Factors Affecting Option Prices

1. The current stock price S_0.

2. The option strike price K.

3. The time to expiration T.

4. The volatility of the stock price σ.

5. The risk-free interest rate r.

6. The value of dividends expected during the life of the option.

Impact on Option Prices

Suppose each relevant factor increased in isolation. How does that impact the value of an *American* option?

variable	call	put
stock price	$+$	$-$
strike price	$-$	$+$
expiration date	$+$	$+$
volatility	$+$	$+$
risk-free rate	$+$	$-$
dividends	$-$	$+$

The confusing one is the impact of increasing the risk-free rate. If interest increases *independently* of stock price, the present cost (value) of my call purchase (put sale) in the future decreases – good for call, bad for put.

In practice, when interest rates go up usually stock prices down, so a rate increase is bad for a call and good for a put.

The only change with European option prices are that there is no certain relationship with expiration date – for example, increasing T might cover an extra dividend payment.

Upper/Lower Bounds on European Option Prices

Let p and c be the value of European put and call options.

We assume no stocks pay dividends.

Since an option to buy a share of stock at any positive price cannot be worth more than the value of the share, so $c \leq S_0$.

Since an option to sell a share of stock at the strike price cannot be worth more than the strike price, so $p \leq K$.

Since the European option is worth at most K at maturity, it cannot be worth more than the discounted value of this amount, so $p \leq Ke^{-rT}$.

A more sophisticated bound, provable by arbitrage arguments about two portfolios, is that

$$c \geq \max(S_0 - Ke^{-rT})$$

Better Bounds

Portfolio A consists of a European call option for one share plus Ke^{-rT} worth of cash to execute it at time T.

At time T, this is worth S_T if $S_T > K$, or K if $K > S_T$ and we don't buy the option, or in other words $\max(S_T, K)$.

Portfolio B consists of one share, and will be worth S_T at time T.

That A is worth more than B at time T, which implies it A is worth more than B now since otherwise an arbitrage possibility exists.

Thus $c+Ke^{-rT} \geq S_0$. Since no option can have negative value,

$$c \geq \max(S_0 - Ke^{-rT})$$

A similar argument for puts shows that

$$p \geq \max(Ke^{-rT} - S_0)$$

Put-Call Parity

In fact, there is a tight *put-call parity* relationship between the value of a European put and call. Consider the following portfolios:

Portfolio A consists of a European call option for one share plus Ke^{-rT} worth of cash to execute it at time T, worth max(S_T, K) at time T by our previous arguments.

Portfolio C consists of a European put option plus one share of stock, also worth max(S_T, K) at time T.

Since they have equal value then they must have equal value now, so

$$c + Ke^{-rT} = p + S_0$$

Similar but weaker bounds hold for American options

$$S_0 - k \leq C - P \leq S_0 - Ke^{rT}$$

Empirical results basically support the bound on over reasonable time scales to the resolution of transaction costs.

Bull/Bear Spreads

By combining put and call options, we can place bets which payoff under a variety of different market conditions.

A *bull spread* consists of a buying a call with strike price K_1 and selling a call with price $K_2 > K_1$.

I get $\max(S_T - K_1, 0)$ for the first call and my payment for the second option, but must pay $\max(S_T - K_2, 0)$

Both options pay off if $S_T > K_2$, so I gain $K_2 - K_1$.

If $K_1 < S_T < K_2$, I gain $S_T - k_1$

If $S_T \leq K_1$, I gain nothing.

This strategy limits both my upside and downside risk/return.

However, if the current spot price $S_0 < K_1$, the options should be cheap and the strategy show high returns if the stock price rises.

A *bear spread* inverts the role of the calls to profit if the stock price declines.

Butterfly Spreads

A *butterfly spread* is designed to profit from prices staying close to the current spot price S_0, i.e. no news is good news.

We buy one call with strike price $K_1 < S_0$ and another call with price $K_3 > S_0$.

We sell two calls with strike price $K_2 \approx S_0 = (K_1 + K_2)/2$.

If $S_T < K_1$, nothing pays off for anybody.

If $S_T > K_3$, I win with the calls I bought but lose with the calls I sold, offsetting each other.

If $K_1 < S_T < K_2$, I win with my first call, for a profit of $S_T - K_1$

If $K_2 < S_T < K_3$, I win with my first call but lose with the calls I sold, for a profit of $S_T - K_1 - 2(S_T - S_2) = K_3 - S_T$

Combinations of Puts and Calls

Other combination strategies follow from buying both calls and puts.

In a *straddle*, I buy both a call and a put at strike price K.

If $S_T < K$, my put pays off $K - S_T$ and my call is worthless.

If $S_T > K$, my call pays off at $S_T - K$ and my put is worthless.

Thus I profit from significant moves in *either direction*.

This might be a good strategy if a drug company were to announce the results of a drug trial (positive or negative) in the near future.

Reversing the roles of the buying and selling on a butterfly spread is an alternate way to create an option which profits from large moves in either direction.

Options on Options

In principle, we can use options to profit under any given market conditions *if* we can predict correctly what those market conditions are.

Interesting algorithm problem: select the optimal set of options to profit from a given price prediction probability distribution.

In principle, such predictions are difficult or impossible, depending upon which theory of the markets you believe.

The use of options for hedging is appropriate for reducing risk, but such hedging also reduces upside potential.

In principle, hedging via options does not raise our expected return. Indeed, since we must pay for the options, our expected return is lower than if we did not hedge.