

Lecture 8: Interest Rates

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Interest Rates

The current “risk-free” interest rate r is a fundamental parameter to pricing financial securities.

However, it should be clear that there are many different interest rates, differing according to:

- The national currency.
- The duration of the loan.
- The credit worthiness of borrower.
- The backing collateral provided.
- The intended use of the money.

The Risk-Free Rate

Treasury rates are the interest rates earned on loans (bonds/bills) to a government in its own currency.

Since governments can print their own money, there is little chance they can default, hence risk-free.

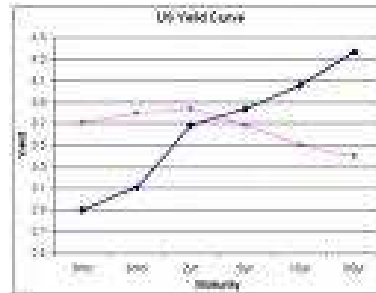
The *London Interbank Offered Rate* (LIBOR) is the interest rate which large AA-rated banks get on deposits from one to the other.

These loans are close to risk-free, and are considered a better proxy to the true risk-free rate by derivatives traders.

The Yield Curve

The **yield curve** defines the relationship between interest rates and their maturities.

Usually interest rates increase with maturity, reflecting the added return required to compensate for increased risk/inconvenience.



But rates are set by supply-and-demand. Difficult economic conditions can yield shallow or even **inverted** yield curves.

The U.S. Treasury Yield Curve

The screenshot shows a web browser window with the URL <http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rat>. The page title is "Daily Treasury Yield Curve Rates". Below the title, there is a "Get e-mail updates when this information changes." link and an "XML" icon with the text "This data is also available in XML format by clicking on the XML icon". A "Historical Data" link is also present. On the right side, there are several links: "Daily Treasury Yield Curve Rates", "Daily Treasury Bill Rates", "Daily Treasury Long-Term Rates", "Daily Treasury Real Yield Curve Rates", and "Daily Treasury Real Long-Term Rates". The main content is a table for "September 2008" with columns for "Date" and maturity terms: "1 mo", "3 mo", "6 mo", "1 yr", "2 yr", "3 yr", "5 yr", "7 yr", "10 yr", "20 yr", and "30 yr".

Date	1 mo	3 mo	6 mo	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	20 yr	30 yr
09/02/08	1.64	1.72	1.93	2.12	2.26	2.51	3.00	3.37	3.74	4.39	4.36
09/03/08	1.57	1.70	1.90	2.08	2.26	2.48	2.95	3.29	3.71	4.36	4.32
09/04/08	1.56	1.69	1.89	2.04	2.20	2.41	2.87	3.21	3.64	4.31	4.27
09/05/08	1.53	1.68	1.90	2.07	2.23	2.44	2.91	3.24	3.66	4.31	4.27
09/08/08	1.59	1.71	1.92	2.12	2.30	2.49	2.96	3.26	3.66	4.30	4.26
09/09/08	1.59	1.66	1.89	2.06	2.23	2.43	2.90	3.21	3.62	4.24	4.20
09/10/08	1.58	1.65	1.87	2.06	2.22	2.42	2.91	3.23	3.65	4.27	4.23
09/11/08	1.53	1.61	1.85	2.01	2.18	2.38	2.87	3.21	3.64	4.25	4.20
09/12/08	1.37	1.49	1.84	2.02	2.23	2.45	2.97	3.32	3.74	4.36	4.32
09/15/08	0.36	1.02	1.55	1.66	1.78	2.01	2.59	2.89	3.47	4.14	4.12
09/16/08	0.23	0.84	1.52	1.72	1.89	2.12	2.64	3.02	3.48	4.12	4.08
09/17/08	0.07	0.03	1.03	1.50	1.64	1.91	2.52	2.93	3.41	4.12	4.08
09/18/08	0.26	0.23	0.79	1.53	1.78	2.05	2.67	3.08	3.54	4.19	4.14
09/19/08	0.75	0.99	1.54	2.05	2.16	2.42	3.01	3.37	3.78	4.42	4.36

Current interest rates reflect the latest sales of both new and *previously issued* bonds.

Bond Sales and Interest Rates

Say a 10 year bond pays 5% interest on \$1000, meaning it will return \$1648.72 ten years from today.

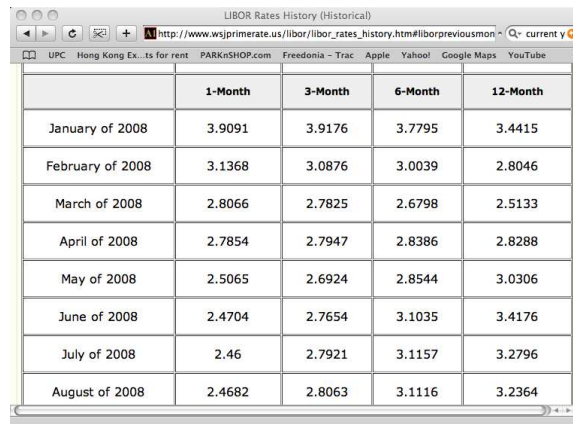
The price that this bond trades for on the open market proves a way to compute the current interest rate, because it reflects the present value of that \$1648.72.

If the bond is selling for more than \$1000, it means ten-year interest rates are **below** 5%.

If the bond is selling for less than \$1000, it means ten-year interest rates are **above** 10%.

The LIBOR Yield Curve

LIBOR rates are indeed higher than Treasury yields; indeed much higher during the peak of the financial crisis.



The screenshot shows a web browser window titled "LIBOR Rates History (Historical)". The address bar contains the URL "http://www.wsjprimerate.us/libor/libor_rates_history.htm#liborpreviousmon". The browser's search bar contains the text "current y". The table below displays the LIBOR rates for each month of 2008, categorized by 1-Month, 3-Month, 6-Month, and 12-Month terms.

	1-Month	3-Month	6-Month	12-Month
January of 2008	3.9091	3.9176	3.7795	3.4415
February of 2008	3.1368	3.0876	3.0039	2.8046
March of 2008	2.8066	2.7825	2.6798	2.5133
April of 2008	2.7854	2.7947	2.8386	2.8288
May of 2008	2.5065	2.6924	2.8544	3.0306
June of 2008	2.4704	2.7654	3.1035	3.4176
July of 2008	2.46	2.7921	3.1157	3.2796
August of 2008	2.4682	2.8063	3.1116	3.2364

Ratings Agencies

Professors assign ratings to students (called grades) to permit quick assessment of overall quality or potential.

Credit ratings are scores issued to measure the quality/strength of bonds and companies – how likely will they be able to pay off their debt.

Standard and Poors rates bonds AAA, AA, A, BBB, BB, B, CCC, CC, and C.

Moody's ratings are analogous: Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C.

Investment grade is BBB (or Baa) and above.

Credit ratings are based on balance sheet strength and fundamental analysis.

Ratings vs. Default

Historical data reporting the default percentage by grade / time permit one to generate default probabilities.

	1	2	3	4	5	7	10
Aaa	0.000	0.000	0.000	0.026	0.099	0.251	0.521
Aa	0.008	0.019	0.042	0.106	0.177	0.343	0.522
A	0.021	0.095	0.220	0.344	0.472	0.759	1.287
Baa	0.181	0.506	0.930	1.434	1.938	2.959	4.637
Ba	1.205	3.219	5.568	7.958	10.215	14.005	19.118
B	5.236	11.296	17.043	22.054	26.794	34.771	43.343
Caa-C	19.476	30.494	39.717	46.904	52.622	59.938	69.178

The interest rate on loans/bonds accordingly varies by rating and term.

Markov Models

Markov models are networks of *states* where there is a given probability of *transition* between each pair of states.

The probability of being in state s at time t is completely a function of (1) the probability of each state at time $t - 1$, and (2) the state transition function giving the probability of mapping each state to s .

The states in a Markov chain can be used to record some knowledge about previous states, but *not* the path used to get there.

Since the transition probabilities from a state are path-independent, the probability of any path/string is simply the product of all transitions on the path.

Markov Models for Default Probability

A Markov model based on the one-year state transition matrix can predict default probability for any grade/duration.

All corporate average one year rating transition matrix 1980-1999

Rating to:	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	WR
Rating from: Aaa	85.88	9.76	0.48	0.00	0.03	0.00	0.00	0.00	3.84
Aa	0.92	84.87	9.64	0.36	0.15	0.02	0.00	0.04	4.01
A	0.08	2.24	86.24	6.09	0.77	0.21	0.00	0.02	4.36
Baa	0.08	0.37	6.02	79.16	6.48	1.30	0.11	0.19	6.30
Ba	0.03	0.08	0.46	4.02	76.76	7.88	0.47	1.40	8.89
B	0.01	0.04	0.16	0.53	5.86	76.07	2.74	6.60	7.98
Caa-C	0.00	0.00	0.00	1.00	2.79	5.38	56.74	25.35	8.73

Source: Moody's

Much of the current credit default crisis is due to rating agencies issuing too high grades to lousy securities.

Default Probabilities Across Portfolios

There is very little chance of getting much less than half tails when flipping many coins, because the events are independent.

Note that the probability of two (say) AA rated companies defaulting over the next Y years is **not independent**, and reflects shared economic conditions.

The higher the correlations between the equity returns and industrial sector, the less independent they are.

Inflation and Interest Rates

That there is close connection between interest rates and inflation can be proven by an arbitrage argument.

If the risk-free rate is too low relative to inflation, people would buy durable goods rather than deposit money in the bank.

Example: During Israel's hyperinflation of the 1980's phone tokens (asimonim) were the most stable form of currency.

If the risk-free rate is too high relative to inflation, people would sell all durable goods and keep the money in the bank.