Lecture 4:
Futures and Options

Steven Skiena

Department of Computer Science
State University of New York
Stony Brook, NY 11794–4400

http://www.cs.sunysb.edu/∼skiena

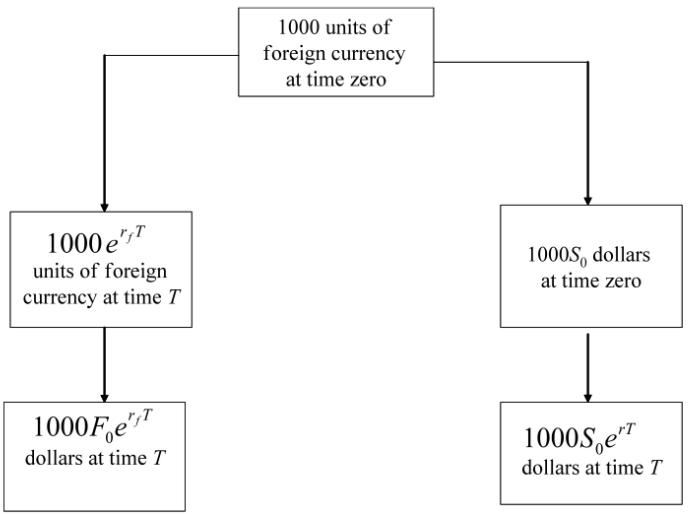
Foreign Currency Futures

Assume that foreign country f has a risk-free interest rate *in its own currency* of r_f.
Let S_0 be the spot price in dollars of one unit of f, and F_0 be the future price in dollars of one unit of f. Then

$$F_0 = S_0 e^{(r-r_f)T}$$

Thus future currency prices are purely a function of the interest rates in the two countries!

Pricing Currency Futures



If one of these options is better, then I can borrow the initial money and guarantee a risk-free profit.

Foreign Currency Arbitrage

Note that I can borrow Y units of currency f at rate r_f, (costing me $Ye^{r_f T}$ in currency f then) convert this to dollars and invest (earning me YS_0e^{rT} in dollars then).
Thus if $F_0 < S_0e^{(r-r_f)T}$, I can go long on a forward contract to get the $Ye^{r_k T}$ in currency f to pay them back at less than I earned on my dollars.
Note that I can borrow Y dollars at rate r, (costing me Ye^{rT} in dollars then), convert this to Y/S_0 units of currency f, and invest this at rate r_k (earning me $(Y/S_0)e^{r_k T}$ in currency f then)
Thus if $F_0 > S_0e^{(r-r_f)T}$ I can go short on a forward contract to sell my currency f, earning more than I paid for my dollars.

Futures Contracts

Closely related are futures contracts for commodities, traded on exchanges like the Chicago Board of Trade (CBOT).

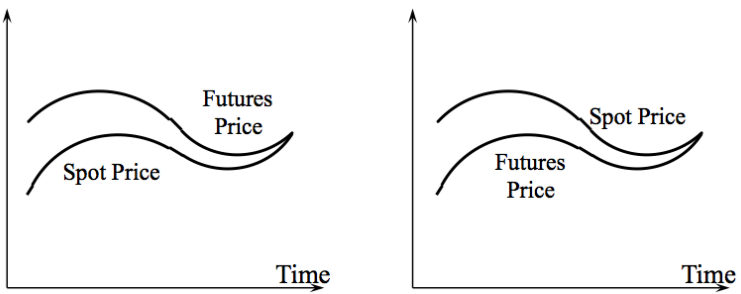
Futures differ from forward contracts in allowing more flexibility on the timing of when the short position must sell.

Futures contracts are completely standardized (e.g. 5,000 bushels of number 2 corn for delivery in July) so they can traded on exchanges.

Most futures contracts are closed prior to delivery, by buying or selling an offseting contract.

Convergence of Future and Spot Prices

We expect that the spot price of an asset converges to that of the futures price as the delivery date of the contract approaches – otherwise an arbitrage opportunity exists.



This convergence means the spot price may go up (down), the futures price may go down (up), or both.

Arbitrage with Futures

If the futures price stays above the spot price, we can buy the asset now and short a futures contract (i.e. agree to sell the asset later at the future price). Then we delivery and clear a profit.

If the futures price stays below the spot price, anyone who wants the asset should go long on a futures contract and accept delivery instead of paying the spot price.

Such arguments do not factor in the cost of storage, convenience, and the durability of the asset.

Expected Future Prices

The gap between the spot and futures prices may contain information about the *expected future price* of the asset.

Keynes and Hicks theorized that the expected future price depends upon the behavior of hedgers. Hedgers seek to reduce risk, and are in principle willing to pay for this.

Speculators will only be in the game if they can expect profits.

Thus if hedgers are going short and speculators are going long, the expected future price should be above the future contract price.

Implementing Short Sales

Many arbitrage arguments assume the ability to short sell an asset, i.e. sell the asset now without owning it.

Unfortunately, such contracts are not available for all assets.

However, if the forward price is too low, anyone who owns the asset should sell the asset, invest the proceeds at the risk-free rate, and buy a forward contract to buy it back later at the fixed price.

Such arbitrage arguments also work if you can rent/borrow the asset for the desired period from someone who already owns it.

Options

Options give the right to do something, but not the obligation.
Call options permit you to *buy* an asset for a certain price by a certain date.
Owning a call option means you want asset prices to go up.
Put options permit you to *sell* an asset for a certain price by a certain date.
Owning a put option means you want asset prices to go down.

Payoff on a Call

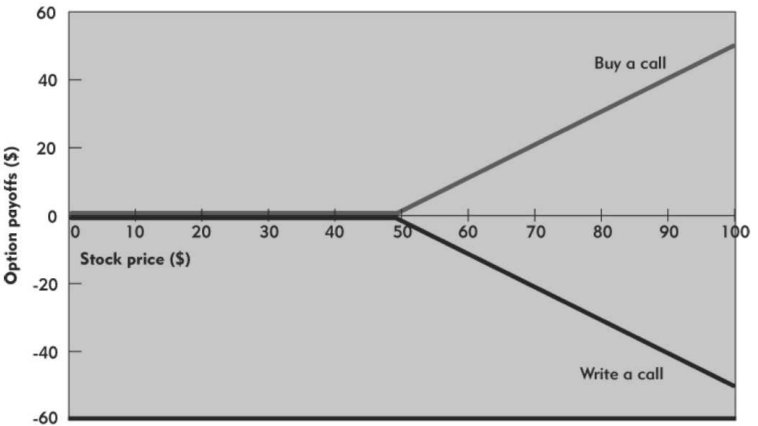
The payoff of a *long position* (buying the option) of a *call* (to buy the asset) at strike price K and current price S_t is $\max(S_t - K, 0)$.

Why? I win if the current price is greater than what I am allowed to buy it for.

My option is worth nothing if the spot price is less than the strike price.

The pay for a short position (selling the option) on the call is $-\max(S_t - K, 0) = \min(K - S_t, 0)$, by a conservation of money argument.

Call Option Payoffs



Payoff on a Put

The payoff of a *long position* (buying the option) of a *put* (to sell the asset) at strike price K is $\max(K - S_t, 0)$.
The corresponding payoff for the *short position* (selling the option) is $\min(S_t - K, 0)$.
The seller of an option make their gain from what they were originally paid for the option.

Put Option Payoffs