CSE 373 Analysis of Algorithms
Fall 2016
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LEC03
ELEMENTARY DATA STRUCTURE (65-77)

Lecture slide courtesy of Prof. Steven Skiena

PROBLEM OF THE DAY

True or False?

1. $2n^2 + 1 = O(n^2)$
2. $\sqrt{n} = O(\log n)$
3. $\log n = O(\sqrt{n})$
4. $n^2(1 + \sqrt{n}) = O(n^2 \log n)$
5. $3n^2 + \sqrt{n} = O(n^2)$
6. $\sqrt{n} \log n = O(n)$
7. $\log n = O(n^{-1/2})$

ELEMENTARY DATA STRUCTURES

- Elementary data structures such as stacks, queues, lists, and heaps are the "off-the-shelf" components we build our algorithm from.
- Changing the data structure does not change the correctness of the program. However, it can change the total performance time.
- There are two aspects to any data structure:
  - The abstract operations which it supports.
  - The implementation of these operations.

DATA ABSTRACTION

- That we can describe the behavior of our data structures in terms of abstract operations is why we can use them without thinking.
- That there are different implementations of the same abstract operations enables us to optimize performance.
- containers, dictionaries, and priority queues

CONTIGUOUS VS. LINKED DATA STRUCTURES

- Data structures can be neatly classified as either *contiguous* or *linked* depending upon whether they are based on arrays or pointers:
  - Contiguously-allocated structures are composed of single slabs of memory
    - Ex> arrays, matrices, heaps, and hash tables.
  - Linked data structures are composed of multiple distinct chunks of memory bound together by *pointers*
    - Ex> lists, trees, and graph adjacency lists.

ARRAYS

- An array is a structure of fixed-size data records such that each element can be efficiently located by its *index* or (equivalently) address.
- Advantages of contiguously-allocated arrays include:
  - Constant-time access given the index.
  - Arrays consist purely of data, so no space is wasted with links or other formatting information.
  - Physical continuity (memory locality) between successive data accesses helps exploit the high-speed cache memory on modern computer architectures.

DYNAMIC ARRAYS

- Unfortunately we cannot adjust the size of simple arrays in the middle of a program's execution.
- Compensating by allocating extremely large arrays can waste a lot of space.
- With *dynamic arrays* we start with an array of size 1, and double its size from *m* to *2m* each time we run out of space.
- How many times will we double for n elements?
  - Only ceil($\log_2$ n).

HOW MUCH TOTAL WORK?

- The apparent waste in this procedure involves the recopying of the old contents on each expansion.
- If half the elements move once, a quarter of the elements twice, and so on, the total number of movements *M* is given by:

$$M = \sum_{i=1}^{lg n} i * \frac{n}{2^i} = n \sum_{i=1}^{lg n} \frac{i}{2^i} \leq n \sum_{i=1}^{\infty} \frac{i}{2^i} = 2n$$

- Thus each of the *n* elements move an average of only twice, and the total work of managing the dynamic array is the same *O(n)* as a simple array.

$$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

POINTERS AND LINKED STRUCTURES

- Pointers represent the address of a location in memory.
- A cell-phone number can be thought of as a pointer to its owner as they move about the planet.
- In C, *p denotes the item pointed to by p, and &x denotes the address (i.e. pointer) of a particular variable x.
- A special NULL pointer value is used to denote structure-terminating or unassigned pointers.

LINKED LIST STRUCTURES

```
typedef struct list {
        item_type item;
        struct list *next;
} list;
```

1. one or more data fields

2. a pointer field to at least one other node

3. pointer to the head of the the structure

SEARCHING A LIST

- Searching in a linked list can be done iteratively or recursively.

Recursive implementation: .

```
list *search_list(list *l, item_type x)
{
        if (l == NULL) return(NULL);

        if (l->item == x)
                return(l);
        else
                return( search_list(l->next, x) );
}
```

INSERTION INTO A LIST

- Since we have no need to maintain the list in any particular order, we might as well insert each new item at the head.

```
void insert_list(list **l, item_type x)
{
        list *p;

        p = malloc( sizeof(list) );
        p->item = x;
        p->next = *l;
        *l = p;
}
```

- Note the **l, since the head element of the list changes.

DELETING FROM A LIST - RECURSIVE

```
list *predecessor_list(list *l, item_type x)
{
        if ((l == NULL) || (l->next == NULL)) {
                printf("Error: predecessor sought on null list.\n");
                return(NULL);
        }

        if ((l->next)->item == x)
                return(l);
        else
                return( predecessor_list(l->next, x) );
}
```

```
delete_list(list **l, item_type x)
{
        list *p;                        /* item pointer */
        list *pred;                     /* predecessor pointer */
        list *search_list(), *predecessor_list();

        p = search_list(*l,x);
        if (p != NULL) {
                pred = predecessor_list(*l,x);
                if (pred == NULL)       /* splice out out list */
                        *l = p->next;
                else
                        pred->next = p->next;

                free(p);                /* free memory used by node */
        }
}
```

1. find a pointer to the *predecessor* of the item to be deleted

2. Reset the pointer to the head of the list (l) when the first element is deleted:

ADVANTAGES OF LINKED LISTS

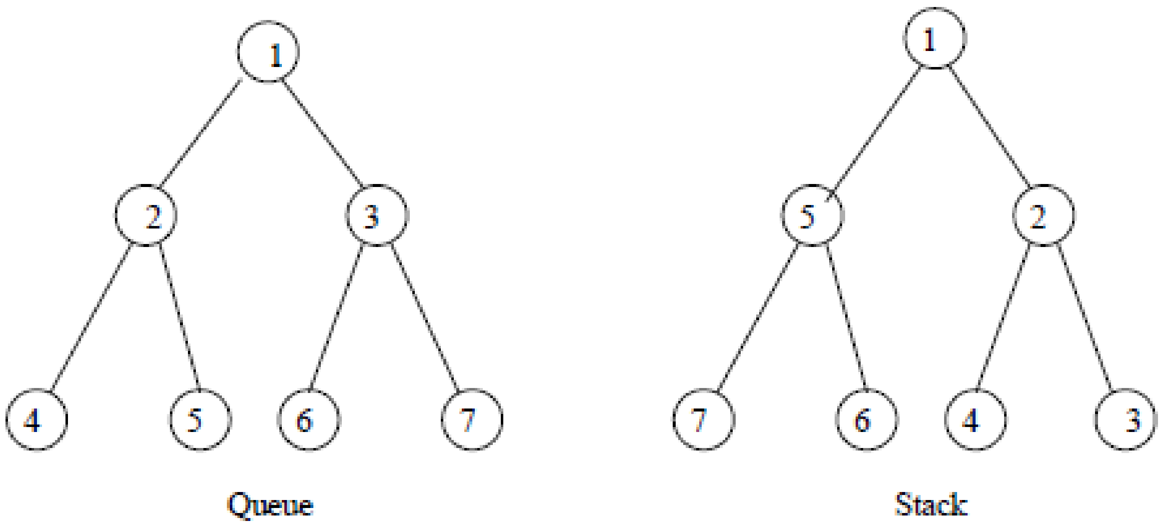
- The relative advantages of linked lists over static arrays include:
  1. Overflow on linked structures can never occur unless the memory is actually full.
  2. Insertions and deletions are *simpler* than for contiguous (array) lists.
  3. With large records, moving pointers is easier and faster than moving the items themselves.
- Dynamic memory allocation provides us with flexibility on how and where we use our limited storage resources.

CONTAINERS: STACKS AND QUEUES

- Sometimes, the order in which we retrieve data is *independent of its content*, being only a function of when it arrived. (DS called *Containers*)
- A *stack* supports last-in, first-out (LIFO) operations:
  - – Push(x,s): Insert item x at the top of stack s.
  - – Pop(s): Return (and remove) the top item of stack s.
- A *queue* supports first-in, first-out (FIFO) operations:
  - – Enqueue(x,q): Insert item x at the back of queue q.
  - – Dequeue(q): Return (and remove) the front item from queue q.
- Lines in banks are based on queues, while food in my refrigerator is treated as a stack.
- Stacks and queues can be effectively implemented using either arrays or linked lists.

IMPACT ON TREE TRAVERSAL

- Both can be used to store nodes to visit in a tree, but the order of traversal is completely different.



- Which order is friendlier for WWW crawler robots?

DICTIONARY / DYNAMIC SET OPERATIONS

The *dictionary* data type permits access to data items by content.

Perhaps the most important class of data structures maintain a set of items, indexed by keys.

- *Search(S,k)* – A query that, given a set S and a key value k, returns a pointer x to an element in S such that key[x] = k, or nil if no such element belongs to S.
- *Insert(S,x)* – A modifying operation that augments the set S with the element x.
- *Delete(S,*x)* – Given a pointer x to an element in the set S, remove *x* from *S*. Observe we are given a pointer to an element *x*, not a key value.

May also have following functions:

- *Min(S), Max(S)* – Returns the element of the totally ordered set S which has the smallest (largest) key.
- *Next(S,k), Previous(S,)* – Retrieve the item from *D* whose key is immediately before (or after) *k* in sorted order.
- These enable us to iterate through the elements of the data structure. There are a variety of implementations of these *dictionary* operations, each of which yield different time bounds for various operations.

ARRAY BASED SETS: UNSORTED ARRAYS

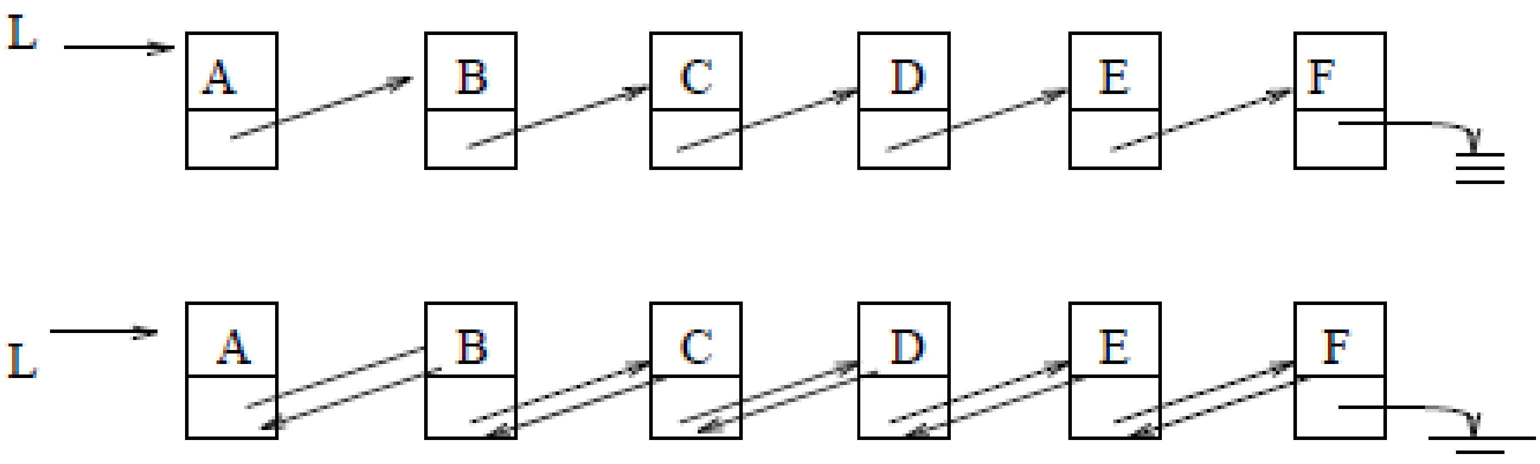
- Search(S,k) - sequential search, O(n)
- Insert(S,x) - place in first empty spot, O(1)
- Delete(S,*x) - copy nth item to the xth spot, O(1)
- Min(S), Max(S) - sequential search, O(n)
- Successor(S,k), Predecessor(S,k) - sequential search, O(n)

ARRAY BASED SETS: SORTED ARRAYS

- Search(S,k) - binary search, O(lg n)
- Insert(S,x) - search, then move to make space, O(n)
- Delete(S,*x) - move to fill up the hole, O(n)
- Min(S), Max(S) - first or last element, O(1)
- Successor(S,k), Predecessor(S,k) - Add or subtract 1 from pointer, O(1)

POINTER BASED IMPLEMENTATION

We can maintain a dictionary in either a singly or doubly linked list.



DOUBLY LINKED LISTS

- We gain extra flexibility on predecessor queries at a cost of doubling the number of pointers by using doubly-linked lists.
- Since the extra big-Oh costs of doubly-linked lists is zero, we will usually assume they are, although it might not be necessary.
- Singly linked to doubly-linked list is as a Conga line is to a Can-Can line.

STOP AND THINK: COMPARING DICTIONARY IMPLEMENTATIONS

Problem: What is the asymptotic worst-case running times for each of the seven fundamental dictionary operations when the data structure is implemented as

- A singly-linked unsorted list.
- A doubly-linked unsorted list.
- A singly-linked sorted list.
- A doubly-linked sorted list.

Dictionary operation	Singly unsorted	Double unsorted	Singly sorted	Doubly sorted
Search(L, k)	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Insert(L, x)	$O(1)$	$O(1)$	$O(n)$	$O(n)$
Delete(L, x)	$O(n)^*$	$O(1)$	$O(n)^*$	$O(1)$
Successor(L, x)	$O(n)$	$O(n)$	$O(1)$	$O(1)$
Predecessor(L, x)	$O(n)$	$O(n)$	$O(n)^*$	$O(1)$
Minimum(L)	$O(n)$	$O(n)$	$O(1)$	$O(1)$
Maximum(L)	$O(n)$	$O(n)$	$O(1)^*$	$O(1)$