Triangular Bezier Patch

- Triangular Bezier Surface

\[ s(u,v) = \sum_{i,j,k \geq 0} p_{i,j,k} B_{i,j,k}^n(r,s,t), \]

where \( i + j + k = n, r + s + t = 1 \), and they are local barycentric coordinates.

- Basis functions are Bernstein polynomials of degree \( n \)

\[ B_{i,j,k}^n(r,s,t) = \frac{n!}{i!j!k!} r^i s^j t^k \]

- How many basis functions: \( \frac{1}{2}(n + 1)(n + 2) \)

- How many control points: \( \frac{1}{2}(n + 1)(n + 2) \)

- \( r, s, t \): local barycentric coordinates in the \( uv \)-plane

- Partition of unity

\[ \sum_{i,j,k \geq 0} B_{i,j,k}^n(r,s,t) = 1 \]
• Positivity

\[ B_{i,j,k}^n(r, s, t) \geq 0, \quad r, s, t \in [0, 1] \]

• Recursion

\[ B_{i,j,k}^n(r, s, t) = rB_{i-1,j,k}^{n-1} + sB_{i,j-1,k}^{n-1} + tB_{i,j,k-1}^{n-1} \]

• Recursive evaluation

\[ p_{i,j,k}^0 = p_{i,j,k} \]

\[ p_{i,j,k}^l = rp_{i+1,j,k}^{l-1} + sp_{i,j+1,k}^{l-1} + tp_{i,j,k+1}^{l-1} \]

where \( i + j + k = n - l \), and \( i, j, k \geq 0 \)

\[ s(u,v) = p_{0,0,0}^n \]

(refer to the diagram)

• Efficient algorithms

• (Directional) derivatives

• Degree elevation

• Subdivision
- Composite surfaces
- Continuity across adjacent patches
- Integral computation
- Triangular splines over regular triangulation
- Transform triangular splines to a set of piecewise triangular Bezier patches
- Interpolation/approximation using triangular splines
Recursive Evaluation
Triangular Domain

uv-plane

rst-plane

(r,s,t)

(u,v)
Basis functions (Cubic)