Points assigned to questions are as in real midterm examination.

TAKE it as a practice - and correct it yourself to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself- but do it ONLY AFTER you complete it all by yourself.

This is the goal of the PRACTICE TEST!

PLEASE SUBMIT your SOLUTIONS that have been CORRECTED BY YOU - you WILL GET 10 points for THAT! even if all problems you solved were first wrong- and then CORRECTED!

The real midterm will have less problems; I will make sure you will be able to complete it within 1 hour and 15 minutes.

BRING YOUR solved-corrected TEST to class on Tuesday, March 22

1 YES/NO questions

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. For any function f from $A \neq \emptyset$ onto $A$, f has property $f(a) \neq a$ for certain $a \in A$.
   Justify:
   y  n

2. For any binary relation $R \subseteq A \times A$, $R^{-1}$ exists.
   Justify:
   y  n

3. All infinite sets have the same cardinality.
   Justify:
   y  n

4. Set $A$ is uncountable iff $R \subseteq A$ ($R$ is the set of real numbers).
   Justify:
   y  n

5. Let $A \neq \emptyset$ such that there are exactly 25 partitions of $A$. It is possible to define 20 equivalence relations on $A$.
   Justify:
   y  n
6. There is a relation that is equivalence and order at the same time.
   Justify: y n

7. Let $A = \{ n \in N : n^2 + 1 \leq 15 \}$. It is possible to define 8 alphabets $\Sigma \subseteq A$.
   Justify: y n

8. There is exactly as many languages over alphabet $\Sigma = \{ a \}$ as real numbers.
   Justify: y n

9. Let $\Sigma = \{ a, b \}$. There are more than 20 words of length 4 over $\Sigma$.
   Justify: y n

10. $L^* = \{ w_1...w_n : w_i \in L, i = 1, 2, .., n, n \geq 1 \}$.
   Justify: y n

11. $L^+ = LL^*$.
    Justify: y n

12. $L^+ = L^* - \{ e \}$.
    Justify: y n

13. If $L = \{ w \in \{ 0, 1 \}^* : w \text{ has an unequal number of } 0\text{'s and } 1\text{'s } \}$, then $L^* = \{ 0, 1 \}^*$.
    Justify: y n

14. For any languages $L_1, L_2$,
    $$L_1^* = L_2^* \text{ if and only if } L_1 = L_2$$
    Justify: y n

15. For any languages $L_1, L_2$, $(L_1 \cup L_2) \cap L_1 = L_1$.
    Justify: y n

16. For any languages $L_1, L_2$, $(L_1 \cup L_2)^* = L_1^*$.
    Justify: y n

17. $((\emptyset^* \cap a) \cup b^*) \cap \emptyset^*$ describes a language with only one element.
    Justify: y n

18. $((\emptyset^* \cap a) \cup b^*) \cap a^*$ is a finite regular language.
    Justify: y n

19. $\{a\} \cup \{e\} \cap \{ab\}^*$ is a finite regular language.
    Justify: y n
20. Any regular language has a finite description.
   **Justify:** y n

21. Any finite language is regular.
   **Justify:** y n

22. Every deterministic automaton is also non-deterministic.
   **Justify:** y n

23. The set of all configurations of a given finite state automaton is always non-empty.
   **Justify:** y n

24. Let $M$ be a finite state automaton, $L(M) = \{ \omega \in \Sigma^* : (q, \omega) \xrightarrow{s,M} (s, e) \}$.
   **Justify:** y n

25. For any automaton $M$, $L(M) \neq \emptyset$.
   **Justify:** y n

26. $L(M_1) = L(M_2)$ iff $M_1$, $M_2$ are deterministic.
   **Justify:** y n

27. DFA and NDFA compute the same class of languages.
   **Justify:** y n

28. Let $M_1$ be a deterministic, $M_2$ be a nondeterministic FA, $L_1 = L(M_1)$ and $L_2 = L(M_2)$ then there is a deterministic automaton $M$ such that $L(M) = (L^* \cup (L_1 - L_2)^*)L_1$
   **Justify:** y n

**TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA**

**BOOK DEFINITION:** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$.
   **OBSERVE** that $\Delta$ is always finite because $K, \Sigma$ are finite sets.

**LECTURE DEFINITION:** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta$ is finite and $\Delta \subseteq K \times \Sigma^* \times K$.
   **OBSERVE** that we have to say in this case that $\Delta$ is finite because $\Sigma^*$ is infinite.

**SOLVING PROBLEMS** you can use any of these definitions.
2 Problems

Problem 1 (10 pts)

Let $L$ be a language defined as follows

$$L = \{w \in \{a, b\}^*: \text{every } a \text{ is either immediately preceded or followed by } b\}.$$

1. Describe a regular expression $r$ such that $L(r) = L$.

2. Construct a finite state automata $M$, such that $L(M) = L$.

State Diagram of $M$ is:

Some elements of $L(M)$ as defined by the state diagram are:

Components of $M$ are:
Problem 2 (5 pts)

1. Let $M$ be a deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

2. Let $M$ be a non-deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

Problem 3 (15 pts) Let

$M = (K, \Sigma, s, \Delta, F)$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_1, q_2, q_3\}$ and
$\Delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_2), (q_1, b, q_1), (q_3, a, q_3), (q_3, b, q_2)\}$

1. Draw the State Diagram of $M$.

2. List some elements of $L(M)$.

3. Write a regular expression for the language accepted by $M$. Simplify the solution.
4. Define a deterministic $M'$ such that $M \approx M'$, i.e. $L(M) = L(M')$.

State Diagram of $M'$ is:

Some elements of $L(M')$ as defined by the state diagram are:

Justify why $M \approx M'$.

Problem 4 (10 pts)
Let $M$ be defined as follows

$$ M = (K, \Sigma, s, \Delta, F) $$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$

$\Sigma = \{a, b, c\}$, $F = \{q_0, q_2, q_3\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, c, q_3), (q_1, bc, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the State Diagram of $M$. 
2. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps. Does $e \in L(M)$?

3. Write down (you can draw the diagram) an automata $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

**Problem 5** (20 pts.) For $M$ defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$

$\Sigma = \{a, b\}$, $F = \{q_2\}$ and

$$\Delta = \{(q_0, a, q_3), (q_0, e, q_3), (q_0, b, q_1), (q_0, e, q_1), (q_1, a, q_2), (q_2, b, q_3), (q_2, e, q_3)\}$$

1. Draw the State Diagram of $M$.

2. Write 2 steps of the general method of transformation a NDFA $M$, into an equivalent $M'$, which is a DFA, where $M$ is given by a following state diagram.

**Step 1:** Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

**Step 2:** Evaluate $\delta$ on all states that result from step 1.

**Reminder:** $E(q) = \{p \in K : (q, e) \xrightarrow{M^*} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q, (q, \sigma, p) \in \Delta\}.$$