DEFINITION A context-free grammar \( G = (V, \Sigma, R, S) \) is called regular, or right-linear iff
\[
R \subseteq (V - \Sigma) \times \Sigma^*(V - \Sigma \cup \{e\})
\]

QUESTION 1 (3pts)
Given a Regular grammar \( G = (V, \Sigma, R, S) \), where
\[
V = \{a, b, S, B\}, \quad \Sigma = \{a, b\},
\]
\[
R = \{S \rightarrow bS \mid B, \quad B \rightarrow baB \mid a \mid b\}
\]

1. Construct a finite automaton \( M \), such that \( L(G) = L(M) \) - must use general construction from the proof of the theorem:
"For any Regular Grammar \( G \), \( L(G) \) is regular”.

You can just draw a diagram.

Solution We construct a non-deterministic finite automata
\[
M = (K, \Sigma, \Delta, s, F)
\]
as follows:
\[
K = (V - \Sigma) \cup \{f\}, \quad \Sigma = \Sigma, \quad s = S, \quad F = \{f\},
\]
\[
\Delta = \{(S, b, S), (S, e, B), (S, e, f), (B, ba, B), (B, a, f), (B, b, f)\}
\]

2. Trace a transitions of \( M \) that lead to the acceptance of the string \( bbbbabab \),
and compare with a derivation of the same string in \( G \).

Solution
The accepting computation is:
\[
(S, bbbbabab) \vdash_M (S, bbbabab) \vdash_M (S, babab) \vdash_M (B, babab)
\]
\[
\vdash_M (B, bab) \vdash_M (B, b) \vdash_M (f, e)
\]

\( G \) derivation is:
\[
S \Rightarrow bS \Rightarrow bbS \Rightarrow bbbS \Rightarrow bbbB \Rightarrow bbbbaB \Rightarrow bbbbabaB \Rightarrow bbbbabab
\]
QUESTION 2

1. Construct a Push Down Automaton $M$ such that $L(M)$ contains all $w \in \{a, b\}^*$, such that $w$ has the same number of $a$’s and $b$’s, i.e. such that

$$L(M) = \{w \in \{a, b\}^* : \#a = \#b\}.$$ 

Justify your answer. You can DRAW A DIAGRAM!

**Solution** in the Lecture Notes.