Artificial Intelligence CSE 537

Assignment 3 (Nov. 12th 2014)

Due date and time: Nov 25th 2:30pm (in class)
Submit in class (hardcopy).

- **103 points max**

(Chapters covered:)

1. (pg.506 Exercise 13.3 [4x3 =12 points])
For each of the following statements, either prove it is true or give a counter example.

a. If P(a | b, c) = P(b | a, c), then P(a | c) = P(b | c)
b. If P(a | b, c) = P(a), then P(b | c) = P(b)
c. If P(a | b) = P(a), then P(a | b, c) = P(a | c)

2. (pg 507 Exercise 13.8 [4x4 = 16 points])
* Uppercase variable name are random variables that can be assigned TRUE or FALSE: P(A) = <P(a), P(¬a)>;
* Lowercase variable names are the assignment of variable to TRUE.

Given the full joint distribution shown in Figure 13.3, calculate the following:

a. P(toothache) .
b. P(Cavity) .
c. P(Toothache | cavity) .
d. P(Cavity | toothache ∨ catch) .

3. (pg 508 Exercise 13.13 [8 points])

Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

HINT: * Computer the posterior probabilities and odds ratios: P(V |A)/P(¬V |A) and P(V |B)/P(¬V |B)

4. (pg 559 Exercise 14.4 [13 points])
Consider the Bayesian network in Figure 14.2.

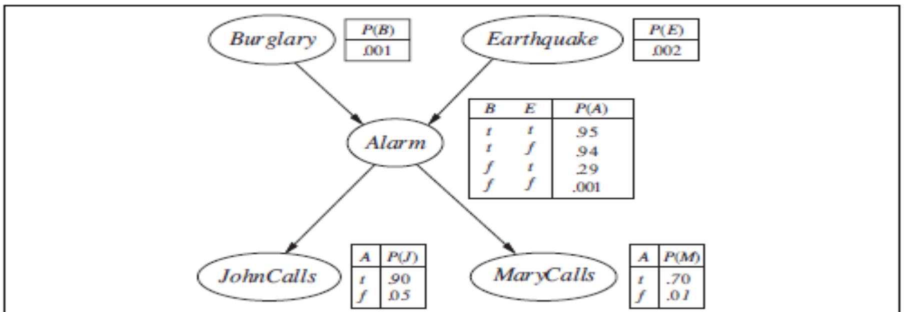

Figure 14.2 A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, and *MaryCalls*, respectively.

a. [5 points] If no evidence is observed, are Burglary and Earthquake independent? Prove this from the numerical semantics and from the topological semantics.
b. [8 points] If we observe Alarm =true, are Burglary and Earthquake independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.

HINTS: * Check numerically whether P(B,E|a) = P(B|a)P(E|a) by computing value for P(B,E|a) and value for P(B|a)P(E|a).
* Just showing example for P(b, e|a) = P(b|a)P(e|a) is enough.

5. (pg 559 Exercise 14.5 a,b,c,d [3x4 =12 points])
Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

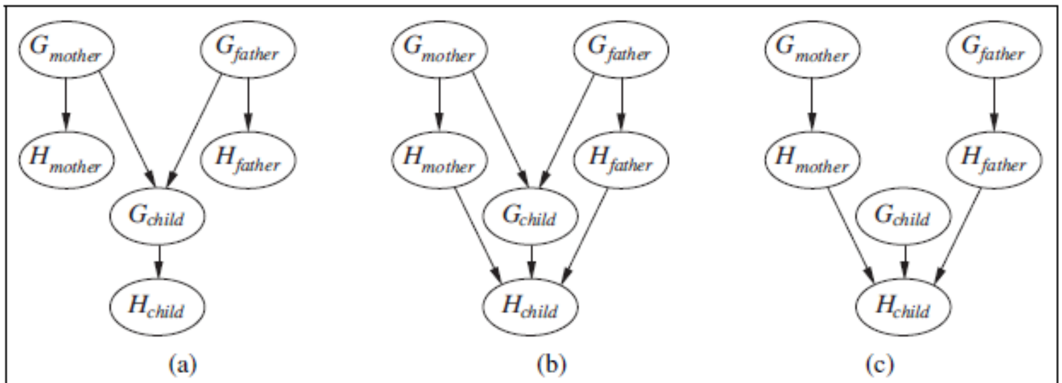

Figure 14.20 Three possible structures for a Bayesian network describing genetic inheritance of handedness.

a. Which of the three networks in Figure 14.20 claim that $P(G_{father},G_{mother},G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?

b. Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

c. Which of the three networks is the best description of the hypothesis?

d. Write down the CPT for the G_{child} node in network (a), in terms of s and m.

6. (pg 561 Exercise 14.11 a,c,d [3x4 = 12 points])
In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), F_A (alarm is faulty), and F_G (gauge is faulty) and the multivalued nodes G (gauge reading) and T (actual core temperature).

a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

c. Supposetherearejust twopossible actual andmeasuredtemperatures, normal andhigh; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.

d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.

7. (pg 563 Exercise 14.14 a [8 points])

Consider the variable elimination algorithm in Figure 14.11 (page 528).

a. Section 14.4 applies variable elimination to the query

P(Burglary | JohnCalls = true, MaryCalls = true) .

Perform the calculations indicated and check that the answer is correct.

8.(pg. 609 Excersize15.13 [12 points])

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

9. (pg 609 Excersize 15.14 [6x3 = 18 points])

For the DBN specified in Exercise 15.13 and for the evidence values

e1 = not red eyes, not sleeping in class

e2 = red eyes, not sleeping in class

e3 = red eyes, sleeping in class

perform the following computations:

a. State estimation: Compute P(EnoughSleept|e1:t) for each of t = 1, 2, 3.

b. Smoothing: Compute P(EnoughSleept|e1:3) for each of t = 1, 2, 3.

c. Compare the filtered and smoothed probabilities for t = 1 and t = 2.