PART 1: Yes/No Questions (2pts each question)
Circle the correct answer. Write ONE-SENTENCE justification
No justification- no credit

1. Set $A$ is uncountable iff $R \subseteq A$ ($R$ is the set of real numbers)
   Justify: $y\ n$

2. All infinite sets have the same cardinality
   Justify: $y\ n$

3. $R^* = R \cup \{(a, b) : \text{there is a path from } a \text{ to } b\}$
   Justify: $y\ n$
4. \((ab \cup a^*b)^*\) is a regular language

Justify: \(\text{y \ or \ n}\)

5. Let \(\Sigma = \phi\), there is \(L \neq \phi\) over \(\Sigma\)

Justify: \(\text{y \ or \ n}\)

6. There are uncountably many languages over \(\Sigma = \{a\}\)

Justify: \(\text{y \ or \ n}\)

7. Let \(\mathbf{R}\) be a set of regular expressions and \(\mathcal{L}\) be a function that maps \(\mathbf{R}\) into set of all subsets of \(\Sigma^*\). Then the following it true.
\[ L \subseteq \Sigma^* \text{ is a regular language } \iff L = \mathcal{L}(r) \text{ for some } r \in \mathbf{R} \]

Justify: \(\text{y \ or \ n}\)

8. \(L^* = \{w \in \Sigma^* : \exists q \in F (s, w) \xrightarrow{\cdot} M (q, e)\}\)

Justify: \(\text{y \ or \ n}\)

9. \((a^*b \cup \phi^*)\) is a regular expression

Justify: \(\text{y \ or \ n}\)

10. \(\{a\}^*\{b\} \cup \{ab\}\) is a regular language

Justify: \(\text{y \ or \ n}\)

11. Let \(L\) be a language defined by \((a^*b \cup ab)\), i.e (shorthand) \(L = a^*b \cup ab\). Then \(L \subseteq \{a, b\}^*\)

Justify: \(\text{y \ or \ n}\)

12. \(\Sigma = \{a\}\), there are \(c\) (continuum) languages over \(\Sigma\)

Justify: \(\text{y \ or \ n}\)

13. \(L^* = L^+ - \{e\}\)

Justify: \(\text{y \ or \ n}\)

14. \(L^* = \{w_1 \ldots w_n, w_i \in L, i = 1, \ldots, n\}\)

Justify: \(\text{y \ or \ n}\)
15. For any languages \( L_1, L_2, L_3 \subseteq \Sigma^* \)
\[
L_1 \cup (L_2 \cap L_3) = (L_1 \cup L_2) \cap (L_1 \cup L_3)
\]
Justify:

16. For any languages \( L_1, L_2 \subseteq \Sigma^* \),
if \( L_1 \subseteq L_2 \), then \( (L_1 \cup L_2)^* = L_2^* \)
Justify:

17. \( ((\phi^* \cap a) \cup (\phi \cup b^*)) \cap \phi^* \) represents a language \( L = \phi \)
Justify:

18. \( L(M) = \{ w \in \Sigma^* : (q, w) \vdash^*_{M} (s, e) \} \)
Justify:

19. For every i deterministic automaton \( M \), \( L(M) \neq \phi \)
Justify:

20. If \( M \) is a nondeterministic FA, then \( L(M) \neq \emptyset \)
Justify:

21. \( L(M_1) = L(M_2) \) iff \( M_1 \) and \( M_2 \) are finite automata
Justify:

22. A language is regular iff \( L = L(M) \) and \( M \) is a deterministic automaton
Justify:

23. If \( M \) is a deterministic automaton, then there is a nondeterministic \( N \), such that \( L(N) = L(M) \)
Justify:

24. Any finite language is CF
Justify:

25. Intersection of any two regular languages is a CF language
Justify:
26. Union of a regular and a CF language is a CF language

Justify:

27. \( L_1 \) is regular, \( L_2 \) is CF and \( L_1, L_2 \subseteq \Sigma^* \), then \( L_1 \cap L_2 \subseteq \Sigma^* \) is a CF language

Justify:

28. If \( L \) is regular, then there is a PDA \( M \) such that \( L = L(M) \)

Justify:

29. If \( L \) is regular, then there is a CF grammar \( G \), such that \( L = L(G) \)

Justify:

30. \( L = \{a^n b^n c^n : n \geq 0\} \) is CF

Justify:

31. \( L = \{a^n b^n : n \geq 0\} \) is CF

Justify:

32. Let \( \Sigma = \{a\} \), then for any \( w \in \Sigma^* \), \( w^R w \in \Sigma^* \)

Justify:

33. \( A \rightarrow Ax, A \in V, x \in \Sigma^* \) is a rule of a regular grammar

Justify:

34. Regular grammar has only rules \( A \rightarrow xA, A \rightarrow x, x \in \Sigma^*, A \in V - \Sigma \)

Justify:

35. Let \( G = (\{S, (.)\}, \{(.)\}, R, S) \) for \( R = \{S \rightarrow SS | (S)\} \)
\( L(G) \) is regular

Justify:

36. \( L = \{w \in \{a, b\}^* : w = w^R\} \) is regular

Justify:
37. We can always show that $L$ is regular using Pumping Lemma

Justify: 

38. $((p, e, \beta), (q, \gamma)) \in \Delta$ means: read nothing, move from $p$ to $q$

Justify: 

39. $L = \{a^n b^m c^n : n, m \in N\}$ is CF

Justify: 

40. If $L$ is not regular, then there is a CF grammar $G$, such that $L = L(G)$

Justify: 

41. There is countably many non CF languages over $\Sigma \neq \phi$

Justify: 

42. Every subset of a regular language is a language

Justify: 

43. Any regular language is accepted by some PD automata

Justify: 

44. $\Delta \subseteq (K \times \Sigma^* \times \Gamma^*) \times (K \times \Gamma^*$ is a transition relation of a pushdown automaton

Justify: 

45. Let $M$ be a pushdown automaton

$$L(M) = \{w \in \Sigma^* : (s, w, e) \vDash^* M (f, e, e)\}$$

Justify: 

46. There is uncountably many regular languages

Justify: 

47. Every subset of a regular language is a regular language.

Justify: 

48. A CF language is a regular language.

Justify: $L = \{a^n b^n : n \geq 0\}$ is CF and not regular

Justify: 

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49. A regular language is a CF language.

Justify: y n

50. Every subset of a regular language is a regular language.

Justify: y n

PART 2: PROBLEMS (100pts + 15 extra pts)

QUESTION 1 (10pts)
Let $\Sigma$ be any alphabet, $L_1, L_2$ two languages over $\Sigma$ such that $e \in L_1$ and $e \in L_2$. Show that

$$(L_1 \Sigma^* L_2)^* = \Sigma^*$$
QUESTION 2 (15pts)

Let $L$ be a language defined as follows

$$L = \{w \in \{a, b\}^*: \text{every } a \text{ is either immediately preceded or followed by } b\}$$

Part 1

Describe a regular expression $r$ such that $L(r) = L$

Part 2  Construct a finite state automata $M$, such that $L(M) = L$.

1. Draw a diagram of $M$

2. List some elements of $L(M)$
QUESTION 3 (15pts)

Given a Regular grammar $G = (V, \Sigma, R, S)$, where
\[ V = \{a, b, S, A\}, \quad \Sigma = \{a, b\}, \]
\[ R = \{S \rightarrow aS \mid A \mid e, \quad A \rightarrow abA \mid a \mid b\}. \]

Part 1

Use the construction in the proof of L-GTheorem:

Language L is regular if and only if there exists a regular grammar G such that $L = L(G)$
to construct a finite automaton $M$, such that $L(G) = L(M)$.

Diagram of $M$

Part 2 Trace a transitions of $M$ that lead to the acceptance of the string $aaaababa$, and compare
with a derivation of the same string in $G$

The accepting computation of $aaaababa$ is:

$G$ derivation of $aaaababa$ is:
QUESTION 4 (15pts)
Define a grammar $G$ such that $L(G) = \{a^kb^j : k < j\}$

Justify your construction

QUESTION 5 (15pts)
Prove that the Class of context-free languages is NOT closed under intersection
QUESTION 6  (15pts)

Construct a **pushdown** automaton $M$ such that

$$L(M) = \{ w \in \{a, b\}^* : w = w^R \}$$

**Draw** a DIAGRAM of $M$

$M$ components are

**Trace a transitions** of $M$ that lead to the acceptance of the string $ababa$. 
**QUESTION 7** (15pts)

Construct a PD automaton $M$, such that

$$L(M) = \{b^n a^{2n} : n \geq 0\}.$$  

Draw a DIAGRAM

**Explain** the construction. Write motivation.

**Trace formal** transitions of $M$ that leads to the acceptance of the string $bbaaaa$, i.e complete:

$$(s, bbaaaa, e) \vdash_M$$
QUESTION 9 (EXTRA CREDIT - 15pts)

Use closure under union for CF languages to show that

\[ L = \{a^n b^n : n \neq m\} \]

is a CF language.