cse371/mat371
LOGIC

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DEFINITIONS and FACTS for QUIZ 1

Definition
Logical Paradoxes, also called Logical Antinomies are paradoxes concerning the notion of a set

FACT
Russell Paradox
Consider the set $A$ of all those sets $X$ such that $X$ is not a member of $X$

Clearly, $A$ is a member of $A$ if and only if $A$ is not a member of $A$

So, if $A$ is a member of $A$, the $A$ is also not a member of $A$; and if $A$ is not a member of $A$, then $A$ is a member of $A$

In any case, $A$ is a member of $A$ and $A$ is not a member of $A$. CONTRADICTION!
FACT
The MAIN difference between classical and intuitionists’ mathematics lies in the interpretation of the word exists.

In classical mathematics proving existence of an object x such that $P(x)$ holds does not mean that one is able to indicate a method of construction of it.

In the intuitionists’ universe we are justified in asserting the existence of an object having a certain property only if we know an effective method for constructing, or finding such an object.
DEFINITIONS and FACTS for QUIZ 1

Definition
Semantic Paradoxes are paradoxes that deal with the notion of truth

FACT
The Liar Paradox:
A man says: I am lying.
If he is lying, then what he says is true, and so he is not lying

If he is not lying, then what he says is not true, and so he is lying

CONTRADICTION!
DEFINITIONS and FACTS for QUIZ 1

Definition
A **non-monotonic inference** is a reasoning in which introduction of a new information can **invalidate** old facts

Example
Consider a statement *Birds fly*. Tweety, we are told, is a bird. From this, and the fact that birds fly, we **conclude** that Tweety can fly

This conclusion is **defeasible**: Tweety may be an ostrich, a penguin, a bird with a broken wing, or a bird whose feet have been set in concrete.

This is a **non-monotonic Inference**: on learning a new fact (that Tweety has a broken wing), we are forced to **retract** our conclusion (that he could fly)
Definitions and Facts for Quiz 1

Definition:
A **default** reasoning is a reasoning that lets us **draw of plausible inferences** from less-than-conclusive evidence in the absence of information to the contrary.

Observe: **non-monotonic** reasoning is an example of the default reasoning.

Definition
Any reasoning about **one’s own knowledge or belief** is called an **auto-epistemic** reasoning.

Auto-epistemic reasoning **models** the reasoning of an ideally rational agent reflecting upon his beliefs or knowledge.
Definition
A propositional language is a pair

\( \mathcal{L} = (\mathcal{A}, \mathcal{F}) \)

where \( \mathcal{A}, \mathcal{F} \) are called an alphabet and a set of formulas, respectively

Definition
Alphabet is a set

\[ \mathcal{A} = \text{VAR} \cup \text{CON} \cup \text{PAR} \]

VAR, CON, PAR are all disjoint sets of propositional variables, connectives and parenthesis, respectively

VAR is a countably infinite set of propositional variables

CON \( \neq \emptyset \) is a finite set of logical connectives
DEFINITIONS and FACTS for QUIZ 1

Definition
The set $\mathcal{F} \subseteq \mathcal{A}^*$ of all formulas of a propositional language $L_{CON}$ is the smallest set for which the following conditions are satisfied:

1. $\text{VAR} \subseteq \mathcal{F}$
2. If $A \in \mathcal{F}$, $\vartheta \in C_1$, then $\vartheta A \in \mathcal{F}$
3. If $A, B \in \mathcal{F}$, $\circ \in C_2$, i.e. $\circ$ is a two argument connective, then $(A \circ B) \in \mathcal{F}$

By (1) propositional variables are formulas and they are called atomic formulas.
DEFINITIONS and FACTS for QUIZ 1

Example
The set $F \subseteq A^*$ of all formulas of a propositional language $L_{\{\neg, \cup\}}$ is the smallest set for which the following conditions are satisfied

1. $\text{VAR} \subseteq F$
2. If $A \in F$, then $\neg A \in F$
3. If $A, B \in F$, then $(A \cup B) \in F$
DEFINITIONS and FACTS for QUIZ 1

Definition
Given a set $S$ of formulas of a language $\mathcal{L}_{\text{CON}}$
Let $CS \subseteq CON$ be the set of all connectives that appear in formulas of $S$
A language

$\mathcal{L}_{CS}$

is called the **language defined** by the set of formulas $S$

Example
Let $S$ be a set
$S = \{((a \Rightarrow \neg b) \Rightarrow \neg a), \Box(\neg \Diamond a \Rightarrow \neg a)\}$
All connectives appearing in the formulas in $S$ are:

$\Rightarrow, \neg, \Box, \Diamond$

The **language defined** by the set $S$ is

$\mathcal{L}\{\neg, \Rightarrow, \Box, \Diamond\}$