Informed search algorithms

Chapter 4, Sections 1–2

Outline

◇ Best-first search

◇ A^* search

◇ Heuristics

Review: Tree search

```
function TREE-SEARCH( problem, frontier) returns a solution, or failure
    frontier ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), frontier)
    loop do
        if frontier is empty then return failure
        node ← REMOVE(frontier)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        frontier ← INSERTALL(EXPAND(node, problem), frontier)
```

A strategy is defined by picking the **order of node expansion**

Best-first search

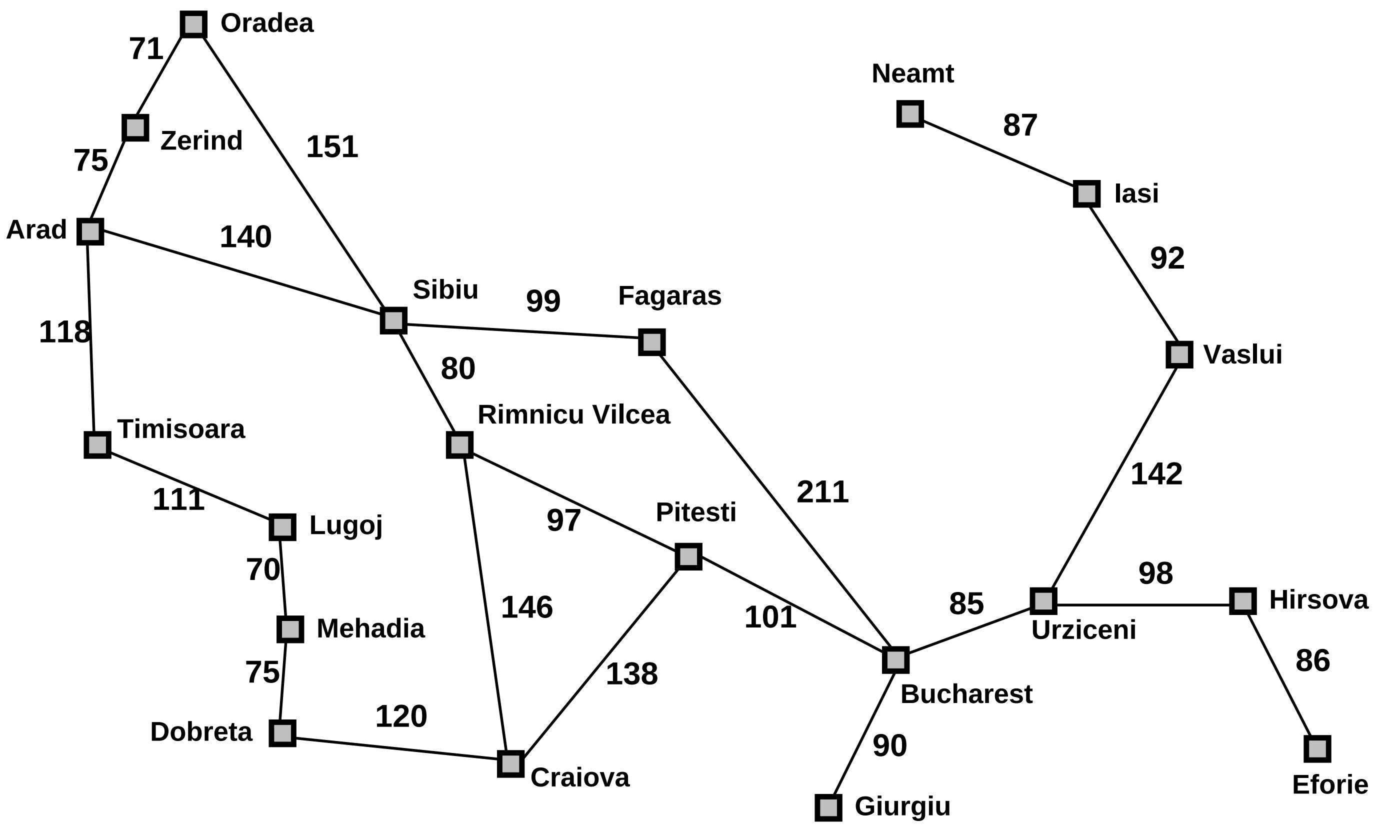
Idea: use an evaluation function for each node
– estimate of "desirability"

$\Rightarrow$ Expand most desirable unexpanded node

Implementation:
$frontier$ is a priority queue sorted in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

Evaluation function $h(n)$ (**h**euristic)

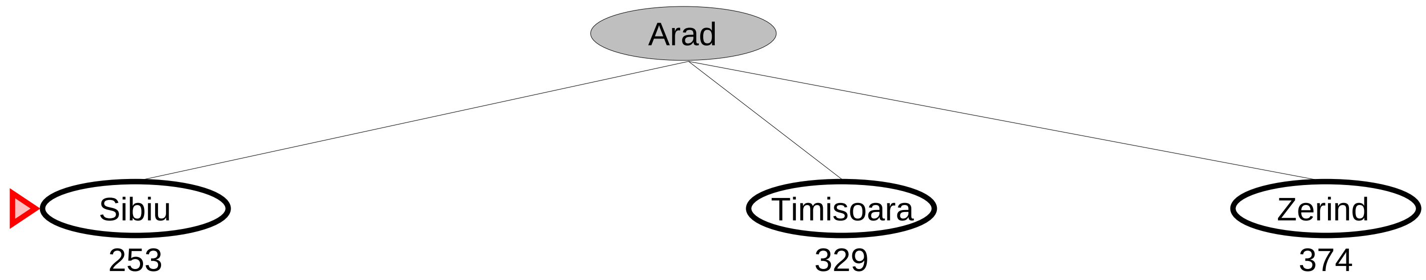
$= $ estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n) = $ straight-line distance from n to Bucharest

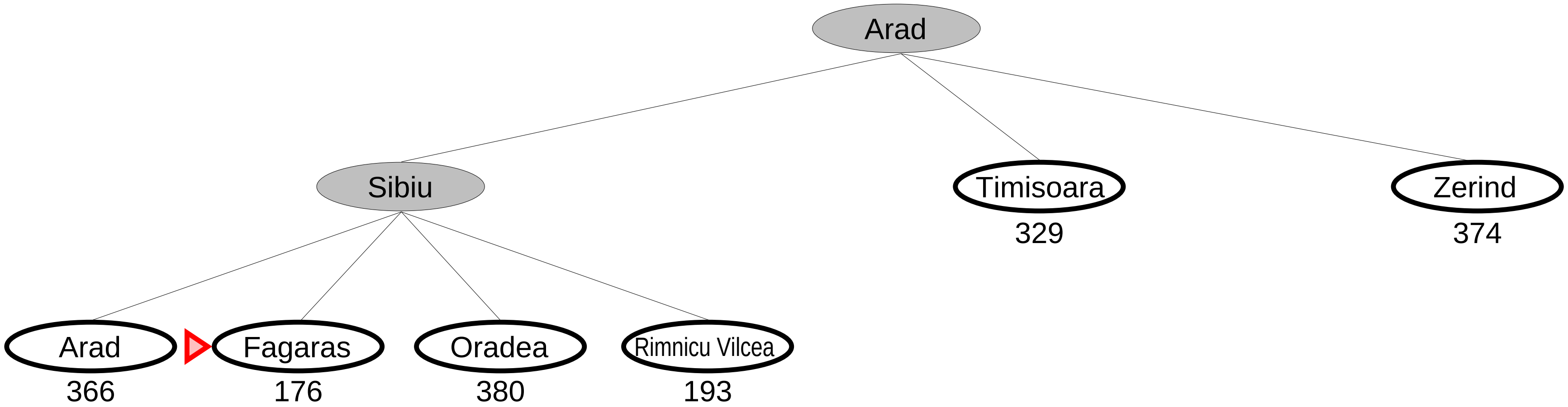
Greedy search expands the node that **appears** to be closest to goal

Greedy search example

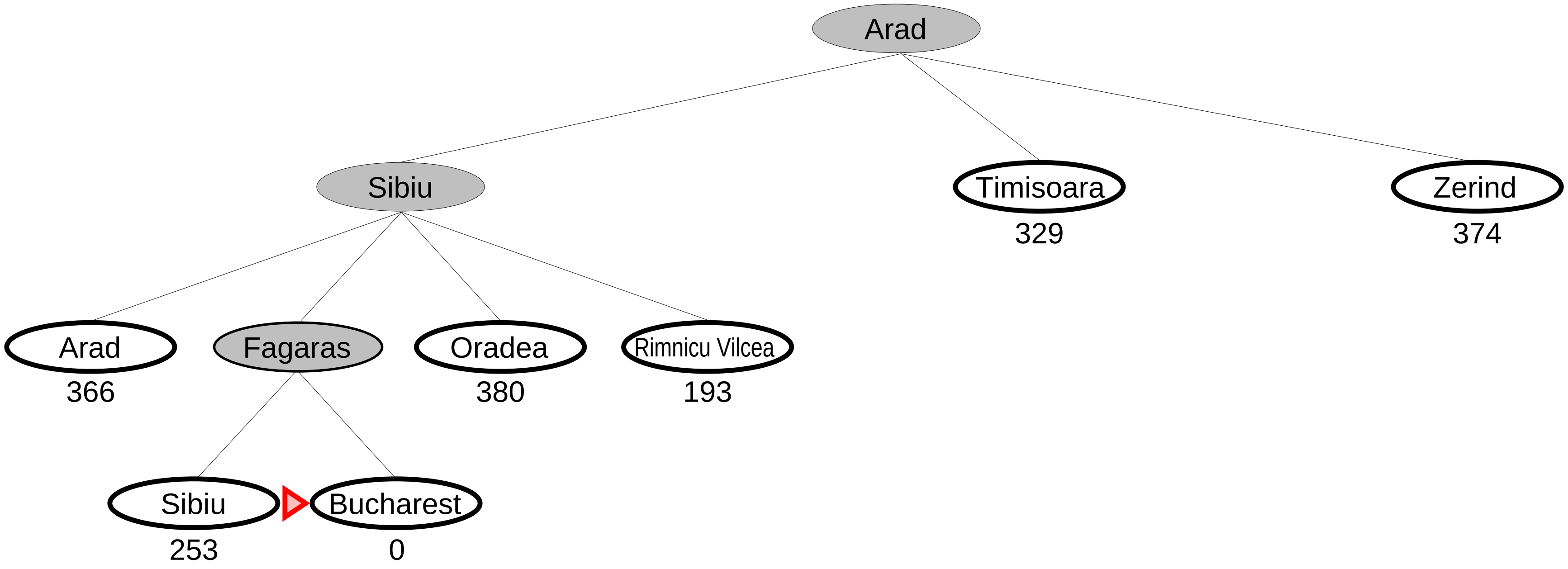
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Time??

Space??

Optimal??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

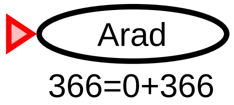
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

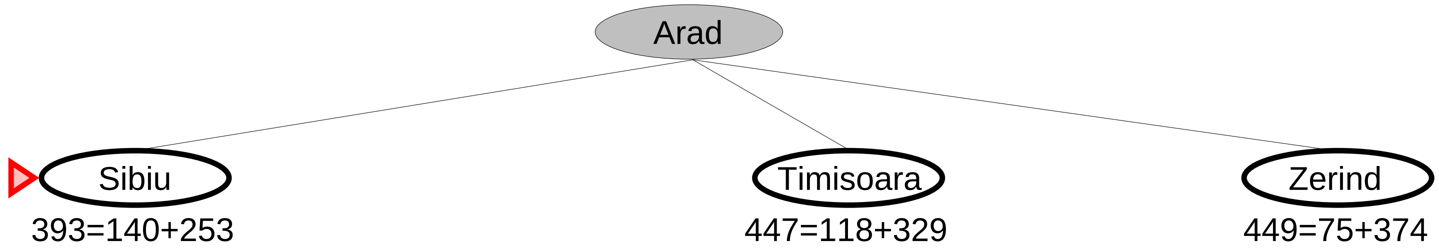
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

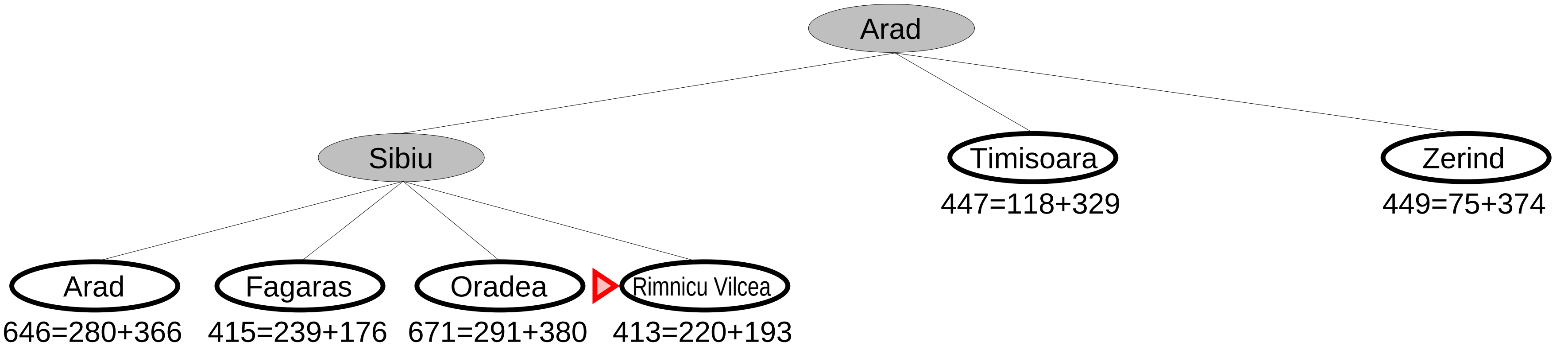
A* search example



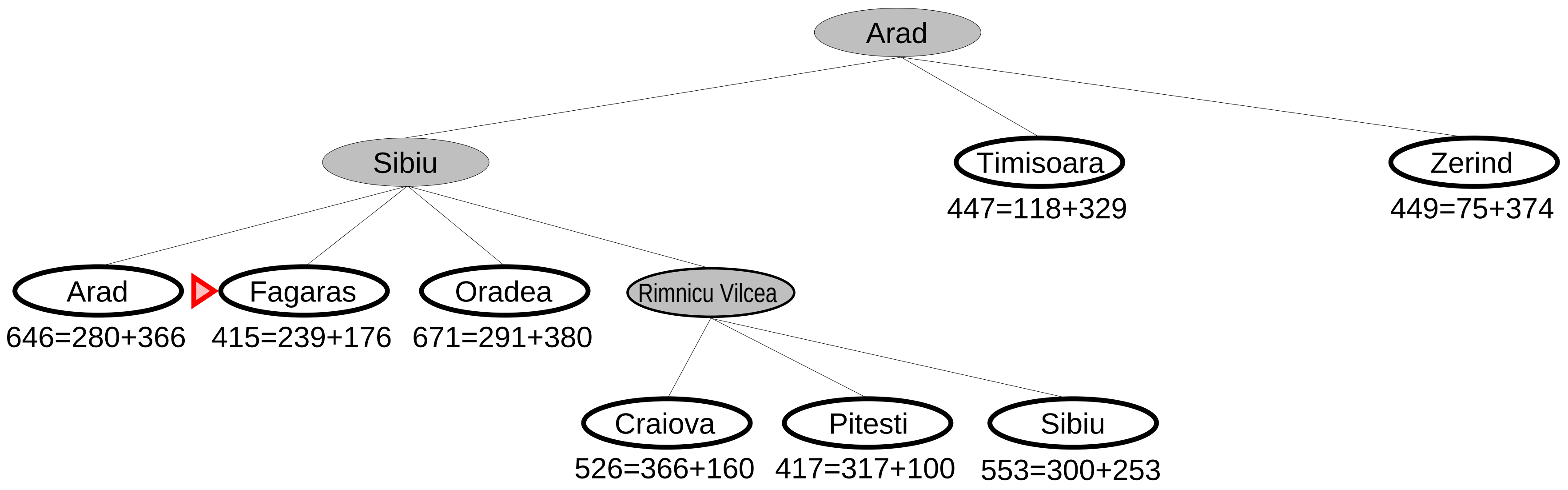
A* search example



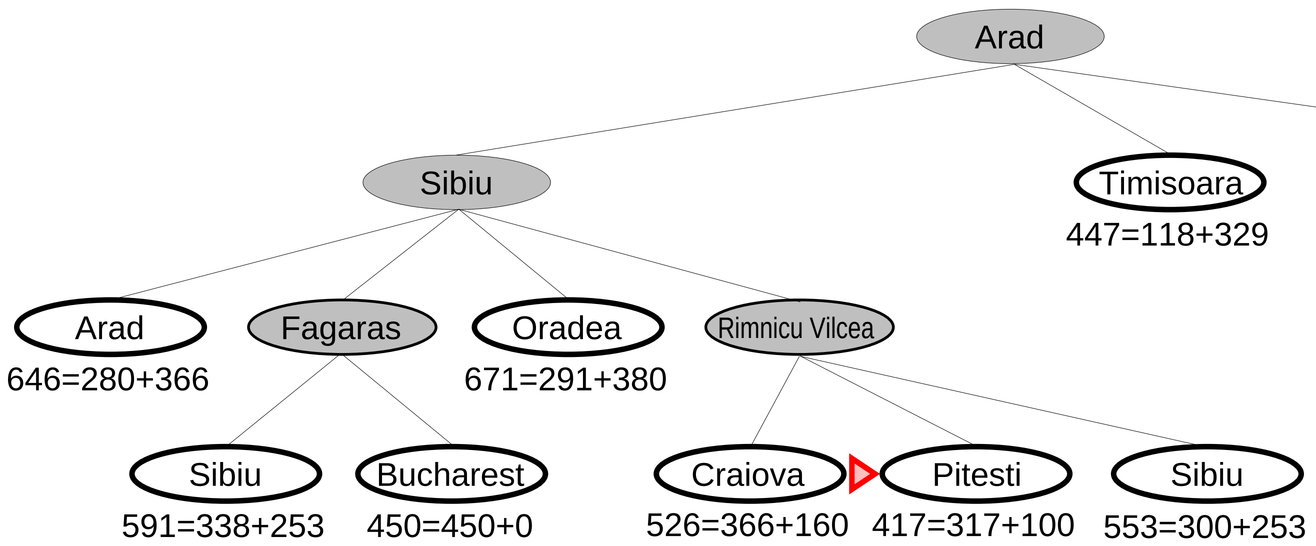
A* search example



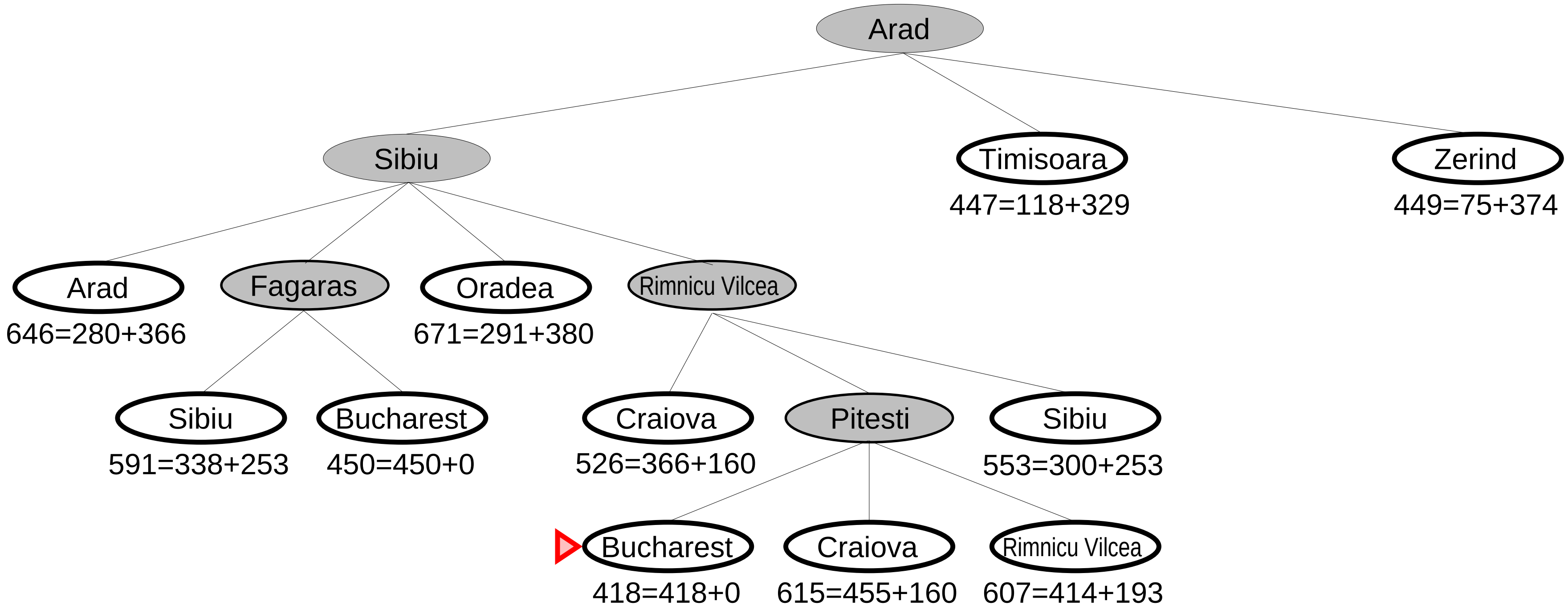
A* search example



A* search example



A* search example



Conditions for optimality of A*

admissible: $h(n)$ is never overestimates the cost to reach the goal. (optimistic) ex¿ straight-line distance.

consistent (or monotonic): (stronger condition) $h(n)$ is required for A* in graph search framework.

$$h(n) \leq c(n, a, n') + h(n')$$

A heuristic $h(n)$ is consistent if, for every node n and every successor n' fo n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of geeting to n' plus the estimated cost of reaching the goal from n'

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$\underline{h_1(S) =}$??
$\underline{h_2(S) =}$??

Admissible heuristics

E.g., for the 8-puzzle:

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(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 **dominates** h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

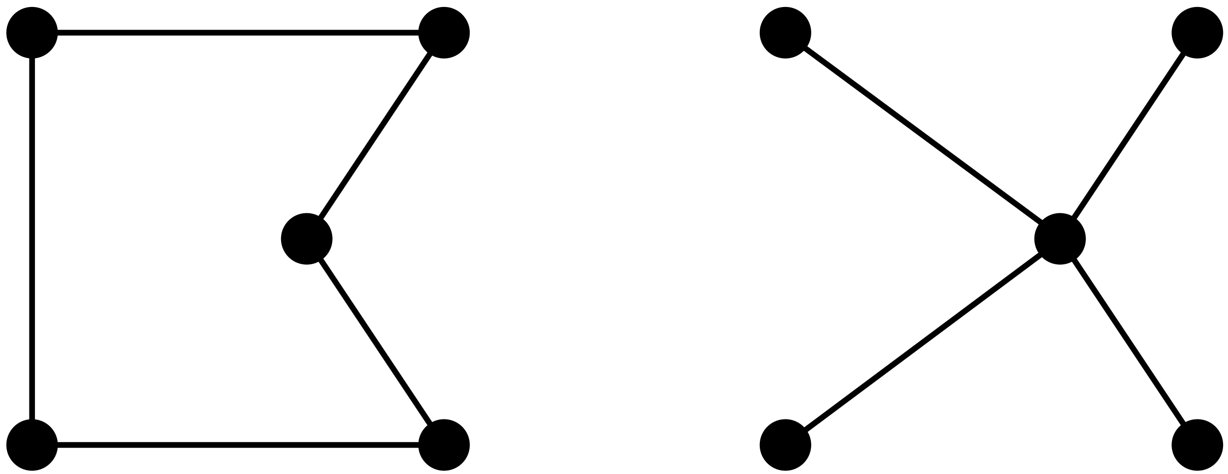
If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once

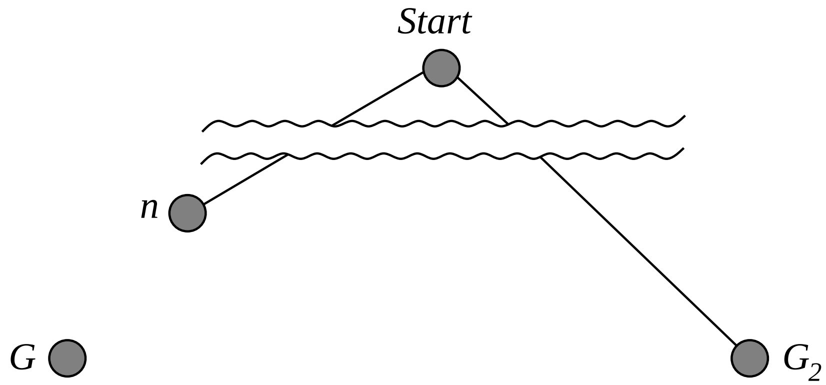


Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Optimality of A* - Tree search

The tree-search version of A is optimal if h(n) is admissible.*

Suppose some suboptimal goal G_2 has been generated and is in the queue.
Let n be an unexpanded node on a shortest path to an optimal goal G.



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

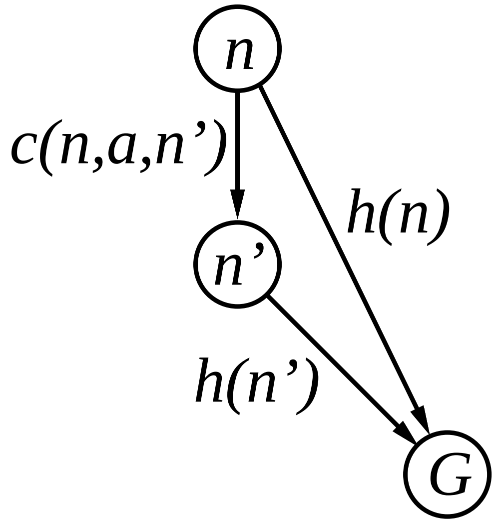
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* - graph search

The graph-search version of A* is optimal if h(n) is consistent.

Lemma: A* using $graph - search$ expands in nondecreasing order of $f(n)$.
1. A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$



If h is consistent, we have

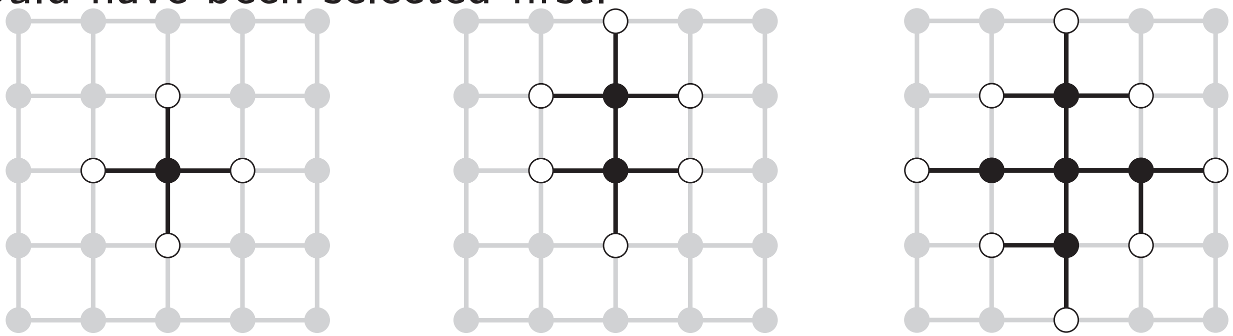
$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.

Optimality of A* - graph search cont.

2. Whenever A* selects a node n for expansion, the opimal path to that node has been found.

(Proof by contradiction) - If optimal path has not been found when A* selects a node, there would have to be another frontier node n' on the optimal path from the start node to n, by the graph separation property of graph-search; because, f is nondecreasing along any path, n' would have lower f-cost than n and would have been selected first.

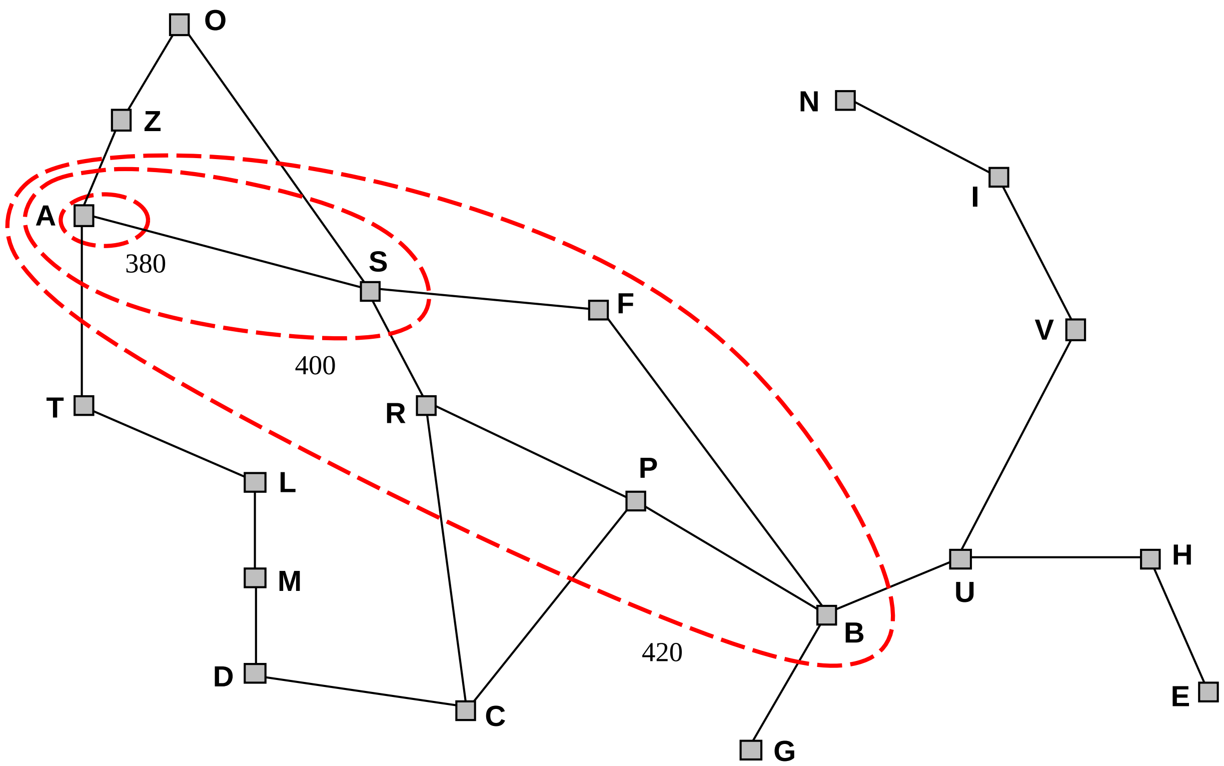


(a) (b) (c)

Optimality of A* - graph search cont.

From 1. and 2. we can see that A* gradually adds "f-contours" of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete??

Time??

Space??

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Summary

Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest h
– incomplete and not always optimal

A* search expands lowest $g + h$
– complete and optimal
– also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems