Lecture 19:
The Capital Assets Pricing Model

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The Capital Asset Pricing Model

The Markowitz portfolio model takes as input the expected returns, volatilities, and pairwise correlations between n assets to design the optimal portfolio.

But accurately estimating these $\binom{n}{2} + 2n$ parameters is difficult.

The Capital Asset Pricing Model (CAPM) simplifies this by relating each asset to an index (the *market portfolio*).

Asset values are assumed to be related to the market but uncorrelated with each other.

The market portfolio is best thought of as a value weighted index of all assets (say the S&P 500).

Beta

The required rate of return for a particular asset j depends on its sensitivity to the movement of the market portfolio M (i.e. the broader market).

This sensitivity is known as the asset β and reflects asset's systematic risk.

$$\beta_j = \frac{Cov[R_j, R_m]}{Var[R_m]} = \frac{E[(R_j - \mu_{R_j})(R_m - \mu_{R_m})]}{E[(R_m - \mu_{R_j})^2]}$$

For the market portfolio $\beta = 1$ by definition.

A risk-free portfolio has $\beta = 0$.

More sensitive (risky) stocks will have a higher beta, less sensitive stocks will have lower betas.

The Single Index Model

Under CAPM, the return on asset j can be decomposed into three parts:

$$R_j = \alpha_j + \beta_j R_m + \epsilon_j$$

where α_j is a constant drift, β_j is the dependence on the market return, and ϵ_j is a random specific risk component of mean zero.

The values of α_j and β_j can be determined by regressing the asset j returns against the index m.

The expected return on j is a function of the expected market return $\mu_j = \alpha_j + \beta_j \mu_m$, with standard deviation $\sigma_j = \sqrt{\beta_j^2 \sigma_m^2 + e_j^2}$ where e_j is the standard deviation of ϵ_j.

Total Risk vs. Systematic Risk

Two assets with the same total risk can have very different systematic risk.
Suppose $\sigma_m = 20\%$

Stock	Business	Market beta	Residual variance
1	Steel	1.5	0.10
2	Software	0.5	0.18

$$\sigma_1^2 = (1.5^2)(0.2)^2 + 0.10 = 0.19$$

$$\sigma_2^2 = (0.5^2)(0.2)^2 + 0.18 = 0.19$$

Required Rates of Returns under CAPM

According to the CAPM the required rate of return for a stock is

$$r_s = \beta(r_m - r_f) + r_f$$

where r_s is the required rate of return on a stock, r_m is the return rate of the market portfolio, r_f is the risk-free interest rate, and β is the beta of the stock.

This says that the relative return of a given stock is a completely a function of its volatility!

Assessing CAPM

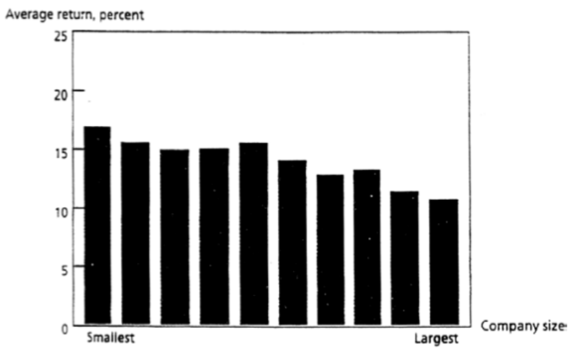
There is some tension between CAPM and arbitrage theory in how assets are priced, but we will accept both.

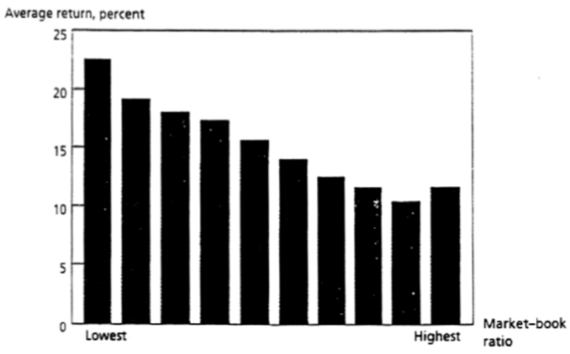
Does CAPM adequately explain the variation in stock returns? As with all simple models of complex phenomena, the answer is not completely.

However, that it remains the most widely used method for calculating the cost of capital means it is worth our respect.

Other Predictors of Returns

In recently decades, the relationship between volatilty and return appears weaker than market capitalization and return.





The book or accounting value of a company is its net asset value as per its balance sheet.

The *market-to-book ratio* compares book value to market capitalization.

Sharpe Ratio

Just considering the return on a portfolio Π over time paints on incomplete picture of the quality of the manager, because it ignores risk.

Better is the Sharpe ratio, which measures reward to volatility:

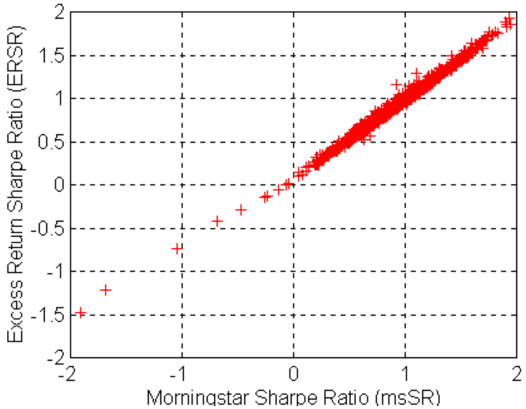
$$\text{Sharpe ratio} = \frac{E[\Pi - r]}{\sqrt{\text{Var}(\Pi - r)}}$$

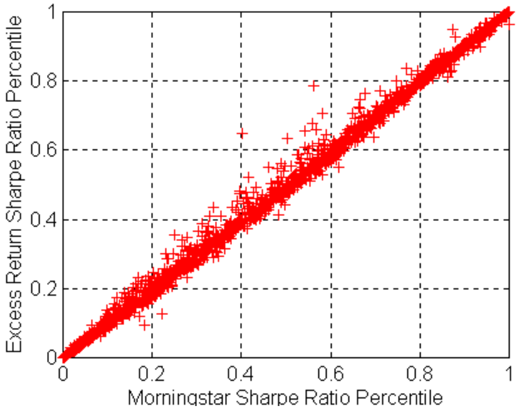
where r is the risk-free rate.

The single portfolio with the best Sharpe ratio is the best with which to employ leverage to boost expected returns.

Sharpe Ratios vs. Morningstar Fund Ratings

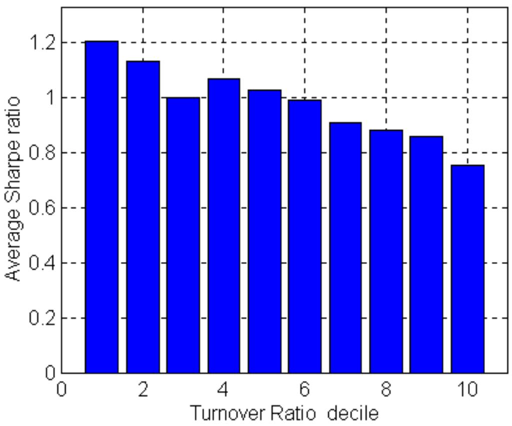
Respected mutual fund ratings rank funds essentially by Sharpe ratio:





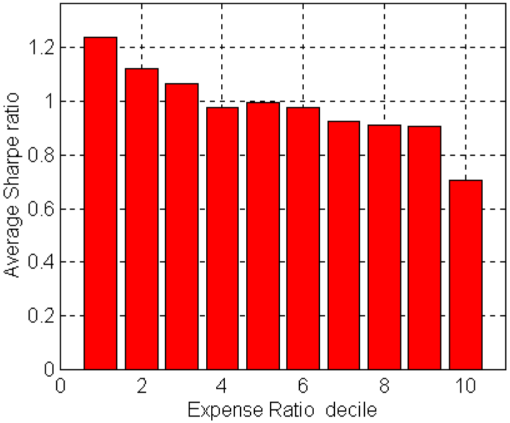
Active vs. Passive Trading Strategies?

Historical data ranking funds by trading activity favor relatively passive funds:



High vs. Low Expense Strategies?

Historical data ranking funds by expenses favors low expense funds, such as index funds:



Large vs. Small Funds?

Historical data ranking funds favor larger funds, presumably because they have less expenses and success attracts investment: