CSE541  Practice Midterm 1  Spring 2015
25 pts extra credit

NAME ID:

QUESTION 1
Write the following natural language statement:
One likes to play bridge, or from the fact that the weather is good we conclude the following: one does not like to play bridge or one likes not to play bridge
as a formula of 2 different languages

1. Formula $A_1 \in F_1$ of a language $L_{\{\neg, L, \cup, \Rightarrow\}}$, where $L_A$ represents statement "one likes $A$", "$A$ is liked".

2. Formula $A_2 \in F_2$ of a language $L_{\{\neg, \cup, \Rightarrow\}}$.

Write carefully all steps of your translation.

QUESTION 2
Write the formal definition of the language $L_{\{\neg, L, \cup, \Rightarrow\}}$ and give examples if is formulas of the degrees 0, 1, 2, 3, and 4.

QUESTION 3
Define formally your OWN 3 valued extensional semantics $M$ for the language $L_{\{\neg, L, \cup, \Rightarrow\}}$ under the following assumptions

1. Assume that the third value is intermediate between truth and falsity, i.e. the set of logical values is ordered and we have the following

Assumption 1 $F < \bot < T$

Assumption 2 $T$ is the designated value

2. Model the situation in which one "likes" only truth; i.e. in which $L_T = T$ and $L_{\bot} = F$, $L_F = F$

3. The connectives $\neg, \cup, \Rightarrow$ can be defined as you wish, but you have to define them in such a way to make sure that

$\models_M (L_A \cup \neg L_A)$

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**REMINDER**

**Formal definition** of many valued extensional semantics follows the pattern of the classical case and consists of giving **definitions** of the following main components:

1. Logical Connectives
2. Truth Assignment
3. Satisfaction Relation, Model, Counter-Model
4. Tautology

**QUESTION 3**

1. Verify whether the formulas $A_1$ and $A_2$ from the **QUESTION 1** have a model/counter model under your semantics $M$. You can use shorthand notation.

2. Verify whether the following set $G$ is $M$-consistent. You can use shorthand notation:

$$G = \{ L_a, (a \cup \neg L_b), (a \Rightarrow b), b \}$$

3. Give an example on an infinite, $M$-consistent set of formulas of the language $\mathcal{L}(\neg, L, \cup, \Rightarrow)$.

**QUESTION 4**

Let $S$ be the following **proof system**:

$$S = (\mathcal{L}(\neg, L, \cup, \Rightarrow), \mathcal{F}, \{A_1, A_2\}, \{r_1, r_2\})$$

for the logical axioms and rules of inference defined for any formulas $A, B \in \mathcal{F}$ as follows:

**Logical Axioms**

$A_1$ \hspace{1em} ($L_A \cup \neg L_A$)

$A_2$ \hspace{1em} ($A \Rightarrow L_A$)

**Rules** of inference:

\[
\begin{align*}
(r1) \quad & A ; B \\
& (A \cup B), \\
(r2) \quad & A \\
& L(A \Rightarrow B)
\end{align*}
\]
1. Write a proof in $S$ with 2 applications of rule (r1) and one application of rule (r2)

You must write comments how each step pot the proof was obtained

2. Show, by constructing a formal proof that

$$\vdash_S ((Lb \cup \neg Lb) \cup L((La \cup \neg La) \Rightarrow b)))$$

3. Verify whether the inference rules r1, r2 are M-sound. You can use shorthand notation

4. Verify whether the system $S$ is M-sound. You can use shorthand notation

EXTRA QUESTION

If the system $S$ is not sound under your semantics M then re-define the connectives in a way that such obtained new semantics N would make $S$ sound.

You can use shorthand notation