CSE 373 Analysis of Algorithms
Fall 2016
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LEC21: REVIEW OF ANALYSIS OF ALGORITHMS

Lecture slide courtesy of Prof. Steven Skiena

COURSE OUTCOME:

The ABET objectives for the course:

1. Ability to perform **worst-case asymptotic algorithm analysis**
2. Ability to **define and use classical combinatorial algorithms** for problems such as sorting, shortest paths and minimum spanning trees
3. Knowledge of **computational intractability** and **NP completeness**

The program objective for the course:

- (S6) have a solid understanding of computational theory and foundational mathematics.

- **Objectives**
- Read about the CS accreditation ABET program. The A BET objectives for the course are
- Provide a rigorous introduction to worst-case asymptot ic algorithm analysis.
- Develop classical graph and combinatorial algorithms for such problems as sorting, shortest paths and mini mum spanning trees.
- Introduce the concept of computational intractability a nd NP completeness.

WHAT IS AN ALGORITHM?

- An algorithmic problem is specified by describing the
  - **set of instances it must work** on and
  - what **desired properties the output must have.**
- Properties of Algorithms
  - Correctness: For any algorithm, we must prove that it ***always*** returns the desired output for **all** legal instances of the problem.
  - Efficient

PROVING CORRECTNESS: INDUCTION AND RECURSION

- Failure to find a counterexample to a given algorithm does not mean *"it is obvious"* that the algorithm is correct.
- Mathematical **induction** is a very useful method for proving the correctness of **recursive algorithms.**
- Recursion and induction are the same basic idea:
  - (1) basis case,
  - (2) general assumption,
  - (3) general case.

  Ex> proving

$$\sum_{i=1}^{n} i = n(n+1)/2$$

THE RAM MODEL OF COMPUTATION

Algorithms are an important and durable part of computer science because they can be studied in a machine/language independent way.

This is because we use the **RAM model of computation** for all our analysis.

- Each "*simple*" operation (+,*, -, =, if, call) takes 1 step.
- Loops and subroutines are *not* simple operations. They depend upon the size of the data and the contents of a subroutine.
    - ex> "Sort" is not a single step operation.

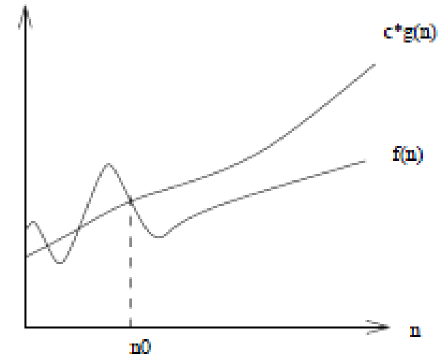
ASYMPTOTIC NOTATIONS: NAMES OF BOUNDING FUNCTIONS

- Big-Oh: $g(n) = O(f(n))$ means $C*f(n)$ is an *upper bound* on $g(n)$.
- Big-Omega: $g(n) = \Omega(f(n))$ means $C*f(n)$ is a *lower bound* on $g(n)$.
- Big-Theta: $g(n) = \Theta(f(n))$ means $C_1*f(n)$ is an upper bound on $g(n)$ and $C_2*f(n)$ is a lower bound on $g(n)$. (a.k.a. *tight bound*)

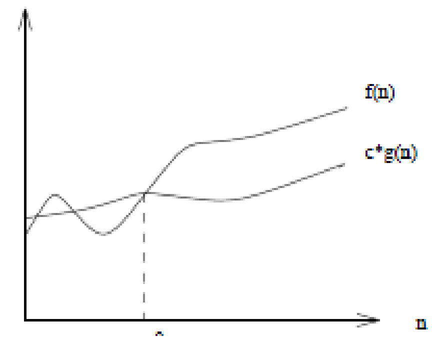
C, C_1, and C_2 are all constants independent of n.

ASYMPTOTIC NOTATIONS

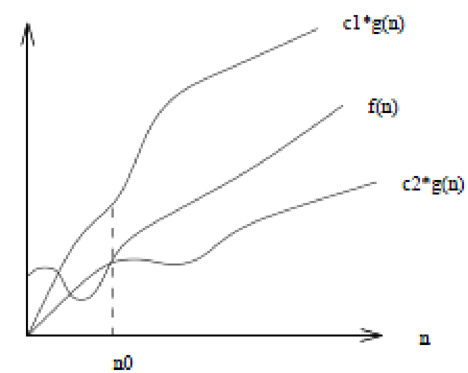
- Big-Oh: $f(n) = O(g(n))$ if there are positive constants n_0 and c such that to the right of n_0, the value of $f(n)$ always lies on or below $c*g(n)$.



- Big-Omega: $f(n) = \Omega(g(n))$ if there are positive constants n_0 and c such that to the right of n_0, the value of $f(n)$ always lies on or above $c*g(n)$.



- Big-Theta: $f(n) = \Theta(g(n))$ if there exist positive constants n_0, c_1, and c_2 such that to the right of n_0, the value of $f(n)$ always lies between $c_1*g(n)$ and $c_2*g(n)$ inclusive.



The definitions imply a constant n_0 *beyond which* they are satisfied. We do not care about small values of n.

DOMINANCE RELATIONS

- Faster-growing function *dominates* a slower-growing one
- Common functions that appear in algorithms analysis order of increasing dominance:

$$n! \gg 2^n \gg n^3 \gg n^2 \gg n \log n \gg n \gg \log n \gg 1$$

  - *Constant functions, $f(n) = 1$*
  - *Logarithmic functions, $f(n) = \log n$*
  - *Linear functions, $f(n) = n$*
  - *Superlinear functions, $f(n) = n \lg n$*
  - *Quadratic functions, $f(n) = n^2$*
  - *Cubic functions, $f(n) = n^3$*
  - *Exponential functions, $f(n) = c^n$ for a given constant $c > 1$*
  - *Factorial functions, $f(n) = n!$*

PROGRAM COMPLEXITY ANALYSIS

- Determining time complexity analysis given a code.
  - EX> Selection Sort Worst Case Analysis
  - EX> Insertion Sort Worst Case Analysis
  - EX> String Pattern Matching Worst Case Analysis
- Properties of Logarithms
  - In relation to Trees - ex> Binary Search
  - Logarithms and Multiplication
  - The Base is not Asymptotically Important
  - Logarithms and Bits

DATA STRUCTURE

- Complexity of an algorithms may differ when using different data structures.
- **Types of DS:**
  - Contiguous vs. Linked Data Structures
  - Containers: Stacks and Queues
- **Dictionary / Dynamic Set Operations & time analysis**
  - *Basic Operations: Search(S,k) Insert(S,x) Delete(S,x)*
  - Binary Search Trees: operations.
  - Balanced Search Trees
  - Hash Tables: Collisions, hash functions, Performance on Set Operations
  - Analysis of Substring Pattern Matching using different dictionary data structures.

SORTING

- **Applications of sorting**
- **Pragmatics of Sorting**
- **Selection Sort:**
  - Data Structure Matters: Heapsort
- **Priority Queue:**
  - operations;
  - implementations;
  - time analysis o f operations based of data structure used
  - Applications
- *Binary Heap:*
  - Constructing Heaps
  - Heap operations and time analysis: Bubble up & Bubble down

SORTING

- MergeSort & Divide-and-conquer
- Analysis of Algorithms that use Divide-and-conquer
  - EX> matrix multiplication
  - Divide-and-Conquer Recurrences
  - Application of Master Theorem
- **Quicksort & Partitioning**
  - Analysis – Best case, worst case, average case analysis
  - Randomized analysis
- **Lower Bound Analysis on Sorting – comparison based**
- **Non-Comparison-Based Sorting**
  - Bucketsort – time complexity

GRAPH DATA STRUCTURES

- Graph data structure – characteristics & operations
  - Adjacency Matrix
  - Adjacency list
- Graph terminology:
  - Degree
  - *Connected & **strongly connected***
- Types of graphs:
  - Directed vs. Undirected Graphs
  - Weighted vs. Unweighted Graphs
  - Simple vs. Non-simple Graphs
  - Sparse vs. Dense Graphs
  - Cyclic vs. Acyclic Graphs

BREADTH-FIRST SEARCH

- Graph traversal: We want to visit every vertex and every edge exactly **once** in some well-defined order.
- **Breadth-first search** is appropriate if we are interested in shortest paths on unweighted graphs.
- How BFS works on graphs.
- Data Structure for BFS – using queue
- Applications of BFS
  - Shortest Paths
  - Connected Components
  - Two-Coloring Graphs – Bipartite

DEPTH-FIRST SEARCH

- BFS v.s. DFS
- How DFS works on graphs.
- Characteristics of DFS algo
  - **Edge Classification for DFS:** *tree edges, back edges, …*
  - Finding ancestor and descendants by time intervals & appli cations
- Data structure for DFS: stack (recursion)
- Applications of DFS
  - Finding Cycles
  - Articulation Vertices
  - Topological Sorting
  - Strongly Connected Components

MINIMUM SPANNING TREES & GREEDY ALGORITHMS

- Input: Edge-weighted graphs
- Characteristic of MSP
- **Applications**
  - **Net Partitioning**
  - provides a good heuristic **for traveling salesman problems**
- **Prim's Algorithm :**
  - **how it works, characteristics, & time analysis**
- **Kruskal's Algorithm :**
  - **how it works, characteristics, and time analysis**

SHORTEST PATH

- Characteristic of shortest path problem
- Dijkstra's Algorithm (single source shortest path)
  - how it works (Dynamic Programing), characteristics, & time analysis
  - Difference between Prim's/Dijkstra's
- The Floyd-Warshall Algorithm (all-pairs shortest path)
  - how it works (Dynamic Programing), characteristics, & time analysis
- Applications:

BACKTRACKING

- What is Backtracking used for?
- How to apply Backtracking
  - Modeling the solution vector
  - Recursive structure – similar to DFS
  - How to model: is_a_solution(a,k,input); process_solution(a,k,input); construct_candidates(a,k,input,c,&ncandidates); make_move(a,k,input);
- Applications:
  - Sudoku
  - Constructing all Subsets
  - Constructing all Permutations
  - The Eight-Queens Problem
  - Can Eight Pieces Cover a Chess Board?

HEURISTIC SEARCH

- Backtracking searches all configurations to find the best of all possible solutions.
- Heuristic methods provide an alternate way to approach difficult combinatorial optimization problems.
  - *Solution space representation*
  - *Cost function*
- Heuristic search methods:
  - Random sampling,
  - local search strategy
    - Gradient-descent search
    - Simulated annealing

DYNAMIC PROGRAMMING

- When DP is appropriate.
- Characteristics & Benefitsof DP
  - <u>systematically search all possibilities</u> (thus guaranteeing cor rectness) <u>while storing results to avoid recomputing</u>
- **Three Steps to Dynamic Programming**
  1. Formulate the answer as a recurrence relation
  2. Show that the number of different instances of your recurrence is bounded by a polynomial.
  3. Specify an order of evaluation for the recurrence so you always have what you need.

DP CONT.

- Examples:
  - Fibonacci Numbers
  - Binomial Coefficients - Pascal's Triangle
- Edit Distance & applications
  - How Edit Distance works & analysis
  - Substring Matching
  - Longest Common Subsequence
  - Maximum Monotone Subsequence (Longest Increasing Sequence)
  - The Partition Problem
  - Minimum Weight Triangulation
- Comparing DP with Recurrence
- Limitations of Dynamic Programming: TSP
  - *Principle of optimality*

NP-COMPLETENESS

- Bandersnatch(G)
  - Convert G to an instance of the Bo-billy problem Y .
  - Call the subroutine Bo-billy on Y to solve this instance.
  - Return the answer of Bo-billy(Y) as the answer to G.
- Now suppose my **reduction** translates G to Y in $O(P(n))$:
  - 1. If my Bo-billy subroutine ran in $O(P'(n))$ I can solve the Bandersnatch problem in $O(P(n) + P'(n'))$
  - 2. If I know that $\Omega(P'(n))$ is a lower-bound to compute Bandersnatch, then $\Omega(P'(n) - P(n'))$ must be a lowerbound to compute Bo-billy.

Concepts: *problem , instance, decision problem*