size of the problem of the quantity of input in \( n \) space

- asymptotic (time, space) complexity

\[ f(n) = \Theta(g(n)) \]

- Asymptotic notation:
  - Big-Oh (asymptotic upper bound)
  - \( f(n) \leq O(g(n)) \)
  - There exists constants \( n_0 \) and \( c > 0 \) s.t. for all \( n \geq n_0 \)

- Running time is \( O(n^2) \)
- Worst case running time is \( O(n^2) \)

- \( S(n) = 2n\log n + 5n \)

  - \( g(n) = \log n \)

  - Choose \( n_0 = 2 \)
  
  \[ 2n\log n + 5n \leq 2n\log n + 5n \]

  - Choose \( c = 2 \)
  
  \[ 2n\log n + 5n \leq 2n^2 + 5n \]

  - \( 2n\log n + 5n = \Theta(n^2) \)

\( \Theta \) [asymptotic tight bound]

- \( S(n) = \Theta(g(n)) \):
  - There exist three constants \( n_0, c, C \) s.t.
    - For all \( n \geq n_0 \)
      
      \[ c_1 g(n) \leq S(n) \leq c_2 g(n) \]

- \( S(n) = 2n\log n + 5n \)

  - \( g(n) = \log n \)

  - Choose \( n_0 = 2 \)

\( \Omega \) [asymptotic lower bound]

- \( S(n) = \Omega(g(n)) \)

  - There exist constants \( n_0, c > 0 \) s.t.
    - For all \( n \geq n_0 \)
      
      \[ 0 \leq c g(n) \leq S(n) \]

\( o \) [little-o]

- \( 2n = o(n^2) \)
- \( 2n^2 = o(n^3) \)
\( (0 \leq g(n) \leq c \cdot g(n)) \quad 2n = o(n^2) \quad 2^n = o(n^2) \)

\[ 0 \leq c g(n) \leq c f(n) \]

\[ 3 \cdot 3 \cdot 3 + \Theta(n) = \Theta(n) + \Theta(n) = \Theta(n^2) \]

\[ 2n^2 + \Theta(n) = \Theta(n^2) = O(n^2) \]

\[ \Omega(n) : \text{logarithmic} \]
\[ \Theta(n) : \text{poly logarithmic} \]
\[ o(n) : \text{linear} \]
\[ \Omega(n^2) : \text{quadratic} \]
\[ \Theta(n^2) : \text{polyomial} \quad (\text{some const. k}) \]
\[ \Omega(c^k) : \text{exponential} \quad c > 1 \]
\[ \Omega(n^k) \]