## Assignment 3 (Oct. 19 ${ }^{\text {th }}$ 2016)

Due date and time: Oct $31^{\text {st }} 17: 00 \mathrm{p} . \mathrm{m}$.
Submit in class (handwritten hardcopy).
You can work in pairs. If so, turn in one copy and write both participants.

1. For the following graph

(a) Report the order of the vertices encountered on a breadth-first search starting from vertex A. Break all ties by picking the vertices in alphabetical order (i.e., A before Z).
(b) Report the order of the vertices encountered on a depth-first search starting from vertex A. Break all ties by picking the vertices in alphabetical order (i.e., A before Z).
2. Do a topological sort of the following graph G :

3. In breadth-first and depth-first search, an undiscovered node is marked discovered when it is first encountered, and marked processed when it has been completely searched. At any given moment, several nodes might be simultaneously in the discovered state.
(a) Describe a graph on $n$ vertices and a particular starting vertex v such that $\Theta(n)$ nodes are simultaneously in the discovered state during a breadth-first search starting from v .
(b) Describe a graph on $n$ vertices and a particular starting vertex v such that $\Theta(n)$ nodes are simultaneously in the discovered state during a depth-first search starting from v .
4. Suppose an arithmetic expression is given as a tree. Each leaf is an integer and each internal node is one of the standard arithmetical operations $(+,-, *, /)$. For example, the expression $2+3 * 4+(3 * 4) / 5$ is represented by the tree in Figure 5.17(a). Give an $O(n)$ algorithm for evaluating such an expression, where there are $n$ nodes in the tree.

5. A vertex cover of a graph $G=(V, E)$ is a subset of vertices $V^{\prime} \in V$ such that every edge in $E$ contains at least one vertex from $V^{\prime}$. Delete all the leaves from any depth-first search tree of $G$. Must the remaining vertices form a vertex cover of G? Give a proof or a counterexample.
6. Consider the problem of determining whether a given undirected graph $G=(V, E)$ contains a triangle or cycle of length 3.
(a) Give an $\mathrm{O}\left(|\mathrm{V}|^{3}\right)$ to find a triangle if one exists.
(b) Improve your algorithm to run in time $\mathrm{O}(|\mathrm{V}| \cdot|\mathrm{E}|)$. You may assume $|\mathrm{V}| \leq|\mathrm{E}|$.
7. Consider a set of movies M1, M2, . . , Mk. There is a set of customers, each one of which indicates the two movies they would like to see this weekend. Movies are shown on Saturday evening and Sunday evening. Multiple movies may be screened at the same time. You must decide which movies should be televised on Saturday and which on Sunday, so that every customer gets to see the two movies they desire. Is there a schedule where each movie is shown at most once? Design an efficient algorithm to find such a schedule if one exists.
8. A mother vertex in a directed graph $G=(V, E)$ is a vertex $v$ such that all other vertices $G$ can be reached by a directed path from v .
(a) Give an $\mathrm{O}(\mathrm{n}+\mathrm{m})$ algorithm to test whether a given vertex v is a mother of G , where $\mathrm{n}=|\mathrm{V}|$ and $\mathrm{m}=|\mathrm{E}|$.
(b) Give an $\mathrm{O}(\mathrm{n}+\mathrm{m})$ algorithm to test whether graph G contains a mother vertex.
