CSE 373 Analysis of Algorithms
Fall 2016
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LEC15: HEURISTIC SEARCH (247-)

Lecture slide courtesy of Prof. Steven Skiena

- Backtracking gave us a method to find the best of all possible solutions, as scored by a given objective function. However, any algorithm searching all configurations is doomed to be impossible on large instances.
- Heuristic methods provide an alternate way to approach difficult combinatorial optimization problems.
- We look at three different heuristic search methods:
  - Random sampling,
  - gradient-descent search, and
  - simulated annealing.

TWO COMMON COMPONENTS

- *Solution space representation:* complete yet concise description of the set of possible solutions for the problem.
  - *TSP:* solution space consists of $(n-1)!$ elements - all possible circular permutations of the vertices, represented using an array S of $n - 1$ vertices.

- *Cost function: cost* or *evaluation* function to access the quality of each element of the solution space.
  - TSP: given candidate solution *S*, *cost function* is just the sum of the costs involved.

RANDOM SAMPLING AKA *MONTE CARLO METHOD*

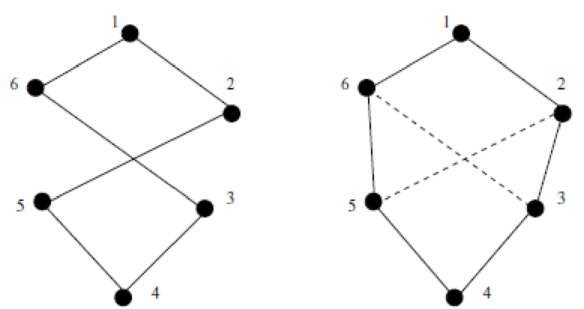
- Repeatedly construct random solutions and evaluate them, stopping as soon as we get a good enough solution, or (more likely) when we are tired of waiting
- Report the best solution found over the course of our sampling.

WHEN MIGHT RANDOM SAMPLING DO WELL?

- *When there are a high proportion of acceptable solutions*
  - *Ex*> Finding prime numbers
- *When there is no coherence in the solution space* - there is no sense of when we are getting *closer* to a solution
  - Ex>Finding who of your friends has a social security number that ends in 00.

LOCAL SEARCH

- A local search heuristic starts from an arbitrary element of the solution space, and then scans the neighborhood looking for a favorable transition to take.
  - EX> TSP, this would be *transition*, which lowers the cost of the tour.
- We want a general transition mechanism that takes us to the next solution by slightly modifying the current one.
  - Ex> TSP: swap the current tour positions of a random pair of vertices *Si* and *Sj* ,



: Improving a TSP tour by swapping vertices 2 and 6

WHEN DOES LOCAL SEARCH DO WELL?

- *When there is great coherence in the solution space*
  - Cost function is either convex or concave
- *Whenever the cost of incremental evaluation is much cheaper than global evaluation*
  - *EX> TSP:* It costs $\Theta(n)$ to evaluate the cost of an arbitrary n-vertex candidate TSP solution. Once that is found, however, the cost of the tour after swapping a given pair of vertices can be determined in constant time.

SIMULATED ANNEALING

- Simulated annealing is a heuristic search procedure that allows occasional transitions leading to more expensive (and hence inferior) solutions – which helps keep our search from getting stuck in **local optima**
- Simulated annealing comes from the physical process of cooling materials down to the solid state.
- A particle's energy state jumps about randomly, with such transitions governed by the temperature of the system where Transition probability *P*(*ei*, *ej*, *T*) from energy *ei* to *ej* at temperature *T:*

$$P(e_i, e_j, T) = e^{\frac{e_i - e_j}{k_b T}}$$

```
Simulated-Annealing()
    Create initial solution S
    Initialize temperature t
    repeat
        for i = 1 to iteration-length do
            Generate a random transition from S to S_i
            If (C(S) ≥ C(S_i)) then S = S_i
            else if (e^((C(S)−C(S_i))/(k·t)) > random[0, 1)) then S = S_i
        Reduce temperature t
    until (no change in C(S))
    Return S
```