CSE303  Q2 PRACTICE SOLUTIONS  Spring 20012

PART 1: YES/NO QUESTIONS  Circle the correct answer. Write SHORT justification.

1. The set $K$ of states of any deterministic finite automaton is always non-empty
   Justify: $s \in K$

2. Alphabet $\Sigma$ of any deterministic finite automaton is always non-empty
   Justify: An alphabet $\Sigma$ is any FINITE set, hence it can be empty.

3. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^*$.
   Justify: this is definition

4. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds
   $(q, aw) \models_M (q', w)$ iff $\delta(q', a) = q$.
   Justify: Proper condition is:
   $(q, aw) \models_M (q', w)$ iff $\delta(q, a) = q'$.

5. Given $M = (K, \Sigma, \delta, s, F)$ we define
   $L(M) = \{w \in \Sigma^* : \exists q \in K ((s, w) \models^*_M (q, e))\}$.
   Justify: Must be: $\exists q \in F ((s, w) \models^*_M (q, e))$.

6. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic.
   Justify: The function $\delta$ is a (special) relation on $K \times \Sigma \times K$, i.e.
   $\delta = \Delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K$.

7. A configuration of a non-deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$.
   Justify: by definition

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA
BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when
$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that $\Delta$ is always finite because $K, \Sigma$ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta$ is finite and
$$\Delta \subseteq K \times \Sigma^* \times K.$$ 

OBSERVE that we have to say in this case that $\Delta$ is finite because $\Sigma^*$ is infinite.

SOLVING PROBLEMS you can use any of these definitions.

A VERY SHORT QUESTION Given the automaton $M$ with the following components:
$$\Sigma = \{a, b, c\}, \quad K = \{q_0, q_1, q_2\}, \quad s = q_0, \quad F = \{q_2\}.$$ 

We define $\Delta$ as follows.
$$\Delta = \{(q_0, abc, q_1), (q_1, e, q_2), (q_0, a, q_2)\}$$

1. State and explain whether $M$ represents a deterministic or a non-deterministic automaton.

Solution $M$ is non-deterministic. $\Delta$ is not a function on $K \times \Sigma$., also $\Delta \subseteq K \times \Sigma^* \times K$ (Lecture definition).

2. Write down a regular expression representing $L(M)$.

Solution
$$L(M) = abc \cup a$$

PART 2: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton $M$ such that
$$L(M) = \{w \in \{a, b\}^* : \text{neither } bb \text{ nor } aa \text{ is a substring of } w\}.$$ 

Draw a state diagram and specify all components $K, \Sigma, \delta, s, F$ of $M$. Justify your construction.

Solution
Components of \( M = (K, \Sigma, \delta, s, F) \) are:

\( \Sigma = \{ a, b \} \), \( K = \{ q_0, q_1, q_2, q_3 \} \), \( s = q_0 \), \( q_3 \) is a trap state, \( F = \{ q_0, q_1 \} \).

We define \( \delta \) on non-trap states as follows.

\( \delta(q_0, a) = q_1 \), \( \delta(q_0, b) = q_2 \),
\( \delta(q_1, b) = q_2 \),
\( \delta(q_2, a) = q_1 \).

\( M \) accepts strings \( a, aba, ababa.. \) or \( b, bab, baba.. \) etc and never \( aa, bb, \) etc...

**QUESTION 2** For the automata \( M \) defined below describe the property defining \( L(M) \).

Components of \( M \) are:

\( \Sigma = \{ a, b \} \), \( K = \{ q_0, q_1, q_2, q_3 \} \), \( s = q_0 \), \( F = \{ q_1 \} \).

We define \( \delta \) as follows.

\( \delta(q_0, a) = q_1 \), \( \delta(q_0, b) = q_2 \),
\( \delta(q_1, a) = q_0 \), \( \delta(q_1, b) = q_3 \),
\( \delta(q_2, a) = q_3 \), \( \delta(q_2, b) = q_0 \),
\( \delta(q_3, a) = q_2 \), \( \delta(q_3, b) = q_1 \).

**Solution**

**Language** of \( M \) is:

\( L(M) = \{ w \in \Sigma^* : w \text{ has an odd number of } a \text{'s and an even number of } b \text{'s} \} \).

**QUESTION 3** Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton \( M \), such that

\( L(M) = (ab)^*(ba)^* \).

Draw a state diagram and specify all components \( K, \Sigma, \Delta, s, F \) of \( M \). Justify your construction by listing some strings accepted by the state diagram.

**Solution 1** We use the lecture definition.

Components of \( M \) are: \( \Sigma = \{ a, b \} \), \( K = \{ q_0, q_1 \} \), \( s = q_0 \), \( F = \{ q_0, q_1 \} \).

We define \( \Delta \) as follows.

\( \Delta = \{ (q_0, ab, q_0), (q_0, c, q_1), (q_1, ba, q_1) \} \).

Strings accepted : \( ab, abab, abba, ababba, abababa, .. \).

**Solution 2** We use the book definition.
Components of $M$ are: $\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_2\}$.

We define $\Delta$ as follows.

$\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}$.

Strings accepted: $ab, abab, abba, ababba, ababbaba, ....$

**QUESTION 4** Let $M$ be defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and $\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$.

1. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps.

**Solution**

$L(M) = (abc)^* abbb \cup abbb (abc)^* baa \cup ba = (abc)^* abbb \cup (abc)^* baa (abc)^* (abbb \cup baa)$.

We used the property:

$$LL_1 \cup LL_2 = L(L_1 \cup L_2)$$

2. Write down (you can draw the diagram) an automata $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

**Solution**

We apply the "stretching" technique to $M$ and the new $M'$ is as follows.

$$M' = (K \cup \{p_1, p_2, ..., p_5\}, \Sigma, s = q_0, \Delta', F' = F)$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_3\}$ and $\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$.

**QUESTION 5** Let $M$ be defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_0, q_2\}$ and $\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}$.
Write 4 steps of the general method of transformation a NDFA $M$, into an equivalent $M^\prime$, which is a DFA. Reminder: $E(q) = \{ p \in K : (q, e)^{\ast} M(p, e) \}$ and

$$\delta(Q, \sigma) = \bigcup \{ E(p) : \exists q \in Q, (q, \sigma, p) \in \Delta \}.$$ 

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step i+1: Evaluate $\delta$ on all states that result from step i.

Solution

Step 1:

$$E(q_0) = \{ q_0 \}, \ E(q_1) = \{ q_1 \}, \ E(q_2) = \{ q_2 \}$$

$$\delta(\{ q_0 \}, a) = E(q_1) = \{ q_1 \} \quad \delta(\{ q_0 \}, b) = \emptyset$$

Step 2:

$$\delta(\emptyset, a) = \emptyset, \ \delta(\emptyset, a) = \emptyset, \delta(\{ q_1 \}, a) = \emptyset, \delta(\{ q_1 \}, b) = E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F^\prime$$

Step 3:

$$\delta(\{ q_0, q_2 \}, a) = E(q_1) \cup E(q_0) = \{ q_0, q_1 \}, \ \delta(\{ q_0, q_2 \}, b) = \emptyset$$

Step 4:

$$\delta(\{ q_0, q_1 \}, a) = \emptyset \cup E(q_1) = \{ q_1 \}, \ \delta(\{ q_0, q_1 \}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F^\prime$$