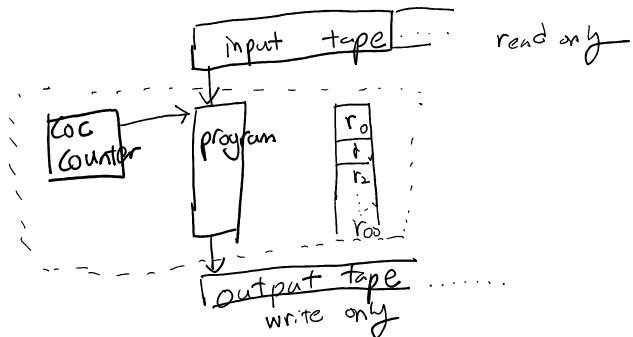


## Random access machine (RAM)



## \*assumptions about memory

- ① reg. can hold integers of arbitrary size
- ② # reg. infinite

## o \*assumption about program.

- ① program does not modify itself
- ② program is seq of opntrn (instructions)

## o two complexities in RAM

- uniform cost criterion for time:  
→ instructions req. one unit of time

- o logarithmic cost criterion for time  
a instruction req.  $l_{ci}$  time on integer  $i$

$$l_{ci} = \begin{cases} 1 & i = 0 \\ \lceil \log_2(i+1) \rceil + l_{c0} & i > 0 \end{cases}$$

- o uniform cost criterion for space  
: each reg. used one unit of space

- o log. cost criterion for space

$$\sum_{\text{location } i} l_{ci} \text{ where } x_i \text{ is the largest value ever stored in } i$$

- Ex) Factoring  $n$  by "trial division"  
 $m \leftarrow n$

$F \leftarrow \emptyset$   
 $i \leftarrow 2$   
 while  $i \leq \lfloor \sqrt{n} \rfloor$  do

if  $i$  divides evenly into  $m$   
 then insert  $i$  into  $F$   
 $m \leftarrow m/i$   
 else  $i \leftarrow i+1$   
 endif  
 endwhile

if  $m > 1$ , then insert  $m$  into  $F$

\* ab

• # times through the while loop

$$\leq \lfloor \sqrt{n} \rfloor + \lg n$$

Ex)  $2^4 = 16 = 2 \times 2 \times 2 \times 2$

$$\begin{array}{ll}
 \left. \begin{array}{l} i=2 \\ i=2 \\ i=2 \\ i=2 \end{array} \right\} m = 2 \times 2 \times 2 \times 2 & \sqrt{2^4} = 4 \\
 \left. \begin{array}{l} i=2 \\ i=2 \\ i=2 \\ i=2 \end{array} \right\} m = 2 \times 2 \times 2 & \\
 \left. \begin{array}{l} \lg 2^4 \\ = 4 \end{array} \right\} i=2 \quad m=2 & \\
 \left. \begin{array}{l} \lg 2^4 \\ = 4 \end{array} \right\} i=2 \quad m=1 & \\
 \left. \begin{array}{l} \sqrt{2^4} \\ = 4 \end{array} \right\} i=2 & 
 \end{array}$$

• max # of factor?  $\lg n$

• uniform cost

time  $\mathcal{O}(\lfloor \sqrt{n} \rfloor + \lg n) = \mathcal{O}(\sqrt{n})$

space  $\mathcal{O}(\lg n)$

• logarithmic cost

trial division

• logarithmic cost

time: worst loop time for each ~~loop~~ trial division

is when  $m$  never gets updated

$\rightarrow \underline{\lg(n)}$  for each trial division  $(\sqrt{n})$

$\therefore O(\sqrt{n} \cdot \underline{\lg(n)})$

Space :

$$F = (n = f_1 \times f_2 \times \dots \times f_k)$$

$$\lg n = \lg f_1 + \lg f_2 + \dots + \lg f_k \quad O(\lg n)$$

$$m \Rightarrow \underline{\lg n}$$

※

$$i = \lg \sqrt{n} = \underline{\frac{1}{2} \lg n}, \text{ Space } O(\lg n)$$