# Lecture 6: Portfolios with Stock Options

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# Portfolios with Options

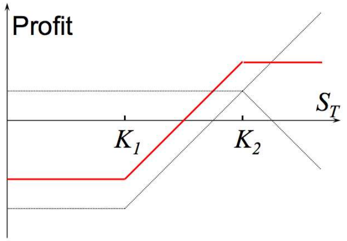
The raw materials of investments we have include borrowing/depositing money, buying/selling options, and buying/selling stocks.

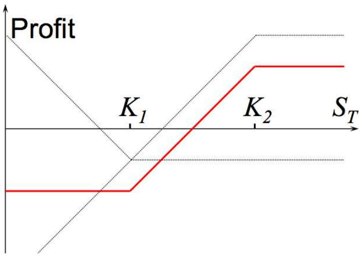
Combinations of options and stocks can be used in clever ways to engineer portfolios exploiting certain opportunities and eliminating particular risks.

By combining put and call options, we can place bets which payoff under a variety of different market conditions.

# Bull/Bear Spreads

A *bull spread* consists of a buying a call with strike price $K_1$ and selling a call with price $K_2 > K_1$.
I get $\max(S_T - K_1, 0)$ for the first call and my payment for the second option, but must pay $\max(S_T - K_2, 0)$
Both options pay off if $S_T > K_2$, so I gain $K_2 - K_1$.
If $K_1 < S_T < K_2$, I gain $S_T - K_1$
If $S_T \leq K_1$, I gain nothing.



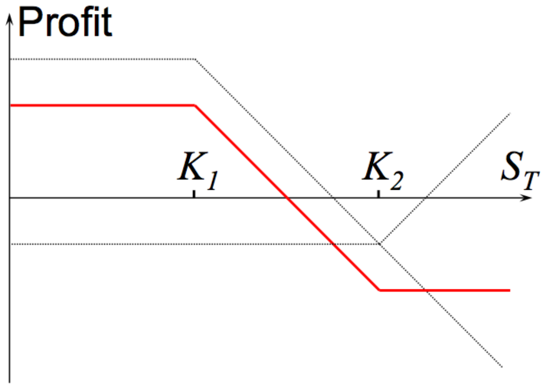
This strategy limits both my upside and downside risk/return. However, if the current spot price $S_0 < K_1$, the options should be cheap and the strategy can show high returns if the stock price rises.

Bull spreads can alternately be constructed by buying and selling puts.

The prices of these options, and hence our profit/loss, depends upon their relations to the strike price.

# Bear Spreads

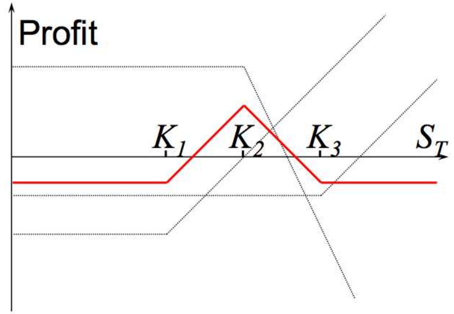
A *bear spread* inverts the role of the calls to profit if the stock price declines.




Sell a call at $K_1$, buy a call at $K_2$, or
Sell a put at $K_1$, buy a put at $K_2$.

# Butterfly Spreads

A *butterfly spread* is designed to profit from prices staying close to the current spot price $S_0$, i.e. no news is good news.
We buy one call with strike price $K_1 < S_0$ and another call with price $K_3 > S_0$.
We sell two calls with strike price $K_2 \approx S_0 = (K_1 + K_2)/2$.



## Case Analysis for the Butterfly

If $S_T < K_1$, nothing pays off for anybody.

If $S_T > K_3$, I win with the calls I bought but lose with the calls I sold, offsetting each other.

If $K_1 < S_T < K_2$, I win with my first call, for a profit of $S_T - K_1$

If $K_2 < S_T < K_3$, I win with my first call but lose with the calls I sold, for a profit of $S_T - K_1 - 2(S_T - S_2) = K_3 - S_T$

# Combinations of Puts and Calls

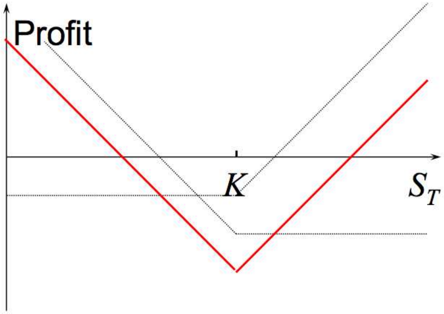
Other combination strategies follow from buying both calls and puts.

In a *straddle*, I buy both a call and a put at strike price $K$.

If $S_T < K$, my put pays off $K - S_T$ and my call is worthless.

If $S_T > K$, my call pays off at $S_T - K$ and my put is worthless.

Thus I profit from significant moves in *either direction*.

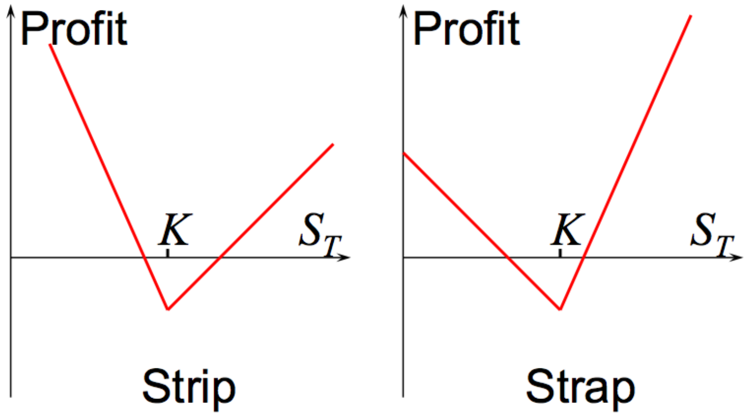


This might be a good strategy if a drug company were to announce the results of a drug trial (positive or negative) in the near future.

Reversing the roles of the buying and selling on a butterfly spread is an alternate way to create an option which profits from movements in either direction.

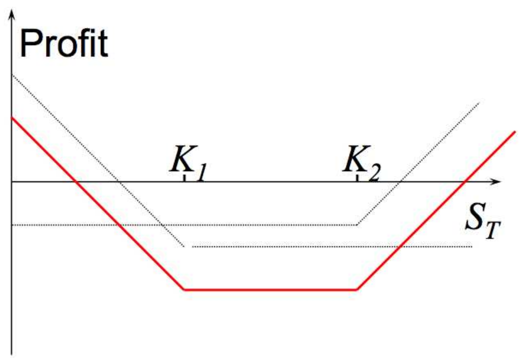
# Strips and Straps

Buying an extra call (strip) or put (strap) on top of a straddle profits from large movements either way, but particularly in one direction.



# Strangle

Buying a put at $K_1$ and a call at $K_2$ provides another way to profit from large movements.




Bigger movements are required than a straddle, but at less risk.

## Options on Options

In principle, we can use options to profit under any given market conditions *if* we can predict correctly what those market conditions are.

We can realize any possible payoff function if and only if options are available at every possible strike price – but this is not the case on options exchanges.

*Algorithm problem*: select the optimal set of options to profit from a given price prediction probability distribution.

In principle, such predictions are difficult or impossible, depending upon which theory of the markets you believe.

# Dangers of Hedging

The use of options for hedging is appropriate for reducing risk, but such hedging also reduces upside potential.

In principle, hedging via options does not raise our expected return. Indeed, since we must pay for the options, our expected return is lower than if we did not hedge.

# Hedging in Time: Calendar Spreads

Consider a portfolio of two options (a put and a call) with the same strike price but different expiration dates.

A *calendar spread* sells a call at strike price $K$ for date $D_1$ and buys one at $K$ for date $D_2 > D_1$.

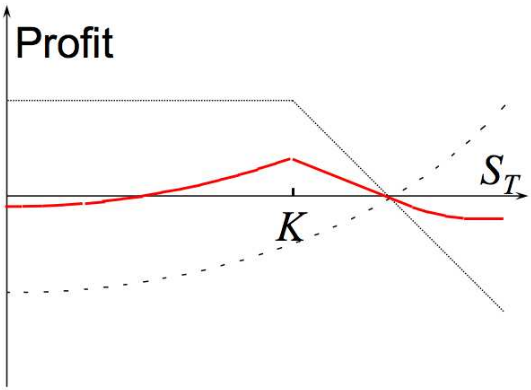
What is the value on $T = D_1$ with spot price $S_T$?

If $S_T << K$, both options are worthless.

If $S_T >> K$, both options are in the money and offset each other.

If $S_T = K$, the short-maturity one is worth near zero but the long-maturity one has great value.

# Calendar Spread Payoffs




The payoff lines are curves reflecting the non-linear nature of options prices.
The overall shape is somewhat like a butterfly spread.

# Exotic Options

Other, more complicated options can be constructed.

*Asian options* have payoffs dependent on the average price over a specified period.

*Bermudan options* can executed on specified dates, and hence are between American and European options.

The more complicated the instrument, the more difficult it is to calculate its true value.

Most futures and options are settled for cash values, instead of delivering the actual goods.