CSE526 Spring 2004
Homework Two
Problem 3

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1 Problem

Using the direct denotational semantics in Sections 2.2-2.4, prove that the equation above equation (2.2) on [Reynolds, p. 28] holds, i.e.,

\[
[[\text{while } b \text{ do } c]] \sigma = [[\text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip}]] \sigma
\]

In other words, when we have defined the meaning of ”while” loops using equation (2.4) on [Reynolds, p. 36], we should be able to prove that the above equation holds, as a way of checking that equation (2.4) has the desired meaning.

2 Proof:

\[
\text{RHS} = \begin{array}{l}
\text{if } [[b]] \sigma \text{ then } [[c; \text{while } b \text{ do } c]] \sigma \text{ else } \sigma \\
= \text{if } [[b]] \sigma \text{ then } ([[\text{while } b \text{ do } c]]_{\bot\bot}([c]) \sigma) \text{ else } \sigma \\
= \text{if } [[b]] \sigma \text{ then } (Y_{\Sigma \rightarrow \Sigma_{\bot}} F)_{\bot\bot}([c]) \sigma \text{ else } \sigma
\end{array}
\]

where \( Ff\sigma = \text{if } [[b]] \sigma \text{ then } f_{\bot\bot}([c]) \sigma \text{ else } \sigma. \)

While for the left-hand-side, we have:

\[
\text{LHS} = [[\text{while } b \text{ do } c]]\sigma \\
= Y_{\Sigma \rightarrow \Sigma_{\bot}} F \sigma
\]

where, \( Ff\sigma = \text{if } [[b]] \sigma \text{ then } f_{\bot\bot}([c]) \sigma \text{ else } \sigma. \)

Note that we have \( Y_D F \sigma = F (Y_D F) \sigma, \) therefore,

\[
\text{LHS} = F (Y_{\Sigma \rightarrow \Sigma_{\bot}} F) \sigma \\
= \text{if } [[b]] \sigma \text{ then } (Y_{\Sigma \rightarrow \Sigma_{\bot}} F)_{\bot\bot}([c]) \sigma \text{ else } \sigma
\]

where, \( Ff\sigma = \text{if } [[b]] \sigma \text{ then } f_{\bot\bot}([c]) \sigma \text{ else } \sigma. \)

Hence, \( \text{RHS} = \text{LHS}. \) Proved.

END OF PROOF