Problem Warmup (0 points)
This is a warmup problem. Try this one before you try the rest of the problems in the set.

First, come up with a relation with some functional dependencies. At least one of the functional dependencies violates BCNF and at least one other of the functional dependencies makes it not dependency-preserving. Now, do lossless-join decomposition into BCNF although it may not be dependency-preserving.

Second, come up with a relation with some functional dependencies that make it dependency-preserving but not lossless-join.

Problem 1 (10 points)
Did you try to do the Warmup Problem? If you have not, I strongly recommend that you do that first.

List non-trivial functional dependencies satisfied by the following relation. You do not need to find all of them. It is enough to identify a set of functional dependencies that imply all functional dependencies that are satisfied by the relation.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b1</td>
<td>c2</td>
</tr>
<tr>
<td>a2</td>
<td>b1</td>
<td>c1</td>
</tr>
<tr>
<td>a2</td>
<td>b1</td>
<td>c3</td>
</tr>
</tbody>
</table>

Problem 2 (20 points)
Consider a relation \( R \) with five attributes \( ABCDE \). You are given the following functional dependencies:

\[ A \rightarrow B \]
\[ BC \rightarrow E \]
\[ ED \rightarrow A \]

(a) List all the candidate keys for \( R \).
(b) Is $R$ in 3NF? Explain.
(c) Is $R$ in BCNF? Explain.

**Problem 3 (20 points)**

Suppose a relation $R$ contains a functional dependency that violates the BCNF condition and another functional dependency whose left side is a superkey for $R$. Is it possible to decompose $R$ into a set of relations in BCNF while preserving all the FD’s of $R$. Explain your answer.

**Problem 4 (35 points)**

Assume the following set of functional dependencies hold for the relation $R(A, B, C, D, E)$:

- $AB \rightarrow C$
- $DE \rightarrow C$
- $B \rightarrow D$

(a) List all the candidate keys.
(b) Is it in BCNF? Explain your answer. If it is not, normalize it into a set of relations to be in BCNF. Show your work while indicating which dependency you are applying in each decomposition.
(c) Is it in 3NF? Explain your answer. If it is not, normalize it into a set of relations to be in 3NF. Show your work while indicating how you obtained each decomposition. Note that the set of functional dependencies given in the problem is a minimal basis for $R$.

**Problem 5 (15 points)**

The algorithm for decomposing a relation $R$ into a set of relations in BCNF is non-deterministic. That is, if there are multiple functional dependencies of $R$ that violate the BCNF condition, we can pick any one of them to decompose the relation $R$. The algorithm eventually produces a set of relations in BCNF. Is it true that the algorithm will always produce the same set of relations in BCNF regardless of the order of the functional dependencies that we apply as we decompose $R$? If your answer is true, show why. If not, give a counter example.