Lecture 2: Financial Markets and Products

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Bond Markets

Bond markets trade bonds ("loans") made to governments and government agencies, as well as companies. Bonds are contracts for a specified party makes a specified sequence of payments according to a specified schedule. The ability to trade debt increases the value and liquidity of such investments. Bond prices vary according to the term (length of time) of the loan, the interest rate and payment schedule, the financial strength of the borrowing party, and the returns available from other investments.
The Time Value of Money

The value $V$ of an asset $A$ after $n$ years of compounding $m$ periods per year at an annual interest rate of $r$ is

$$V = A(1 + \frac{r}{m})^{mn}$$

In the case of continuous compounding, $m \to \infty$ and

$$V = Ae^{rn}$$

This exponential growth explains why compound interest is a good thing.

The time value of money is a fundamental principle regulating how the world works.
Einstein discovers that time is actually money.
Perpetual Annuities

What is the value of a security which pays you $1 per year forever? You will eventually receive an infinite number of dollars, so is it infinite?

The present value of the dollar received in the second year is the amount \( V_2 \) we can put in the bank now to have $1 then, so given the available interest rate \( r \) so

\[
V_2(1 + r) = $1 \rightarrow V_2 = \frac{1}{(1 + r)} \quad \text{(annual compounding)}
\]

\[
V_2(e^r) = $1 \rightarrow V_2 = \frac{1}{e^r} \quad \text{(continuous compounding)}
\]
The General Case

\[ V_m(1 + r)^m = $1 \rightarrow V_m = \frac{1}{(1 + r)^m} \quad \text{(annual compounding)} \]

\[ V_m(e^{mr}) = $1 \rightarrow V_m = \frac{1}{e^{mr}} \quad \text{(continuous compounding)} \]

Using the formula for the sum of an infinite series,

\[ 1 + a + a^2 + \ldots = \frac{1}{1 - a} \]

yields a present value of \(1/r\) (annual) or \(1/(e^r - 1)\) (continuous).

Thus a 5% interest rate and annual compounding prices this annuity at only $20.
Fixed-Rate Mortgage Payments

The *present value* of the stream of payments in a fixed rate mortgage must equal the value of the principal \((L)\) borrowed. Now we only care about summation of the first \(m\) terms of a geometric series, so

\[
S_m(a) = 1 + a + a^2 + \ldots + a^{(m-1)} = \frac{1 - a^m}{1 - a}
\]

There will be \(m = 12y\) equal monthly payments of \(\$x\) over \(y\) years, so

\[
L = x \left( \sum_{i=0}^{12y-1} \frac{1}{(1 + r/12)^i} \right)
\]

and

\[
x = \frac{L}{S_{12y}(1/(1 + r/12))}
\]
Mortgage Amortization and Valuation

Amortization means “killing” the loan. The amount I still owe the bank is equal the present value of the remaining payments. Thus the last payment is all principal, and the first all interest.

The value of this payment stream as a bond is a function of the current interest rate and the trustworthiness of the borrower.
Commodity Markets

*Commodities* are types of goods which can be defined so that they are largely indistinguishable in terms of quality (e.g. orange juice, gold, cotton, oil, pork bellies). Commodities markets exist to trade such products, from before they are produced to the moment of shipping. Commodity futures are agreements to buy or sell a certain amount of a commodity at a particular price at a particular point in the future. The *spot price* is the cost of buying a good now, while *future price* concerns some future date.
Hedging Against Risk

The existence of commodity futures gives suppliers and consumers ways to protect themselves from unexpected changes in prices.

- A farmer might want to lock in a price now for corn they are growing by selling a future contract.
- Airlines might want to protect themselves against fuel price changes by buying future contracts in oil.

The prices of commodities are affected by changes in supply and demand resulting from weather, political, and economic forces.
Currency Markets

The largest financial markets by volume trade different types of currency, such as dollars, Euros, and Yen. Typically, each seller has a buy and sell price for a given currency, and makes their money from the *spread* between these two prices. Buying equals selling, or else the prices change.

Currency markets are used to (a) acquire funds for international trade, (b) hedge against risks of currency fluctuations, (c) speculate on future events.

As we will see, the “right” relative price between two currencies is a function of their respective interest rates.
Arbitrage versus Speculation

- **Speculators** are investors who deliberately take risks by betting on future events. For example, they will buy a stock because they think it will go up.

- **Hedgers** are investors who trade so as to reduce their exposure to risk. For example, they will both buy and short a stock simultaneously.

- **Arbitrage** is a trading strategy which takes advantage of two or more securities being inconsistently priced relative to each other.

Advanced arbitrage techniques involve sophisticated mathematical analysis and rapid trading.
Derivatives

Derivatives are financial instruments whose value derives from the values of other, more basic variables. Options give the owner the right, but not the obligation, to buy or sell a security at a specified price on (or perhaps before) a specified date. The Chicago Board of Exchange (www.cboe.com) trades options on over 1200 stocks and stock indices. Futures contracts gives one the right and obligation to buy or sell a commodity at a given price at a given time.