2. Adversary Argument

Adversary

Algo

⇒ devise a strategy to construct a worst case input for an unknown correct algorithm that solves the problem.

⇒ Adversary interacts with some algorithm in such a way that the worst case input for the algo is generated.

Ex > Element Uniqueness in a comparison based mode.

Adversary, private DB,

⇒ initially require all the values in set be within interval \(0, n\)
all the value $v_i$ will have interval $[a_i, b_i]$ associated with.

each time a comparison is done certain interval may be shrunk.

if intervals of two values overlaps, then the value could be equal $v_i \cap v_j$. 

if the algorithm would halt before all intervals are pairwise nonoverlapping, assuming the adversary has not yet said "$=$", then the adversary could answer either "$>$" or "$<". Thus the algorithm is not correct.

**Adversary Action For Corp. EUP.**

"when algo ask : compare $v_i$ vs $v_j$"

"Adversary Actin:

$$\text{if } i = j \text{ then answer } v_i = v_j \text{ or } t$$
\[ m_i = \frac{c_i + b_i}{2} \quad \text{and} \quad m_j = \frac{c_j + b_j}{2} \]

\[ [v_i] = \frac{c_i - b_i}{v_j - v_i} \]

\[ [v_j] = \frac{c_j - b_j}{v_j - v_i} \]

\[
\begin{align*}
\text{if } m_i \leq m_j & \text{ then} \\
& b_i + m_i \\
& a_i + m_i \\
& b_j + m_j \\
\text{else } & \text{ (if } m_i > m_j \text{)} \\
& a_i + m_i \\
& b_j + m_j \\
& \text{answer } "v_j < v_i" \quad \text{answer } "v_j < v_i" \\
\end{align*}
\]

\text{endif}

\text{Analysis of Adversary Action}

When the intervals are non-overlapping, the total length is \(N\):

\[ \sum_{i=1}^{n} l_i \leq N \quad l_i = \text{length of } [c_i, b_i] \]

- When \( l_i = \frac{N}{2c_i} \) where

\[ c_i \text{ is # of interesting comparisons for } v_i \]

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\[
\sum_{i=1}^{n} \frac{1}{2} c_i \leq n 
\]

\[
\sum_{i=1}^{n} \frac{1}{2} c_i \leq 1 
\]

**Claim**

\[
\sum_{i=1}^{n} c_i \text{ is minimized when}
\]

\[
 \text{time complexity.}
\]

\[
C_j = C_k \text{ for all } j, k
\]

**assume**

\[
\frac{1}{2} c \leq \frac{1}{n} 
\]

\[
2^{c} \geq n 
\]

\[
C_i \geq 1/\ln n 
\]

\[
\sum_{i=1}^{n} c_i \geq n \ln n 
\]

\[\text{Thus, etc. U.P. is } \sum (n \ln n)\]