Tangible Math Applications

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ABSTRACT

Math manipulatives, math puzzles, and physical math activities provide numerous benefits in the learning environment. First, they provide concrete representations of abstract concepts, thereby helping more children to understand those concepts. Second, they provide an opportunity for children to explore and test their understanding of math concepts. Third, they enable groups of children to work together, talk about the problem at hand, and learn from one another. Fourth, they give active young children the opportunity to be physically engaged in their lessons, as opposed to sitting through a “boring” lecture. Finally, these physical activities give children a fun way to practice their math skills, which helps with retention and transfer as cognitive scientists have shown.

Because of all these benefits, schools and museums alike have embraced the use of physical learning activities to supplement learning. Teachers will often divide their classes into small groups and have them work on a particular manipulative. Other times, the teacher will use overhead displays to talk about the manipulatives while the children work at their desks with their own pieces. At the Goudreu Museum for Mathematics in Art and Science, children look forward to being able to play with the math puzzles after their more formal activities. The trouble comes in trying to direct and help all of the students at once. Students often need an attentive guide who will ask questions and make suggestions that will help children to think about the problems in the right ways. If the teacher is busy with one group, other groups that are "stuck" will quickly get bored and often find inappropriate activities to occupy themselves with.

At the New York Hall of Science, student “explainers” are available to provide help as needed. This museum has discovered that having a knowledgeable “guide on the side” helps their visitors to get more out of the exhibits and activities. Yet in some settings, this is not always practical. Tangible user interfaces provide a perfect solution for this. By making the math manipulatives part of the user interface, the computer application is aware of what the children are doing, and can therefore provide appropriate feedback as needed.

In this paper, we describe several new math applications that take advantage of advances in physical computing. The first two use computer vision to "watch" what the students are doing with the puzzle pieces. The latter two use sensors to provide a physical computing environment. All use multimedia feedback to guide the students through their activities and give them the encouragement that they need. We will also discuss how these applications can be used in the classroom and museum settings to enhance learning among children.

Tower of Hanoi

The Tower of Hanoi is a classic math problem known and loved by many. For younger children, it is a challenging puzzle that can be made successively harder by adding more and more disks to the tower. For older children, it is an ideal tool for demonstrating the concept of recurrence and recurrence functions.

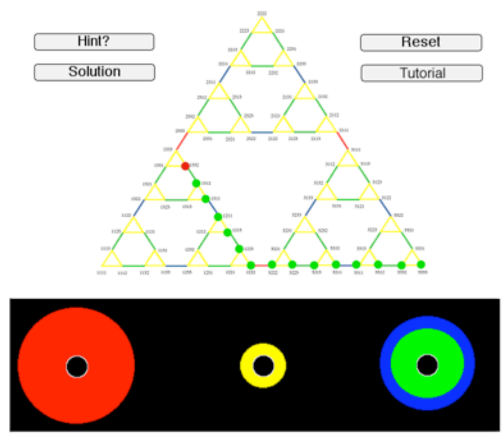

Figure 1. Display for the Tower of Hanoi application shows the arrangement of disks literally (at the bottom) and represented in a graph (at the top).

We have developed a physical Tower of Hanoi that uses computer vision to keep track of how the disks are arranged. In this application a camera is mounted overhead, looking down on a set of three poles and four different sized disks. These disks are distinguished by very different fluorescent colors. While the students work with the tower, the screen shows the current state of the puzzle in two ways (figure 1). Disks at the bottom of the screen show the arrangement of disks on the three poles. A diagram of Sierpinski's triangle shows the moves that have been made (thus far) as a path through a graph. The optimal solution to the problem can be represented in this graph by the shortest path from one corner of the triangle to another.

Additional multimedia content, such as a narrated and animated 3D representation of the puzzle's solution, provide students an overview of the math principles demonstrated by the Tower of Hanoi puzzle.

Pattern Blocks

Patten blocks are simple polygonal shapes that include a hexagon, a trapezoid (half of the hexagon), a rhombus (one third of the hexagon), and an equilateral triangle (one half of the rhombus). These blocks can be used in a wide variety of ways to teach several different concepts. For example, because the smaller blocks can be combined to create larger blocks, these can be used to teach fractions, multiplication, and division. Pattern blocks are also used to teach geometry concepts such as tessellation.

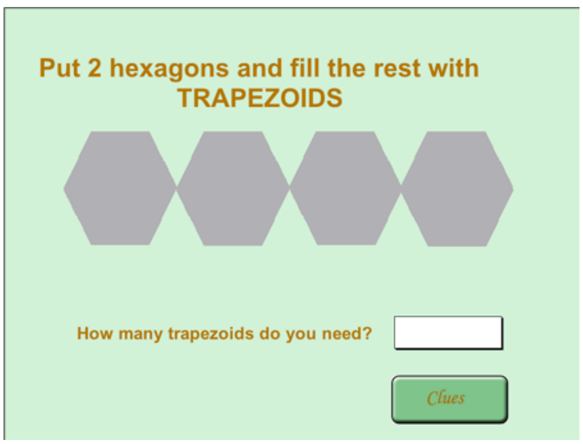

Figure 2. Children are asked to fill in the pattern with specific shapes, and then count the number used.

We have developed an application that uses computer vision to track the positions of wooden pattern blocks on a TICLE (tangible interfaces for collaborative learning environments) table. An internal representation of an isometric grid indicates how the various pattern pieces are filling the space. This activity has various levels of play for differing skill levels. Initially, children are asked to fill in a pattern using the four geometric shapes, and count how many were used to create the pattern (figure 2). Later on, they are asked to estimate how many of each will be needed. Numerous animated hints are provided to help children understand how the various shapes relate to one another both geometrically and proportionally. This activity can be used to help children to practice and test their understanding of geometry, fractions, multiplication, and division.

For the more advanced children, the application teaches them about tessellation. After working with various sample patterns, the children are asked to create their own tessellation pattern and devise rules for placing the patterns side-by-side (figure 3). These rules are expressed as a rotation (in increments of 60) followed by a sequence of translations. The application then fills the space with the pattern, complaining if the pattern contains overlaps or holes. If the pattern truly fills the space, children have the option of printing out a page filled with their tessellation pattern to take home.

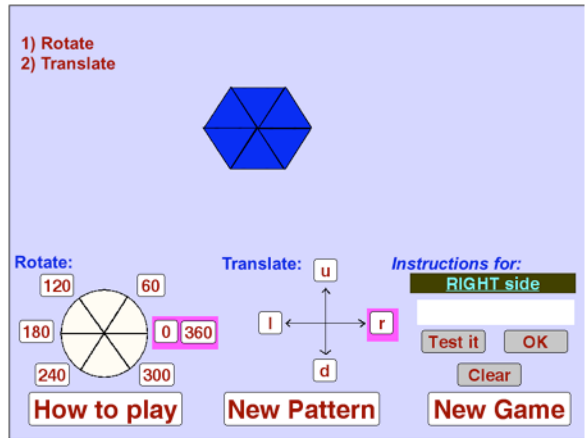

Figure 3. After creating a tiling pattern, the child is asked to define rules for placing copies of that pattern to the right, left, above, and below.

SmartStep

Similar to hopscotch or jump rope, this application uses physical activity to reinforce basic math skills like skip counting, while honing motor skills, pattern recognition, rhythm and coordination. The hardware component consists of nine, differently colored, 12" square floor tiles arranged in a 3 X 3 grid, connected to the computer through a MIDI interface.

The student interface is a 3 X 3 grid of cells in a table that mirrors the arrangement of the floor tiles. Each cell presents a sequence of numbers required to play the math game. Visual and auditory feedback accompany each hop a student makes on the floor tiles.

The teacher's interface allows them to create math games and set their parameters through easy to use forms and pull-down menus. Teachers are able to save these games for reuse, building a library of activities for individuals or groups of learners.

Figure 4. SmartStep interface indicates what number is represented by each square, and which square the child is currently on.

Figure 5. Child practicing addition with SmartStep.

Floor Math

A number line can help children to understand abstract number concepts by providing a visual representation of the number system. In Kindergarten classrooms, children are commonly given number lines to help them count up and down, add, and subtract.

We have incorporated touch sensors into a "floor math" floor mat to create a discrete number line that children can walk on. Each number on the line triggers a key press response to a keyboard emulator, which is fed to a multimedia application.

One application of this number line teaches numerical concepts by having children walk up and down the line. Very young children can use this to count up and down. Slightly older children also use this for "skip counting" (a precursor to multiplication!). The number line is also used to teach addition and subtraction. To add two numbers, the child counts up to the first number and then takes additional steps to reach the sum. To subtract, the child starts at the first number and then walks down the number line.

Figure 6. Child learning to "skip count" with Floor Math.

We have also used this number line to create an application that teaches older students about binary numbers. In this case, each sensor position represents a bit position in a binary number. Students step on these positions to toggle between values of 0 and 1. The computer asks them to enter the binary representation of a decimal number that has been selected randomly.