cse303
ELEMENTS OF THE THEORY OF COMPUTATION

Professor Anita Wasilewska
LECTURE 6a
REVIEW for Q2

Q2 covers Lecture 5 and Lecture 6

Chapter 2 - Deterministic Finite Automata DFA

Chapter 2 - Nondeterministic Finite Automata NDFA

1. Some YES-NO Questions
2. Some Very Short Questions
3. Some Homework Problems
CHAPTER 2 PART 1
Deterministic Finite Automata DFA
Nondeterministic Finite Automata NDFA
Short YES/NO Questions

Here are solutions to some short YES/NO Questions for material covered in Chapter 2, Part 1
Solving Quizzes and Tests you have to write a short solutions and circle the answer
You will get 0 pts if you only circle your answer without providing a solution, even if it is correct

Here are some questions

Q1  Alphabet \( \Sigma \) of any deterministic finite automaton \( M \) is always non-empty
no  An alphabet \( \Sigma \) is, by definition, any finite set, hence it can be empty

Q2  The set \( K \) of states of any deterministic finite automaton is always non-empty
yes  \( s \in K \)
Short YES/NO Questions

Q3 A configuration of a DF Automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$
no Configuration is any element $(q, w) \in K \times \Sigma^*$

Q4 Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a transition relation iff the following condition holds

$(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$
no Proper condition is:

$(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$

Q5 A configuration of a non-deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$

yes by definition
Short YES/NO Questions

Q6  Given $M = (K, \Sigma, \delta, s, F)$ we define
$L(M) = \{ w \in \Sigma^* : ((s, w) \vdash^* M(q, e)) \text{ for some } q \in K \}$

no  Must be: for some $q \in F$

Q7  Given $M = (K, \Sigma, \delta, s, F)$ we define
$L(M) = \{ w \in \Sigma^* : \exists q \in K ((s, w) \vdash^* M(q, e)) \}$

no  Must be: $\exists q \in F ((s, w) \vdash^* M(q, e))$

Observe that Q7 is really the Q6 written in symbolic way correctly using the symbol of existential quantifier
Short YES/NO Questions

Q8  If \( M = (K, \Sigma, \delta, s, F) \) is a deterministic, then \( M \) is also non-deterministic

yes  The function \( \delta \) is a (special) relation on \( K \times \Sigma \times K \), i.e.
\[
\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K
\]

Q9  For any automata \( M \), we have that \( L(M) \neq \emptyset \)

no  Take \( M \) with \( \Sigma = \emptyset \) or \( F = \emptyset \) then we get \( L(M) = \emptyset \)

Q10  For any DFA \( M = (K, \Sigma, \delta, s, F) \), \( e \in L(M) \) if and only if \( s \in F \)

yes  this is the DFA Theorem
Short YES/NO Questions

Q11  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times \Sigma^* \times K$

no  we must say: $\Delta$ is finite

Q12  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$

yes  this is book definition; do not need to say $\Delta$ is a finite set, as the set $K \times (\Sigma \cup \{e\}) \times K$ is always finite

Q13  The set of all configurations of any non-deterministic state automata is always non-empty

yes  the set of all configuration of NDFA is by definition $K \times \Sigma^* = \{(q, w) : q \in K, w \in \Sigma^*\}$ and we have that $(s, e) \in K \times \Sigma^*$ even when $\Sigma = \emptyset$ as always $s \in K$, $e \in \Sigma^*$
Short YES/NO Questions

Q14  We say that two automata $M_1, M_2$ (deterministic or nondeterministic) are the same, i.e. $M_1 = M_2$ if and only if $L(M_1) = L(M_2)$

no  we say that $M_1, M_2$ are equivalent, i.e. $M_1 \approx M_2$ if and only if $L(M_1) = L(M_2)$

Q15 For any DFA $M$, there is is a NDFA $M'$, such that $M \approx M'$

yes  This is the **Equivalency Theorems 1**

Q16 For any NDFA $M$, there is is a DFA $M'$, such that $M \approx M'$

yes  This is the **Equivalency Theorems 2**
Short YES/NO Questions

Q17  If \( M = (K, \Sigma, \Delta, s, F) \) is a non-deterministic as defined in the book, then \( M \) is also non-deterministic, as defined in the lecture

**yes** \( \Sigma \cup \{e\} \subseteq \Sigma^* \)

Q18  We define, for any (deterministic or non-deterministic \( M = (K, \Sigma, \Delta, s, F) \)) a **computation** of the length \( n \) from \((q, w)\) to \((q', w')\) as a **sequence**

\[(q_1, w_1), (q_2, w_2), ..., (q_n, w_n), \quad n \geq 1\]

of **configurations**, such that

\( q_1 = q, \; q_n = q', \; w_1 = w, \; w_n = w' \) and

\((q_i, w_i) \vdash^* M (q_{i+1}, w_{i+1}) \) for \( i = 1, 2, .. n - 1 \)

**Statement:** For any \( M \) a computation \((q, w)\) exists

**yes**  By definition a **computation of length one** (case \( n=1 \)) always exists
Very Short Questions

For all **short questions** given on Quizzes and Tests you will have to do the following

1. Decide and **explain** whether the **diagram** represents a **DFA**, **NDFA** or does not

2. List all components of **$M$** when it represents **DFA**, **NDFA**

3. Describe **$L(M)$** as a **regular expression** when it represents **DFA**, **NDFA**
Consider a diagram $M_1$

1. Yes, it represents a DFA; $\delta$ is a function on $\{q_0, q_1\} \times \{a\}$ and initial state $s = q_0$ exists

2. $K = \{q_0, q_1\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \{q_1\}$,
   $\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_1$

3. $L(M_1) = aa^*$
Very Short Questions

Consider a diagram $M_2$

1. Yes, it represents a DFA; $\delta$ is a function on $\{q_0\} \times \{a\}$ and initial state $s = q_0$ exists
2. $K = \{q_0\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \emptyset$, $\delta(q_0, a) = q_0$
3. $L(M2) = \emptyset$
Very Short Questions

Consider a diagram $M_3$

1. Yes, it represents a DFA; initial state $s = q_0$ exists
2. $K = \{q_0\}$, $\Sigma = \emptyset$, $s = q_0$, $F = \emptyset$, $\delta = \emptyset$
3. $L(M_3) = \emptyset$
Very Short Questions

Consider a diagram $M_4$

1. Yes, it represents a DFA; initial state $s = q_0$ exists
2. $K = \{q_0\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \{q_0\}$, $\delta(q_0, a) = q_0$
3. $L(M_4) = a^*$

Remark $e \in L(M_4)$ by DFA Theorem, as $s = q_0 \in F = \{q_0\}$
1. NO! it is NOT neither DFA nor NDFA - initial state does not exist
Very Short Questions

Consider a diagram M6

1. It is not a DFA; Initial state does exist, but $\delta$ is not a function; $\delta(q_0, b)$ is not defined and we didn’t say ”plus trap states”
2. It is a NDFA
3. $L(M6) = \emptyset$
Consider a **diagram** M7

1. Yes! it is a DFA with trap states
   Initial state **exists** and we can complete definition of $\delta$ by adding a **trap state** as pictured below.
Very Short Questions

Consider again diagram M7

2. If we do not say "plus trap states" it represents a NDFA with
   $\Delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_1, b, q_1)\}$

3. $L(M7) = \emptyset$ as $F = \emptyset$
Very Short Questions

There is much more **Short Questions** examples in the section SHORT PROBLEMS at the end of Lecture 5
Some Homework Problems

Problem 1
Construct deterministic $M$ such that

$$L(M) = \{ w \in \Sigma^* : w \text{ has an odd number of } a \text{'s}$$

and an even number of $b$ 's }

Solution
Here is the short diagram - we must say: plus trap states
Some Homework Problems

Problem 2

Construct a DFA $M$ such that

$L(M) = \{ w \in \{a, b\}^* : \text{every substring of length 4 in word } w \text{ contains at least one } b \}$

Solution Here is a short pattern diagram (the trap states are not included)
Problem 3

Construct a DFA $M$ such that

$$L(M) = \{ w \in \{a, b\}^* : \text{every word } w \text{ contains an even number of sub-strings } ba \}$$

Solution

Here is a pattern diagram

Zero is an even number so we must have that $e \in L(M)$, i.e. we have to make the initial state also a final state
Some Homework Problems

Problem 4

Construct a DFA $M$ such that

\[ L(M) = \{ w \in \{a, b\}^* : w \text{ has } abab \text{ as a substring} \} \]

Solution  The essential part of the diagram must produce abab and it can be surrounded by proper elements on both sides and can be repeated.

Here is the essential part of the diagram:

[a] -> b -> a -> b
Problem 4 Solutions

We complete the essential part following the fact that it can be surrounded by proper elements on both sides and can be repeated.

Here is the diagram of M:

Observe that this is a pattern diagram; you need to add names of states only if you want to list all components. M does not have trap states.
Some Homework Problems

Problem 5
Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$$L(M) = (ab)^*(ba)^*$$

Specify all components $K, \Sigma, \Delta, s, F$ of $M$ and draw a state diagram.

Justify your construction by listing some strings accepted by the state diagram.
Problem 5 Solutions

Solution 1
We use the lecture definition
Components of $M$ are:

$$\Sigma = \{a, b\}, \quad K = \{q_0, q_1\}, \quad s = q_0, \quad F = \{q_0, q_1\}$$

We define $\Delta$ as follows

$$\Delta = \{(q_0, ab, q_0), (q_0, e, q_1), (q_1, ba, q_1)\}$$

Strings accepted: $ab, \ abab, \ abba, \ babba, \ ababbaba, \ ...$
Problem 5 Solutions

Solution 2
We use the book definition
Components of $M$ are:

$$
\Sigma = \{a, b\}, \quad K = \{q_0, q_1, q_2, q_3\}, \quad s = q_0, \quad F = \{q_2\}
$$

We define $\Delta$ as follows

$$
\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}
$$

Strings accepted: $ab, abab, abba, babba, ababbaba, \ldots$
Problem 6

Let \( M = (K, \Sigma, s, \Delta, F) \) for \( K = \{q_0, q_1, q_2, q_3,\} \), \( s = q_0 \), \( \Sigma = \{a, b, c\} \), \( F = \{q_3\} \) and

\[
\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}
\]

Find the regular expression describing the \( L(M) \).
Simplify it as much as you can. Explain your steps

**Solution**

\[
L(M) = (abc)^*abbb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa(abc)^*(abbb \cup baa)
\]

We used the property: \( LL_1 \cup LL_2 = L(L_1 \cup L_2) \)
Some Homework Problems

Problem 7

Let $M = (K, \Sigma, s, \Delta, F)$ for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and $\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$

Write down (you can draw the diagram) an automaton $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

Solution

We apply the "stretching" technique to $M$ and the new $M'$ is as follows.

$M' = (K \cup \{p_1, p_2, ..., p_5\} \Sigma, s = q_0, \Delta', F' = F)$

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$
Some Homework Problems

I will NOT include this problem on Q2, but you have to know how to solve similar problems for Midterm

Problem 8
Let \( M \) be defined as follows

\[
M = (K, \Sigma, s, \Delta, F)
\]

for \( K = \{q_0, q_1, q_2\} \), \( s = q_0 \), \( \Sigma = \{a, b\} \), \( F = \{q_0, q_2\} \) and

\[
\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}
\]

Write 4 steps of the general method of transformation a NDFA \( M \), into an equivalent \( M' \), which is a DFA.
Problem 8

Reminder

\[ E(q) = \{ p \in K : (q, e) \vdash^* M(p, e) \} \] and

\[ \delta(Q, \sigma) = \bigcup_{p \in K} \{ E(p) : \exists q \in Q (q, \sigma, p) \in \Delta \} \]

Step 1

Evaluate \( \delta(E(q_0), a) \) and \( \delta(E(q_0), b) \)

Step i+1]

Evaluate \( \delta \) on all states that result from Step i
Problem 8

Solution

\[ \delta(Q, \sigma) = \bigcup_{p \in K} \{ E(p) : \exists q \in Q (q, \sigma, p) \in \Delta \} \]

Step 1

\[ E(q_0) = \{ q_0 \}, \ E(q_1) = \{ q_1 \}, \ E(q_2) = \{ q_2 \} \]

\[ \delta(\{ q_0 \}, a) = E(q_1) = \{ q_1 \}, \ \delta(\{ q_0 \}, b) = \emptyset \]

Step 2

\[ \delta(\emptyset, a) = \emptyset, \ \delta(\emptyset, a) = \emptyset, \ \delta(\{ q_1 \}, a) = \emptyset, \]

\[ \delta(\{ q_1 \}, b) = E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F' \]
Problem 8

Solution

$$\delta(Q, \sigma) = \bigcup_{p \in K} \{E(p) : \exists q \in Q \ (q, \sigma, p) \in \Delta\}$$

Step 3

$$\delta(\{q_0, q_2\}, a) = E(q_1) \cup E(q_0) = \{q_0, q_1\}, \quad \delta(\{q_0, q_2\}, b) = \emptyset$$

Step 4

$$\delta(\{q_0, q_1\}, a) = \emptyset \cup E(q_1) = \{q_1\},$$

$$\delta(\{q_0, q_1\}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$