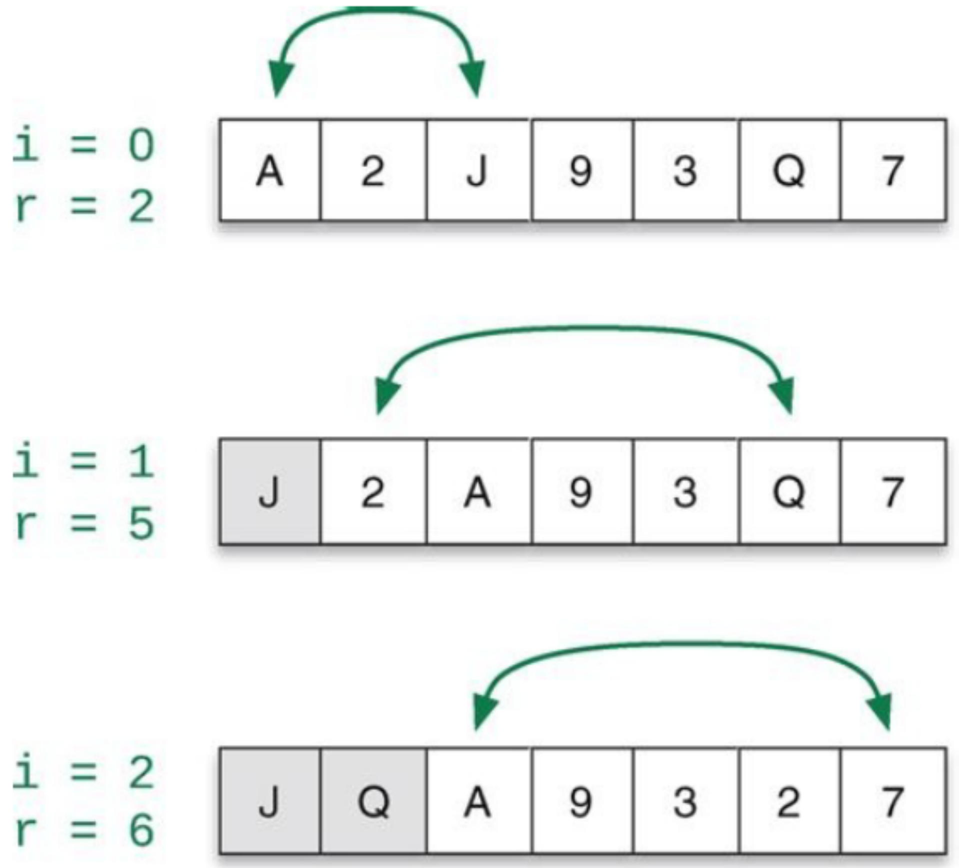
We want to define a function that will let us permute a list to generate a more random ordering of the items in the list

A simple algorithm for permuting the items in a list is to iterate over the list and exchange each element with a random element to its right

- This is most easily seen by example, as on the next slide

Shuffling Card Objects

Iterate over the entire list **deck** (with **i** as the loop variable and index), swapping a random item to the right of **i** with **deck[i]**



Shuffling Card Objects

This shuffling algorithm is easy to implement with the help of a function that will choose a random item to the right of **deck[i]**

The function **randint(low, high)** from the **random** module generates a random integer in the range **low** through **high** (inclusive of both **low** and **high**)

The **permute** function will shuffle any list of items:

```
import random
def permute(a):
  for i in range(0, len(a)-1):
    r = random.randint(i, len(a)-1)
    a[i], a[r] = a[r], a[i]   # swap items
```

Shuffling Card Objects

```
import random
def permute(a):
    for i in range(0, len(a)-1):
        r = random.randint(i, len(a)-1)
        a[i], a[r] = a[r], a[i]    # swap items
```

r = random.randint(i, len(a)-1) picks the random index, **r**, that is to the right of **i** (or might choose **i** itself, meaning that **a[i]** doesn't move)

a[i], a[r] = a[r], a[i] swaps **a[i]** with the randomly chosen item to its right

We would call this function with **permute(deck)** to shuffle our list of **Card** objects

Defining New Objects

The **Card** class we have been working with defines a new kind of object we can use in programs

In object-oriented programming, a class determines the data and operations associated with an object

For example, for a playing card object we need some way to store the rank and suit of a card

- These are the card's data attributes

Operations for a playing card might include code that lets us print a playing card on the screen or retrieve the card's rank and suit

Defining New Objects

The data values associated with a particular object are called **instance variables**

We say that an object is an **instance** of a class

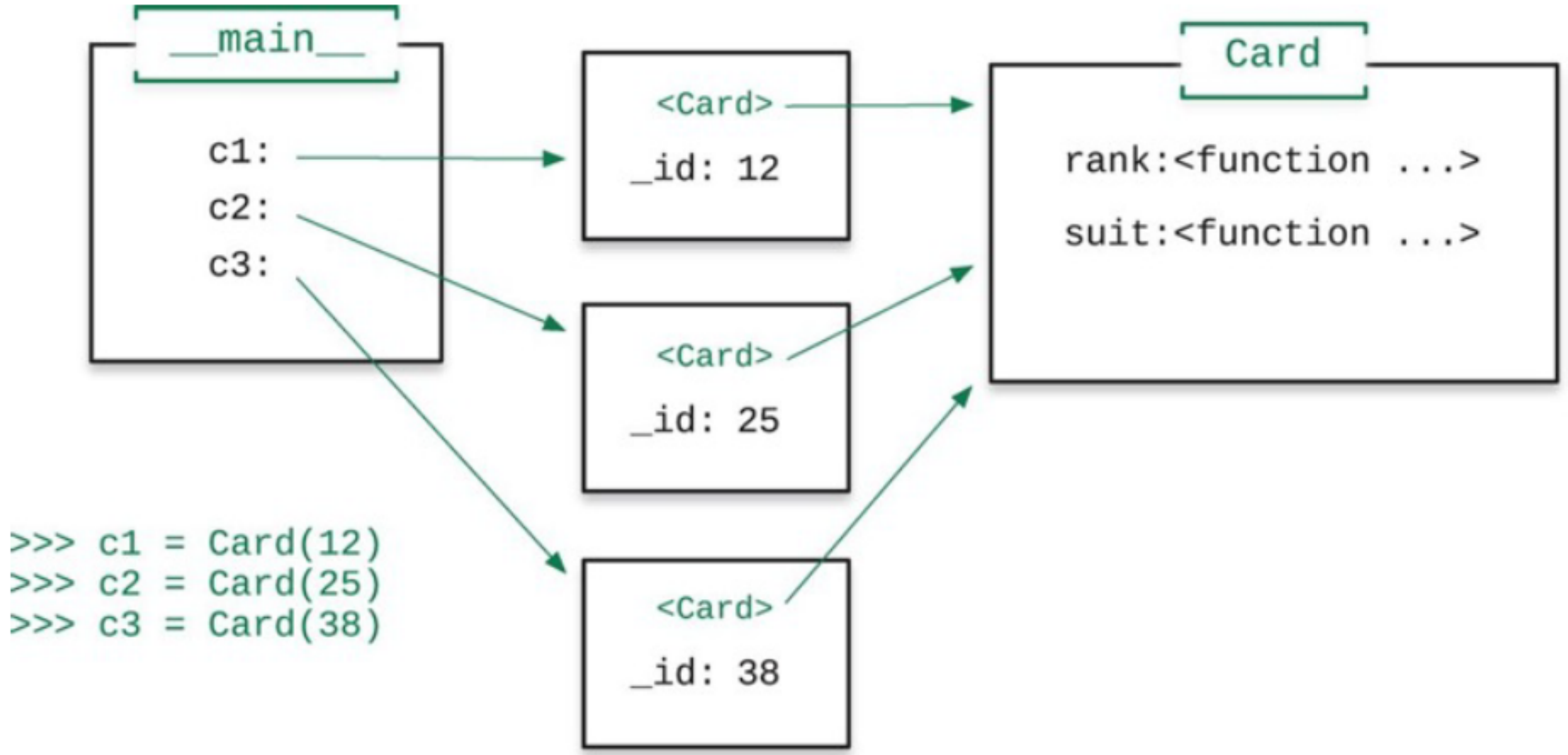
- For example, each of the 52 **Card** objects is an independent instance of the **Card** class
- As such, each **Card** object has its own copies of the instance variable that store the object's rank and suit

The operations associated with an object are called **methods**

So, a class defines the data properties and methods that an object of the class has

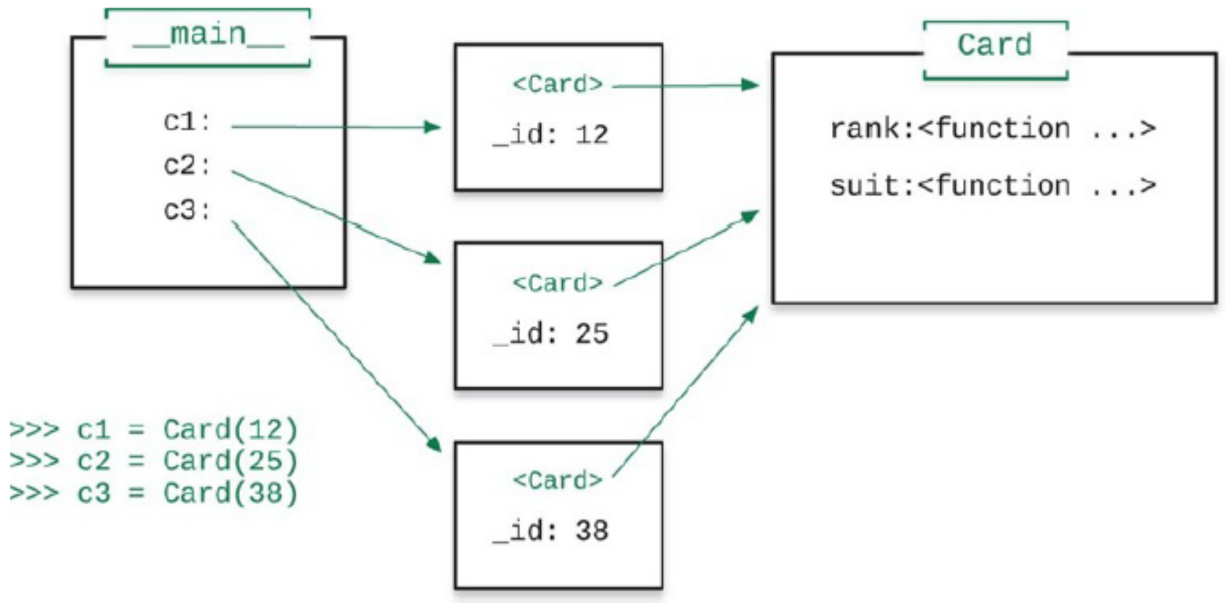
Defining New Objects

Let's see an example where we create three distinct **Card** objects in a program:



```
>>> c1 = Card(12)
>>> c2 = Card(25)
>>> c3 = Card(38)
```

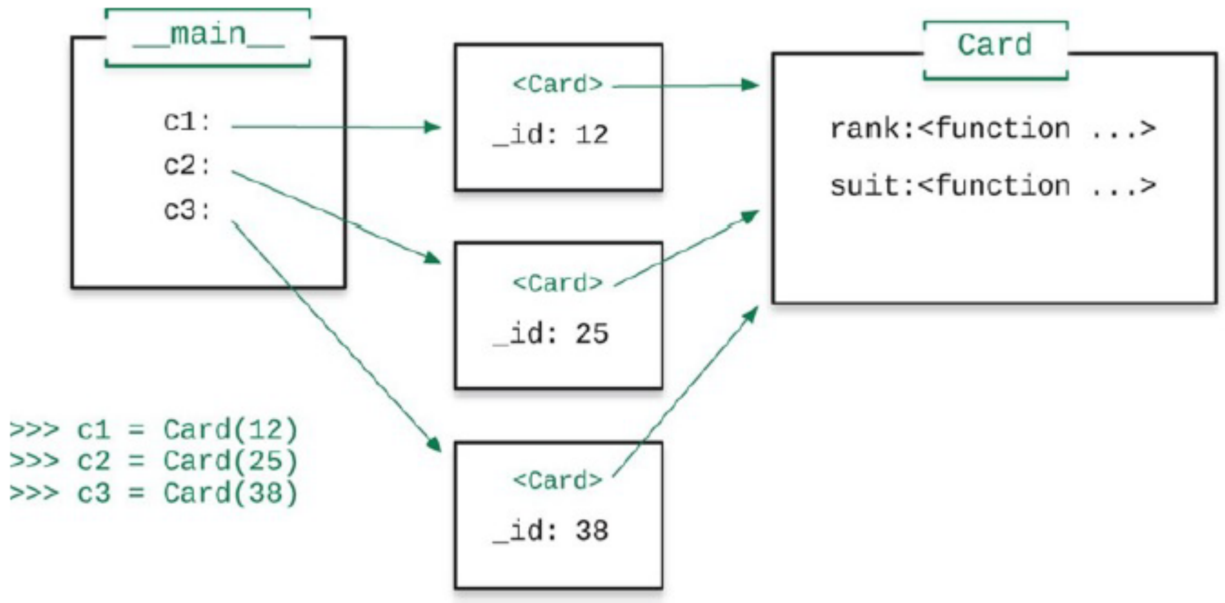
Defining New Objects



```
>>> c1 = Card(12)
>>> c2 = Card(25)
>>> c3 = Card(38)
```

- Three **Card** objects were constructed. They are referenced using the variables **c1**, **c2** and **c3** in **main** as shown on the left
- The objects as they might exist in the computer memory are shown in the middle of the diagram
  - Rather than storing the rank and suit separately, they are combined into a single integer called **_id**

Defining New Objects



```
>>> c1 = Card(12)
>>> c2 = Card(25)
>>> c3 = Card(38)
```

- Prepending an underscore to a variable indicates that **_id** is an instance variable; this is a naming convention, not a strict rule
- To retrieve the rank or suit, we need to call the methods **rank()** or **suit()**, as depicted on the right
  - Example call: **c1.rank()** since **rank()** is a method, not a function

Defining New Objects

To define a new class we usually include the following aspects:

- One or more instance variables
- One or more methods that perform some operation or execute some algorithm
- A __**init**__ method, which initializes (gives starting values to) the instance variables
- A __**repr**__ method, which defines a string representation of an object that is suitable for printing on the screen

Let's step through building the **Card** class from the ground up

Building the Card Class

The code we will build up will all eventually be saved in a file named **Card.py**

We begin by writing a class statement:

```
class Card:
```

Next we write the **__init__** method. This method is to "initialize" or construct new objects of the class when Card() is called.

```
def __init__(self, n):
    self._id = n      # _id is an instance variable to represent the suit and rank
```

The **self** keyword refers to the object itself.

Building the Card Class

Now let's write a simple __**repr**__ method, which stands for "representation"

- This define what happens if we try to print a **Card** object

```
def __repr__(self):
   return 'Card #' + str(self._id)
```

Suppose we create card #43 and print it:

```
cl = Card(43)
print(cl)        # Prints out 'Card #43"
```

That's not very informative, but we we will improve this later

Building the Card Class

Now we can write the **rank()** and **suit()** methods

They translate the **_id** number into the rank and suit of a card

```
def suit(self):
    return self._id // 13
def rank(self):
    return self._id % 13
```

This encoding ensures that all 13 cards of a single suit are placed together in consecutive order

Building the Card Class

The **Card** class so far:

```
class Card:

    def __init__(self, n):
        self._id = n

    def suit(self):
        return self._id // 13

    def rank(self):
        return self._id % 13

    def __repr__(self):
        return 'Card #' + str(self._id)
```

Building the Card Class (next)

We can write a function **new_deck()** that creates a list of 52 playing-card objects.

```
def new_deck():
    return [Card(i) for i in range(52)]
```

Note that this function is not part of the **Card** class itself, it just uses the **Card** class.

An example call to this function:

```
deck = new_deck()
```

Building the Card Class

Another improvement we can make is to add special methods that allow us to compare **Card** objects

If we want to be able to sort **Card** objects, we must provide the **__lt__()** method, which tells us if one object is "less than" another:

```
def __lt__(self, other):
    return self._id < other._id
```

__eq__() defines what it means for two **Card** objects to be "equal to" each other:

```
def __eq__(self, other):
    return self._id == other._id
```

Building the Card Class

For example, consider the following objects:

```
c1 = Card(1)
c2 = Card(4)
```

The expression **c1 < c2** would be **True**, but **c1 == c2** would be **False**

Now that we can compare **Card** objects, we can sort them using the **sorted** function

- **sorted** makes a copy of a list and then sorts the copy:

```
cards_sorted = sorted(cards)
```

Building the Card Class

The **Card** class defines an **application program interface** (API): a set of related methods that other programmers can use to build other software

- We have been using APIs that others built all throughout this course

We have applied the concept of encapsulation by gathering all the code that defines a **Card** object in one place

Building the Card Class

For our **Card** class it would be useful to print symbols representing the suits:

- Since every **Card** object would need to know the suit, we can define **class variables** for this
- Class variables are values that pertain to a particular class but are not instance variables

In Python we have access to many thousands of symbols

- We can access them by giving the correct numeric codes

Let's add two class variables: **suit_sym** and **rank_sym** to **Card** class

Building the Card Class

```
suit_sym = {0: '\u2663', 1: '\u2666',
            2: '\u2665', 3: '\u2660'}
```

If we were to print **suit_sym**, we would get this output:

```
{0: '   ', 1: '   ', 2: '   ', 3: '   '}
```

The codes for various symbols can be found on the Internet by searching for "**Unicode characters**"

Likewise, we can define a dictionary for all the ranks:

```
rank_sym = {0: '2', 1: '3', 2: '4', 3: '5', 4: '6',
            5: '7', 6: '8', 7: '9', 8: '10',
            9: 'J', 10: 'Q', 11: 'K', 12: 'A'}
```

Our goal now is to be able to print a **Card** object in a form like '2   ' – let's do that next

Building the Card Class

We will change our definition of the __**repr**__ method to this:

```
def __repr__(self):
   return Card.rank_sym[self.rank()] + Card.suit_sym[self.suit()]
```

Now, when we print a **Card** object, we will get output like 2 , A , 8 , J , etc.

Exceptions and Exception-handling

What if another programmer using our class inadvertently gives a value outside the range 0 through 51 for **n** when constructing a **Card** object?

- The __**init**__ method will accept the value, but it really shouldn't

We can solve this problem by adding **exception handling** to our code

- An ***exception*** is an unexpected event or error that has been detected
- We say that the program has **raised** an exception

Let's have the __**init**__ method raise an exception if we have an invalid value for **n**

Exceptions and Exception-handling

```
def __init__(self, n):
    if n in range(0, 52):
        self._id = n
    else:
        raise Exception('Card number must be in the range 0-51.')
```

The new version of **__init__** verifies that the argument **n** is valid

If not, it raises the exception and includes a diagnostic message of sorts

Exceptions and Exception-handling

Consider a function now that a programmer might use to make new cards that catches any exception that might be thrown by the __**init**__ method:

```
def make_card(n):
   try:
      return Card(n)
   except Exception as e:
      print('Invalid card: ' + str(e))
      return None
```

If we call **make_card(55)**, we get this output:

```
Invalid card: Card number must be in the range 0-51.
```

Exceptions and Exception-handling

This concludes our development of the **Card** class

See card.py for the completed **Card** class and use_card.py and use_card2.py for some tests

Note: To run, drag use_card.py into PyCharm and run it. Be sure that card.py is in the same folder where use_card.py is located

Questions?