LECTURE 1
GENERAL INFORMATION
Course Web Page
www.cs.stonybrook.edu/~cse541

The webpage contains
Course Syllabus
Book Chapters
Lectures slides
Sample Exercises and Problems with solutions
Sample Tests
Course Text Book

AN INTRODUCTION TO CLASSICAL and NON-CLASSICAL LOGICS
Anita Wasilewska

Book Chapters and Lecture Slides are in Downloads on the course web page

This is a book in writing; all correction, remarks by students are welcome!

Additional Books:
A Friendly Introduction to Mathematical Logic, C.C. Leary, Prientice Hall, 2000
Course Goal

The goal of the course is to make student understand the need of, and to learn the formality of logic as scientific field. The book is written with students on my mind so that they can read and learn by themselves, even before coming to class. The goal the course and of the book is to teach not only understanding of classical and some non-classical logics but to teach also what is a formal logic as scientific subject, with its languages, definitions, problems and basic theorems.
Tests

There will be **TWO MIDTERMS** and a **FINAL** examination
I will also give **TWO PRACTICE TESTS** for extra credit

**Midterms 1, 2** (100pts each)

**Final** (100pts)

**Practice Midterm 1** (10 extra points) - given in class

**Practice Final** (10 extra points) - take home

Course Webpage contains many examples and exercises from the text book and solutions for posted Exercises-Homework Problems for you to **study** from

I also posted some **previous TESTS** (with solutions) so you could see the **format** of tests

**GRADES** for the tests will depend on the form, details, and carefulness of written solutions.
Grading

During the semester you can earn 300pts or more (in the case of extra points). The grade will be determined in the following way:

# of earned points divided by 3 = % grade

The % grade is translated into letter grade in a standard way i.e.

100 - 90 % is A range;
A (100-96%), A- (95- 90%)
89 - 80 % is B range:
B- (80 - 83%), B (84 -86%), B+ (87 -89%)
79 - 70 % is C range:
C- (70- 72%), C (73-76%), C+(77-79%)
69 - 60 % is D range
F is below 60%

None of grades will be curved
Course Contents and Schedule

The course will follow the book very closely and in particular we will cover some, or all of the following subjects chapters

Part one
Motivation, history, syntax and semantics for classical propositional calculus.

Formal symbolic propositional languages, formal definitions of model, counter model, tautology for propositional logic

Part two
Some Many Valued Extensional Semantics
Course Contents and Schedule

Part three
Formal deductive systems, called also proof systems
General definition and examples. Definition of a formal proof.
Relationship between proof systems and their semantics, i.e.
general definition of notions of soundness and completeness of a given proof systems relatively to given semantics.
Definition of a logic as a complete proof system

Part four
Hilbert style proof systems for classical propositional logic.
Proofs of DEDUCTION theorem, and two different proofs of the COMPLETENESS theorem for propositional classical logic.
Course Contents and Schedule

Part five
  Automated Gentzen type proof systems 1:
  RS proof system for classical propositional logic
  Examples of the automatic proof-search
  Automated Gentzen type proof systems 2:
  Original Gentzen proof system

Part six
  A Hilbert style proof system for Intuitionistic Logic
  Relationship between Intuitionistic and Classical logics
Course Contents and Schedule

Part seven
Automated proof systems 3:
Gentzen proof system for Intuitionistic Logic. Heuristic decision procedures.

Part eight
Languages and semantics for classical predicate logic
Hilbert Proof systems and proof of completeness theorem
Course Contents and Schedule

Part nine
  Automated Gentzen type proof systems 4:
  **QRS proof system** for classical predicate logic.
  Examples of the automatic proof-search.
  Constructive proof of **COMPLETENESS theorem**
  Original Gentzen proof system for classical and Intuitionistic predicate Logics.

Part ten
  A Hilbert style proof systems for **Modal Logics S4 and S5**
  Relationships with Intuitionistic Logic.
Chapter 1
INTRODUCTION

PART 1: Logic for Mathematics: Logical Paradoxes
PART 2: Logic for Mathematics: Semantical Paradoxes
PART 3: Non-Classical Logics and Logics for Computer Science
PART 4: Computer Science Puzzles
Chapter 1
PART1: Mathematical Paradoxes

Early Intuitive Approach:

Until recently, till the end of the 19th century, mathematical theories used to be built in the intuitive, or axiomatic way. Historical development of mathematics has shown that it is not sufficient to base theories only on an intuitive understanding of their notions.
Example

Consider the following.
By a set, we mean intuitively, any collection of objects.
For example, the set of all even integers or the set of all students in a class.
The objects that make up a set are called its members (elements)
Sets may themselves be members of sets for example, the set of all sets of integers has sets as its members
Example

Sets may themselves be members of sets for example, the set of all sets of integers has sets as its members

Most sets are not members of themselves; the set of all students, for example, is not a member of itself, because the set of all students is not a student

However, there may be sets that do belong to themselves - for example, the set of all sets
Russell Paradox, 1902

**Russell Paradox**

Consider the set $A$ of all those sets $X$ such that $X$ is not a member of $X$.

Clearly, $A$ is a member of $A$ if and only if $A$ is not a member of $A$.

So, if $A$ is a member of $A$, then $A$ is also not a member of $A$; and if $A$ is not a member of $A$, then $A$ is a member of $A$.

In any case, $A$ is a member of $A$ and $A$ is not a member of $A$.

**CONTRADICTION!**
Russell Paradox Solution

Russel proposed his Theory of Types as a solution to the Paradox

The idea is that every object must have a definite non-negative integer as its type assigned to it

An expression $x$ is a member of the set $y$ is meaningful if and only if the type of $y$ is one greater than the type of $x$
Russell Paradox Solution

Russell’s theory of types guarantees that it is meaningless to say that a set belongs to itself.
Hence Russell’s solution is:
The set $A$ as stated in the Russell paradox does not exist.
The Type Theory was extensively developed by Whitehead and Russell in years 1910 - 1913.
It is successful, but difficult in practice and has certain other drawbacks as well.
LOGICAL PARADOXES

Logical Paradoxes, also called Logical Antinomies are paradoxes concerning the notion of a set.

A modern development of Axiomatic Set Theory as one of the most important fields of modern Mathematics, or more specifically Mathematical Logic, or Foundations of Mathematics resulted from the search for solutions to various Logical Paradoxes.

First paradoxes free axiomatic set theory was developed by Zermello in 1908.
LOGICAL PARADOXES

Two of the most known antinomies, other than Russell’s Paradox, are Cantor and Burali-Forti antinomies.

They were stated at the end of 19th century.

Cantor Paradox involves the theory of cardinal numbers.
Burali-Forti Paradox is the analogue to Cantor’s but in the theory of ordinal numbers.
Cardinality of Sets

We say that sets $X$ and $Y$ have the same cardinality, $\text{card}X = \text{card}Y$ or that they are equinumerous if and only if there is one-to-one correspondence that maps $X$ onto $Y$

$\text{card}X \leq \text{card}Y$ means that $X$ is equinumerous with a subset of $Y$

$\text{card}X < \text{card}Y$ means that $\text{card}X \leq \text{card}Y$ and $\text{card}X \neq \text{card}Y$
Cantor and Schröder- Berstein Theorems

Cantor Theorem
For any set $X$, $\text{card}X < \text{card}\mathcal{P}(X)$

Schröder- Berstein Theorem
For any sets $X$ and $Y$, 
If $\text{card}X \leq \text{card}Y$ and $\text{card}Y \leq \text{card}X$, then $\text{card}X = \text{card}Y$.

Ordinal numbers are the numbers assigned to sets in a similar way as cardinal numbers but they deal with ordered sets
Cantor Paradox, 1899

Let $C$ be the universal set - that is, the set of all sets.

Now, $\mathcal{P}(C)$ is a subset of $C$, so it follows easily that

$$\text{card}\mathcal{P}(C) \leq \text{card}C$$

On the other hand, by Cantor Theorem,

$$\text{card}C < \text{card}\mathcal{P}(C) \leq \text{card}\mathcal{P}(C)$$

so also

$$\text{card}C \leq \text{card}\mathcal{P}(C).$$

From Schröder- Berstein theorem we have that $\text{card}\mathcal{P}(C) = \text{card}C$, what contradicts Cantor Theorem.

Solution: Universal set does not exist.
Burali-Forti Paradox, 1897

Given any ordinal number, there is a still larger ordinal number.

But the ordinal number determined by the set of all ordinal numbers is the largest ordinal number.

Solution: the set of all ordinal numbers do not exist
Logical Paradoxes

Another solution to Logical Paradoxes:

Reject the assumption that for every property $P(x)$, there exists a corresponding set of all objects $x$ that satisfy $P(x)$.

Russell’s Paradox then simply proves that there is no set $A$ defined by a property $P(x)$: of all sets that do not belong to themselves.
Logical Paradoxes

Cantor Paradox shows that
there is no set $A$ defined by a property
$P(X)$: there is an universal set $X$

Burali-Forti Paradox shows that
there is no set $A$ defined by a property
$P(x)$: there is a set that contains all ordinal numbers
Intuitionism

A more radical interpretation of the paradoxes has been advocated by Brower and his intuitionist school.

Intuitionists refuse to accept the universality of certain basic logical laws, such as the law of excluded middle: A or not A.

For intuitionists the excluded middle law is true for finite sets, but it is invalid to extend it to all sets. The intuitionists’ concept of infinite set differs from that of classical mathematicians.
Intuitionists’ Mathematics

The basic difference between classical and intuitionists’ mathematics lies in the interpretation of the word exists.

In classical mathematics proving existence of an object $x$ such that $P(x)$ holds does not mean that one is able to indicate a method of construction of it.

In the intuitionists’ universe we are justified in asserting the existence of an object having a certain property only if we know an effective method for constructing, or finding such an object.
Intuitionists’ Mathematics

In intuitionist’ mathematics the paradoxes are not derivable, or even meaningful.

The Intuitionism, because of its constructive flavor, has found a lot of applications in computer science. For example in the theory of programs correctness.

Intuitionistic Logic (to be studied in this course) reflects intuitionists ideas in a form a formalized deductive system.
PART 2: SEMANTIC PARADOXES
SEMANTIC PARADOXES

The development of axiomatic theories solved some, but not all problems brought up by the Logical Paradoxes.

Even the consistent sets of axioms, as the following examples show, do not prevent the occurrence of another kind of paradoxes, called Semantic Paradoxes that deal with the notion of truth.
Berry Paradox, 1906:

Let $A$ denote the set of all positive integers which can be defined in the English language by means of a sentence containing at most 1000 letters.

The set $A$ is finite since the set of all sentences containing at most 1000 letters is finite. Hence, there exist positive integer which do not belong to $A$.

Consider a sentence: $n$ is the least positive integer which cannot be defined by means of a sentence of the English language containing at most 1000 letters.

This sentence contains less than 1000 letters and defines a positive integer $n$.

Therefore $n \in A$ - but $n \notin A$ by the definition of $n$

CONTRADICTION!
Berry Paradox Analysis

The paradox resulted entirely from the fact that we did not say precisely what notions and sentences belong to the arithmetic and what notions and sentences concern the arithmetic.

Of course we didn’t talk about and examine arithmetic as a fix and closed deductive system.

We also incorrectly mixed the natural language with mathematical language of arithmetic.
Berry Paradox Solution

We have to distinguish always the language of the theory (arithmetic) and the language which talks about the theory, called a metalanguage.

In general we must distinguish a theory from the meta-theory.

In well and correctly defined theory the such paradoxes cannot appear.
The Liar Paradox

A man says: I am lying.
If he is lying, then what he says is true, and so he is not lying

If he is not lying, then what he says is not true, and so he is lying

CONTRADICTION!
Liar Paradoxes

These paradoxes arise because the concepts of the type

” I am true”,  ” this sentence is true”,  ” I am lying”

should not occur in the language of the theory

They belong to a metalanguage of the theory
It it means they belong to a language that talks about the theory
Cretan Paradox

The Liar Paradox is a corrected version of a following paradox stated in antiquity by a Cretan philosopher Epimenides.

Cretan Paradox

The Cretan philosopher Epimenides said: All Cretans are liars. If what he said is true, then, since Epimenides is a Cretan, it must be false. Hence, what he said is false. Thus, there is a Cretan who is not a liar. CONTRADICTION with what he said: "All Cretans are liars"
GENERAL REMARKS;
The Goal of the Course

FIRST TASK when one builds mathematical logic foundations of mathematics or of computer science is to define formally and proper **symbolic language**
This is called building a proper **syntax**

SECOND TASK is to extend the **syntax** to include a **notion of a proof**
It allows us to find out what can and cannot be proved if certain axioms and rules of inference are assumed
This part of syntax is called **PROOF THEORY**
GENERAL REMARKS;
The Goal of the Course

THIRD TASK is to define formally what does it mean that formulas of our formal language defined in the TASK ONE are true. It means that we have to define what we formally call a semantics for our language.

For example, the notion of truth i.e. the semantics for the classical and intuitionistic approaches are different.
GENERAL REMARKS;
The Goal of the Course

FOUTH TASK is to investigate the relationship between proof theory (part of the syntax) and semantics for the given language.

It means to establish correct relationship between notion of a proof and the notion of truth, i.e. to answer the following questions

Q1: Is (and when) everything one proves is true?
Q2: Is it possible (and when it is possible) to guarantee provability of everything we know to be true?
GENERAL REMARKS;
The Goal of the Course

The GOAL of this course is to formally define and develop the above Four Tasks in case of the Classical Logic and in case of Non-Classical Logics like Intuitionistic Logic, some Modal Logics, and some Many Valued Logics
Chapter 1
PART 3: Non-Classical Logics and Logics for Computer Science
Logics in Computer Science

The use of Classical Logic in computer science is known, indisputable, and well established. The existence of PROLOG and Logic Programming as a separate field of computer science is the best example of it.

Intuitionistic Logic in the form of Martin-Löf’s theory of types (1982), provides a complete theory of the process of program specification, construction, and verification. A similar theme has been developed by Constable (1971) and Beeson (1983)
In 1918, an American philosopher, C.I. Lewis proposed yet another interpretation of lasting consequences, of the logical implication.

In an attempt to avoid, what some felt, the paradoxes of implication (a false sentence implies any sentence) he created a modal logic.

The idea was to distinguish two sorts of truth: necessary truth and mere possible (contingent) truth.

A possibly true sentence is one which, though true, could be false.
Modal Logics in Computer Science are used as a tool for analyzing such notions as knowledge, belief, tense. Modal logics have also been employed in a form of Dynamic logic (Harel 1979) to facilitate the statement and proof of properties of programs.
Non-classical Logics for Computer Science

**Temporal Logics** were created for the specification and verification of concurrent programs (Harel, Parikh, 1979, 1983),
for a specification of hardware circuits (Halpern, Manna, Maszkowski, 1983),
and also to specify and clarify the concept of causation and its role in commonsense reasoning Shoham, 1988

**Fuzzy Sets, Rough Sets, Many valued logics** were created and developed to reasoning with incomplete information.
Non-classical Logics for Computer Science

The development of new logics and the applications of logics to different areas of Computer Science and in particular to Artificial Intelligence is a subject of a course in itself but is beyond the scope of class.

In class we will examine in detail the Classical Logic and some aspects of the Intuitionistic Logic. We also introduce some of the most standard many valued, and some modal logics.
Chapter 1
PART 4: Computer Science Puzzles
Computer Science Puzzles
Reasoning Artificial Intelligence
Reasoning in Artificial Intelligence

Assumption 1:
Flexibility of reasoning is one of the key property of intelligence

Assumption 2:
Commonsense inference is defeasible in its nature;

Assumption 3:
we are all capable of drawing conclusions, acting on them, and then retracting them if necessary in the face of new evidence
Reasoning in Artificial Intelligence

If Computer programs are to act intelligently, they will need to be similarly flexible.

Goal:

development of formal systems (logics) that describe commonsense flexibility.
Flexible Reasoning Example

Reiter, 1987
Consider a statement **Birds fly**. Tweety, we are told, is a bird. From this, and the fact that birds fly, we **conclude** that **Tweety can fly**

This conclusion is **defeasible**: Tweety may be an ostrich, a penguin, a bird with a broken wing, or a bird whose feet have been set in concrete.

**Non-monotonic Inference:**
on learning a new fact (that Tweety has a broken wing), we are forced to **retract** our conclusion (that he could fly)
Non-monotonic Logics

Definition:
A non-monotonic logic (reasoning) is a logic in which the introduction of a new information (axioms) can invalidate old facts (theorems)

Definition:
A default reasoning (logic) is a reasoning that let us draw of plausible inferences from less-then- conclusive evidence in the absence of information to the contrary

Observe: non-monotonic reasoning is an example of the default reasoning
Non-monotonic Logics

Here is what is happening in our CS department NOW!

Tiantian Gao will give his RPE presentation on Controlled Natural Languages and Default Reasoning this Thursday Aug 28 at 1:30pm in Room 1310

Part of the Abstract

Controlled natural languages (CNLs) are effective languages for knowledge representation and reasoning.

Over the past 20 years, a number of machine-oriented CNLs emerged and have been used in many application domains for problem solving and question answering.

However, few of them support nonmonotonic inference.

In our work, we propose nonmonotonic extensions of CNL to support defeasible reasoning.
Example: Moore, 1983

Consider my reason for believing that I do not have an older brother.

It is surely not that one of my parents once casually remarked, You know, you don’t have any older brothers, nor have I pieced it together by carefully sifting other evidence.

I simply believe that if I did have an older brother I would know about it;

therefore since I don’t know of any older brothers of mine, I must not have any
Auto-epistemic, Logics

The brother example reasoning is not default reasoning nor non-monotonic reasoning.
It is a reasoning about one’s own knowledge or belief.

Definition
Any reasoning about one’s own knowledge or belief is called an auto-epistemic reasoning.

Auto-epistemic reasoning models the reasoning of an ideally rational agent reflecting upon his beliefs or knowledge.
Logics which describe it are called auto-epistemic logics.
Here is the old Cannibals Problem:
Three missionaries and three cannibals come to the river.
A rowboat that seats two is available.
If the cannibals ever outnumber the missionaries on either bank of the river, the missionaries will be eaten.
How shall they cross the river?
Traditionally the puzzler is expected to devise a strategy of rowing the boat back and forth that gets them all across and avoids the disaster.
Traditional Solution

A state is a triple comprising the number of missionaries, cannibals and boats on the starting bank of the river. The initial state is 331, the desired state is 000. A solution is given by the sequence:

331, 220, 321, 300, 311, 110, 221, 020, 031, 010, 021, 000.
Missionaries and Cannibals Revisited

Imagine now giving someone a problem, and after he puzzles for a while, he suggests going upstream half a mile and crossing on a bridge.

What a bridge? you say.

No bridge is mentioned in the statement of the problem. He replies: Well, they don’t say the isn’t a bridge.

So you modify the problem to exclude the bridges and pose it again.

He proposes a helicopter, and after you exclude that, he proposes a winged horse....
Finally, you tell him the solution.

He attacks your solution on the grounds that the boat might have a leak.

After you rectify that omission from the statement of the problem, he suggests that a see monster may swim up the river and may swallow the boat.

Finally, you must look for a mode of reasoning that will settle his hash once and for all.
McCarthy Solution

McCarthy proposes circumscription as a technique for solving his puzzle.

He argues that it is a part of common knowledge that a boat can be used to cross the river unless there is something with it or something else prevents using it.

If our facts do not require that there be something that prevents crossing the river, the circumscription will generate the conjecture that there isn’t.

Lifschits has shown in 1987 that in some special cases the circumscription is equivalent to a first order sentence.

In those cases we can go back to our secure and well known classical logic.