Lecture 18:
Portfolio Theory

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Investment Strategies

Thus far we have studied the basic mechanics of markets, options pricing, and financial data analysis.

A *money* or *investment manager* uses these tools to develop a strategy to invest capital to best achieve financial goals.

Such goals include some combination of maximizing returns, subject to limiting risk and providing adequate liquidity over a given time period.

A *trader* seeks to implement a given strategy with maximum efficiency and minimum costs.

In this section of the course we will study several aspects of trading and investment strategies.

Managing Portfolios

The view that investment strategies involve picking the right stock is naive.

More important is constructing and managing portfolios of sizable numbers of different securities, and adjusting the weighting to accomplish different objectivies.

Often messy factors outside pure risk-return come into play such as (1) tax liabilities and (2) liquidity/market impact.

Portfolio theory provides the tools to reason about constructing optimal collections of holdings.

Portfolio Theory

Both Markowitzian portfolio theory and the simplified *Capital Assets Pricing Model* (CAPM) relate returns on an asset to the level of risk associated with the asset.

Risk is a notion associated with the *variance* or *volatility* of the investment.

The returns from a *risk-free* investment such as U.S. Treasury bills are guaranteed, but fairly low.

The expected returns from a more volatile investment such as stocks is higher, but the variance in returns is also higher.

The theory assumes investors are *risk-adverse*, that they will take riskier investments only if the expected returns are higher.

Types of Risk

The risk of a given portfolio can be partitioned into *systematic risk* and *specific risk*.

The *specific risk* or *non-systematic risk* is the risk associated with a given asset. For example, will the given company's management perform worse than it should?

The *systematic risk* or *market risk* is the risk common to all securities. For example, will the Fed raise interest rates?

Note that the specific risk can be diversified away by buying a variety of different assets, since the specific risks will cancel each other out.

Volatility and Value

If several stocks have the same expected return, we are better off owning all of them than of them, because we remove specific risk.

The expected return is the same, but the variance of the combined portfolio is lower. Thus the combined portfolio will be superior to a risk-adverse investor.

If assets A and B have the same expected return, but A is more volatile than B, which should be more desirable?

What does this say about the future price/return of A and B?

Volatility also impacts options prices, since volatile assets are more likely to significantly change in value over time.

Diversification and Portfolio Theory

The theroy assumes that the risk-return profile of a portfolio can be optimized, where an optimal portfolio displays the lowest possible level of risk for its level of return.

Since adding a new asset to a portfolio further diversifies it, the optimal portfolio in theory contains all possible assets appropriately value-weighted.

There is an optimal portfolio for *any* desired rate of return (up to the highest of any security) because we can always invest some fraction of capital at the risk-free rate and the rest in an optimal portfolio at some higher rate.

Diversification in Practice

Such *diversification* explains why stock index funds perform better than holding individual stocks.

Much of the difference in the performance of different investment managers is due to variable outcomes in their specific risk more than investment skill.

In practice, about 15 well-selected stocks can provide much the same level of diversification as a market index.

Uncorrelated Returns

To maximize diversification, we should select securities whose performance is *uncorrelated* or (even better) *negatively correlated*.

Suppose stock A returns 20% in summer/fall and 0% in winter/spring and stock B returns 0% in summer/fall and 20% in winter/spring.

Each stock has a 10% return with high variance, but because they are perfectly negatively correlated buying half of each gives us 10% return with zero variance.

Unfortunately all asset returns become correlated in a crisis…

Portfolios of Two Stocks

Suppose there are only two stocks (IBM and Merck), offering returns of 1.49% and 1.00% respectively with variance of 0.007770 and 0.003587, respectively.

By adjusting the fraction of our portfolio (weight) devoted to IBM (w_1, where $w_1 + w_2 = 1$), we can modulate our return to any value between 1.00% and 1.49%.

The variance of returns also changes as a function of w_1, w_2, and the covariance between the two stock's returns:

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

Portfolios with Short Sales

Returns higher than the highest asset are possible if you allow short sales – sell short sell some of the lower return asset and use the proceeds to buy the higher return one.

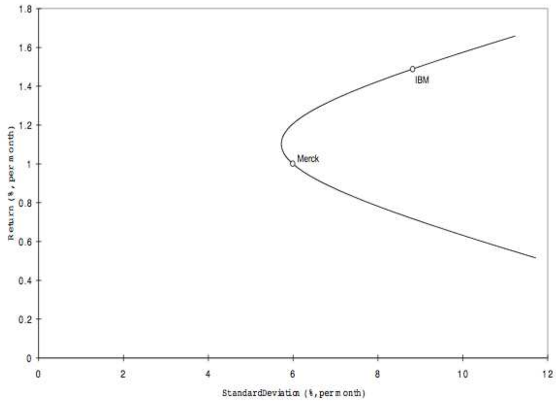
Like using leverage (borrowed money), the tradeoff is increased risk.

Covariances	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Merck}}$
$\tilde{r}_{\text{IBM}}$	0.007770	0.002095
$\tilde{r}_{\text{Merck}}$	0.002095	0.003587
Mean	1.49%	1.00%
StD	8.81%	5.99%

Portfolios of IBM and Merck

Weight in IBM (%)	-40	-20	0	20	40	60	80	100	120	140
Mean return (%)	0.80	0.90	1.00	1.10	1.20	1.29	1.39	1.49	1.59	1.69
StD (%)	7.70	6.69	5.99	5.72	5.95	6.62	7.61	8.81	10.16	11.60

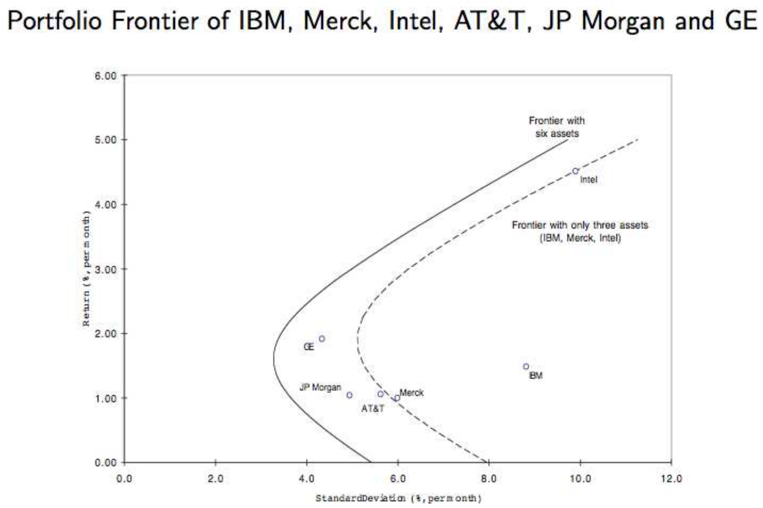
The Portfolio Frontier



Note that I can earn a higher return than Merck at less risk in a portfolio.

The frontier describes the space of optimal portfolios. The best point on this portfolio frontier depends upon one's appetite for risk or *utility function*.

Larger Numbers of Assets



Adding more securities can only improve the frontier.

Portfolio Frontier with a Safe Assest

When there exists a risk-free asset, each portfolio consists of the risk-free asset and risky assets.

Viewing the risky assets as a sub-portfolio, it is clear this sub-portfolio should lie on the optimal frontier.

In fact, linear combinations of the tangent portfolio and the risk-free asset dominate the portfolio frontier.