1 YES/NO questions

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. For any binary relation $R \subseteq A \times A$, $R^*$ exists.
   Justify: y n

2. $R^* = R \cup \{(a, b) : there \ is \ a \ path \ from \ a \ to \ b\}$.
   Justify: y n

3. $R^* = R$ for $R = \{(a, b), (b, c), (a, c)\}$.
   Justify: y n

4. All infinite sets have the same cardinality.
   Justify: y n

5. Set $A$ is uncountable iff $R \subseteq A$ ($R$ is the set of real numbers).
   Justify: y n

6. Let $A \neq \emptyset$ such that there are exactly 25 partitions of $A$. It is possible to define 20 equivalence relations on $A$.
   Justify: y n

7. There is a relation that is equivalence and order at the same time.
   Justify: y n

8. Let $A = \{n \in N : n^2 + 1 \leq 15\}$. It is possible to define 8 alphabets $\Sigma \subseteq A$.
   Justify: y n

9. There is exactly as many languages over alphabet $\Sigma = \{a\}$ as real numbers.
   Justify: y n
10. Let $\Sigma = \{a, b\}$. There are more than 20 words of length 4 over $\Sigma$.

Justify: 

11. $L^* = \{w_1..w_n : w_i \in L, i = 1, 2, ..n, n \geq 1\}$.

Justify: 

12. $L^+ = LL^*$.

Justify: 

13. $L^+ = L^* - \{e\}$.

Justify: 

14. If $L = \{w \in \{0,1\}^* : w$ has an unequal number of 0's and 1's $\}$, then $L^* = \{0,1\}^*$.

Justify: 

15. For any languages $L_1, L_2$, 

$L_1^* = L_2^*$ if $L_1 = L_2$

Justify: 

16. For any languages $L_1, L_2$, $(L_1 \cup L_2) \cap L_1 = L_1$.

Justify: 

17. For any languages $L_1, L_2$, $(L_1 \cup L_2)^* = L_1^*$.

Justify: 

18. $((\emptyset^* \cap a) \cup b^*) \cap \emptyset^*$ describes a language with only one element.

Justify: 

19. $((\emptyset^* \cap a) \cup b^*) \cap a^*$ is a finite regular language.

Justify: 

20. $\{\{a\} \cup \{e\}\} \cap \{ab\}^*$ is a finite regular language.

Justify: 

21. Any regular language has a finite description.

Justify: 

22. Any finite language is regular.

Justify:
23. Every deterministic automata is also non-deterministic.
   Justify: y n

24. The set of all configurations of a given finite state automata is always non-empty.
   Justify: y n

25. Let $M$ be a finite state automaton, $L(M) = \{ \omega \in \Sigma^* : (q, \omega) \xrightarrow{s, e} (s, e) \}$.
   Justify: y n

26. For any automata $M$, $L(M) \neq \emptyset$.
   Justify: y n

27. $L(M_1) = L(M_2)$ iff $M_1, M_2$ are deterministic.
   Justify: y n

28. DFA and NDFA compute the same class of languages.
   Justify: y n

29. Let $M_1$ be a deterministic, $M_2$ be a nondeterministic FA, $L_1 = L(M_1)$ and $L_2 = L(M_2)$ then there is a deterministic automaton $M$ such that $L(M) = (L^* \cup (L_1 - L_2)^*)L_1$
   Justify: y n

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$.
   OBSERVE that $\Delta$ is always finite because $K, \Sigma$ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta$ is finite and $\Delta \subseteq K \times \Sigma^* \times K$.
   OBSERVE that we have to say in this case that $\Delta$ is finite because $\Sigma^*$ is infinite.

SOLVING PROBLEMS you can use any of these definitions.
2 Problems

Problem 1 (10 pts)

Let $L$ be a language defines as follows

$$L = \{ w \in \{a,b\}^* : \text{every } a \text{ is either immediately proceeded or followed by } b \}.$$ 

1. Describe a regular expression $r$ such that $L(r) = L$.

2. Construct a finite state automata $M$, such that $L(M) = L$.

State Diagram of $M$ is:

Some elements of $L(M)$ as defined by the state diagram are:

Components of $M$ are:
Problem 2 (5 pts)

1. Let $M$ be a deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

2. Let $M$ be a non-deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

Problem 3 (15 pts) Let

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_1, q_2, q_3\}$ and
$\Delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_2), (q_1, b, q_1), (q_3, a, q_3), (q_3, b, q_2)\}$

1. Draw the State Diagram of $M$.

2. List some elements of $L(M)$.

3. Write a regular expression for the language accepted by $M$. Simplify the solution.
4. Define a deterministic $M'$ such that $M \approx M'$, i.e. $L(M) = L(M')$.

State Diagram of $M'$ is:

Some elements of $L(M')$ as defined by the state diagram are:

Justify why $M \approx M'$.

Problem 4 (10 pts)

Let $M$ be defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$

$\Sigma = \{a, b, c\}$, $F = \{q_0, q_2, q_3\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, e, q_3), (q_1, bc, q_1), (q_1, b, q_3), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the State Diagram of $M$. 
2. Find the regular expression describing the \( L(M) \). Simplify it as much as you can. Explain your steps. Does \( e \in L(M) \)?

3. Write down (you can draw the diagram) an automata \( M' \) such that \( M' \equiv M \) and \( M' \) is defined by the BOOK definition.

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**Problem 5** (20 pts.) For \( M \) defined as follows

\[
M = (K, \Sigma, s, \Delta, F)
\]

for \( K = \{q_0, q_1, q_2, q_3\} \), \( s = q_0 \)
\( \Sigma = \{a, b\} \), \( F = \{q_2\} \) and
\[
\Delta = \{(q_0, a, q_3), (q_0, e, q_3), (q_0, b, q_1), (q_0, e, q_1), (q_1, a, q_2), (q_2, b, q_3), (q_2, e, q_3)\}
\]

1. Draw the State Diagram of \( M \).

2. Write 2 steps of the general method of transformation a NDFA \( M \), into an equivalent \( M' \), which is a DFA, where \( M \) is given by a following state diagram.

**Step 1:** Evaluate \( \delta(E(q_0), a) \) and \( \delta(E(q_0), b) \).

**Step 2:** Evaluate \( \delta \) on all states that result from step 1.

Reminder: \( E(q) = \{ p \in K : (q, e) \xrightarrow{M} (p, e) \} \) and
\[
\delta(Q, \sigma) = \bigcup \{ E(p) : \exists q \in Q, (q, \sigma, p) \in \Delta \}.
\]