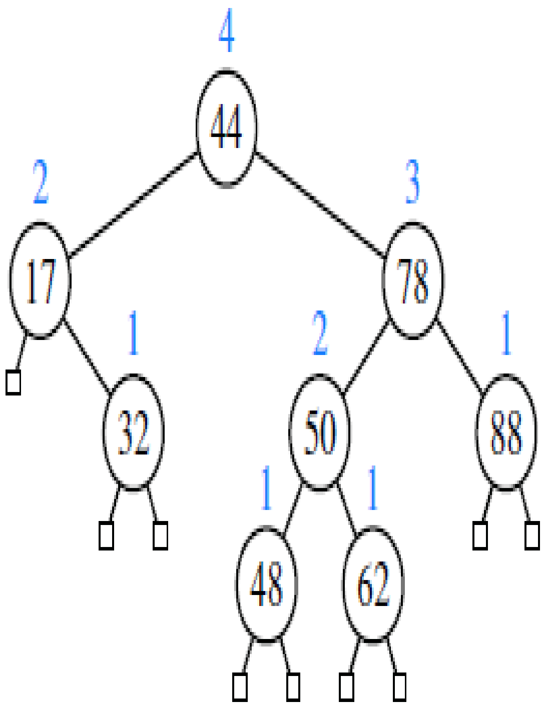
AVL TREE

Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

DEFINITION OF AN AVL TREE

- Any binary search tree *T* that satisfies the height-balance property is said to be an ***AVL tree***, named after the initials of its inventors: Adel'son-Vel'skii and Landis.
- ***Height-Balance Property***: For every internal position *p* of *T*, the heights of the children of *p* differ by at most 1.



PROPERTIES OF AVL TREE

- height-balance property allows
  - subtree of an AVL tree is itself an AVL tree.
  - The height of an AVL tree storing *n* entries is *O*(log*n*).

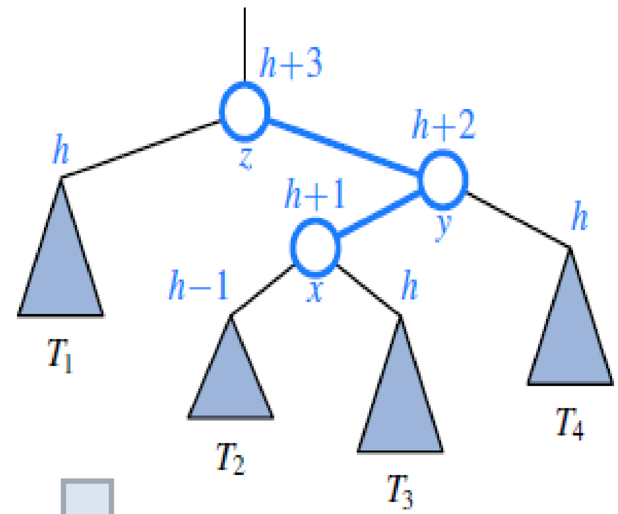
  (view 11.3 for the proof)
- height-balance property characterizing AVL trees is

  equivalent to saying that every position is balanced.
- Given a binary search tree *T*, we say that a position is ***balanced*** if the absolute value of the difference between the heights of its children is at most 1,
- ***AVL tree*** guarantees worst-case logarithmic running time for all the fundamental map operations

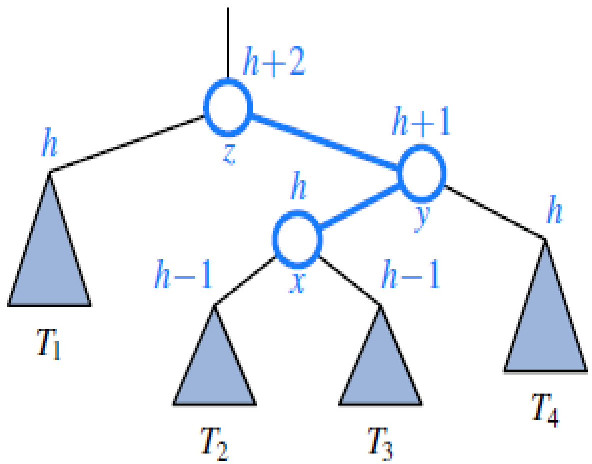
UPDATE OPERATIONS: INSERTION

- The insertion and deletion operations starts off with corresponding operations of (standard) binary search trees, but with post-processing for each operation to restore the balance
  - After insertion, the height-balance property may violated
  - Restructure *T* to fix any unbalance with a "search-and-repair" strategy.
- Any ancestor of *z* that became temporarily unbalanced becomes balanced again, and this one restructuring restores the height-balance property ***globally***.

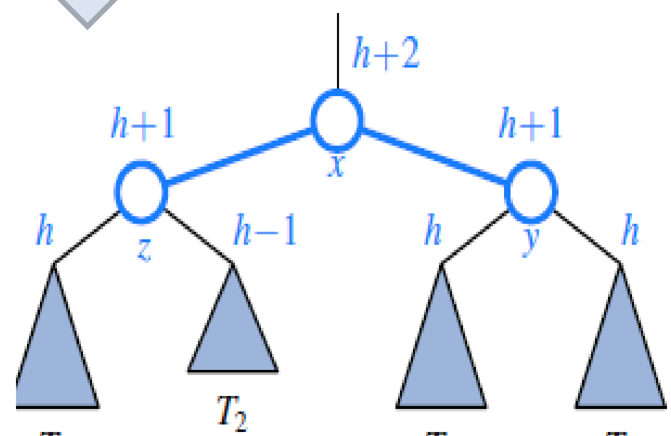
- Let *z* be the first position we encounter in going up from *p* toward the root of *T* such that *z* is unbalanced
- let *y* denote the child of *z* with greater height
- let *x* be the child of *y* with greater height (there cannot be a tie)
- Perform restructure(*x*)



after an insertion in subtree *T*3 causes imbalance at *z*



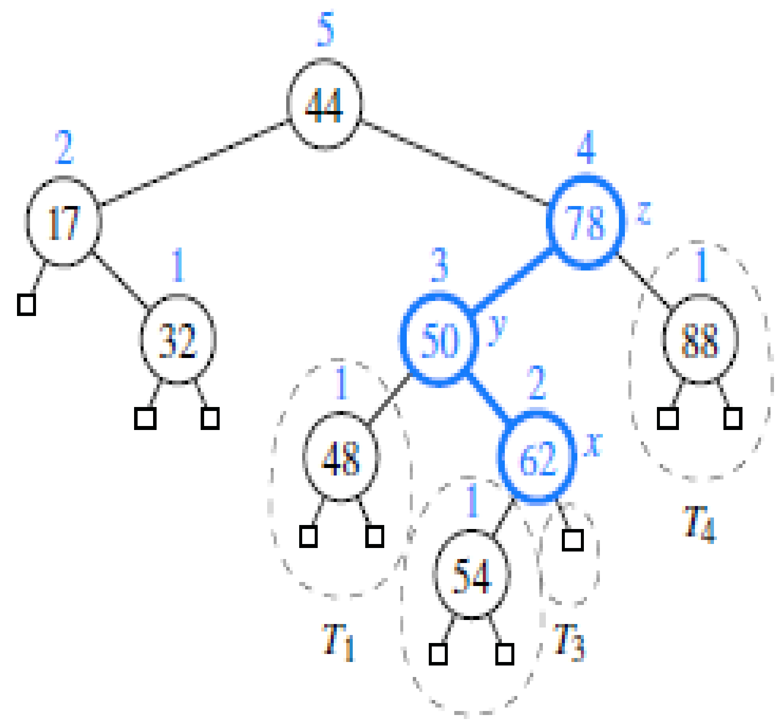
before the insertion



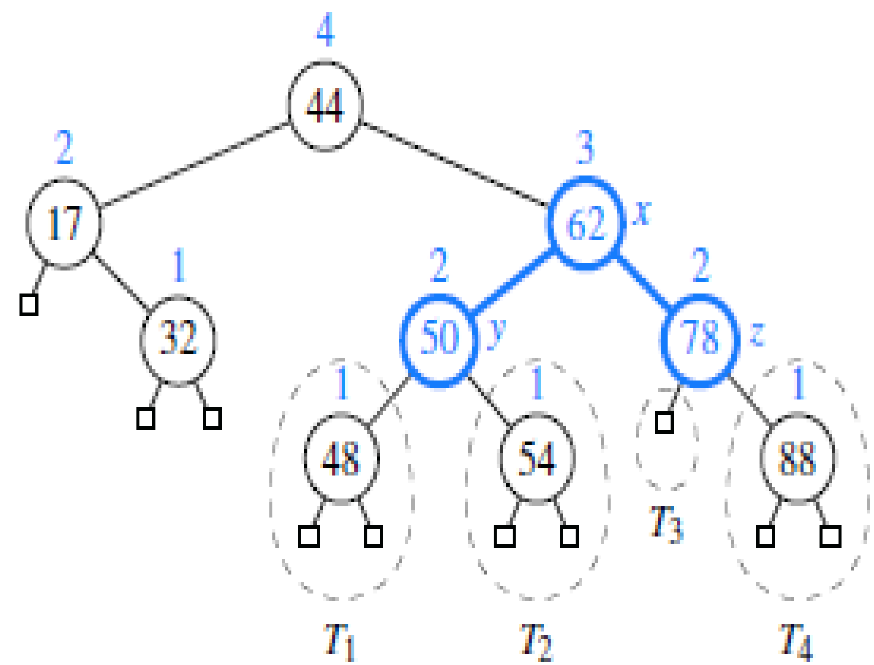
after restoring balance with trinode restructuring

EXAMPLE OF INSERT

insertion of an entry with key 54 in the AVL tree



after adding a new node for key 54, the nodes storing keys 78 and 44 become unbalanced;



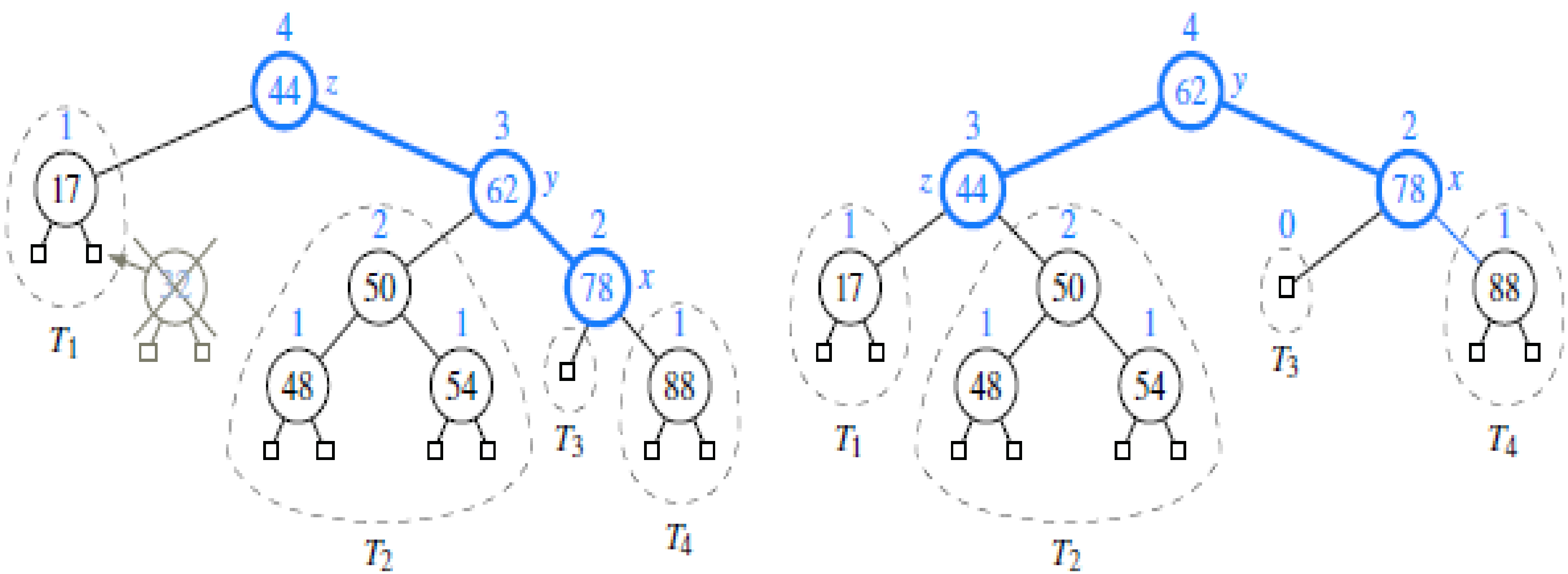
a trinode restructuring restores the height-balance property

UPDATE OPERATIONS: DELETION

- As with insertion, we use trinode restructuring to restore balance in the tree *T* after deletion.
- let *z* be the first unbalanced position encountered going up from *p* toward the root of *T*,
- let *y* be that child of *z* with greater height
- let *x* be the child of *y* defined as follows:
  - if one of the children of *y* is taller than the other, let *x* be the taller child of *y*;
  - else (both children of *y* have the same height), let *x* be the child of *y* on the same side as *y*
- Run restructure(*x*) operation.
- After rebalancing *z*, we continue walking up *T* looking for unbalanced positions
  - The height-balance property is <u>guaranteed to be ***locally*** restored within the subtree</u> of *b* but not globally.

EXAMPLE

Deletion of the entry with key 32 from the AVL tree



after removing the node storing key 32, the root becomes unbalanced

A trinode restructuring of *x*, *y*, and *z* restores the height-balance property.

PERFORMANCE OF AVL TREES

- the height of an AVL tree with *n* entries is guaranteed to be *O*(log*n*).

Method	Running Time
size, isEmpty	$O(1)$
get, put, remove	$O(\log n)$
firstEntry, lastEntry	$O(\log n)$
ceilingEntry, floorEntry, lowerEntry, higherEntry	$O(\log n)$
subMap	$O(s + \log n)$
entrySet, keySet, values	$O(n)$

JAVA IMPLEMENTATION OF AVL TREE 1.

```java
/** An implementation of a sorted map using an AVL tree. */
public class AVLTreeMap<K,V> extends TreeMap<K,V> {
  /** Constructs an empty map using the natural ordering of keys. */
  public AVLTreeMap() { super(); }
  /** Constructs an empty map using the given comparator to order keys. */
  public AVLTreeMap(Comparator<K> comp) { super(comp); }
  /** Returns the height of the given tree position. */
  protected int height(Position<Entry<K,V>> p) {
    return tree.getAux(p);
  }
  /** Recomputes the height of the given position based on its children's heights. */
  protected void recomputeHeight(Position<Entry<K,V>> p) {
    tree.setAux(p, 1 + Math.max(height(left(p)), height(right(p))));
  }
  /** Returns whether a position has balance factor between –1 and 1 inclusive. */
  protected boolean isBalanced(Position<Entry<K,V>> p) {
    return Math.abs(height(left(p)) – height(right(p))) <= 1;
  }
```

AVLTreeMap uses the node's auxiliary balancing variable to store the height of the subtree rooted at that node, with leaves having a balance factor of 0 by default.

JAVA IMPLEMENTATION OF AVL TREE 3

```
/** Returns a child of p with height no smaller than that of the other child. */
protected Position<Entry<K,V>> tallerChild(Position<Entry<K,V>> p) {
  if (height(left(p)) > height(right(p))) return left(p);      // clear winner
  if (height(left(p)) < height(right(p))) return right(p);     // clear winner
  // equal height children; break tie while matching parent's orientation
  if (isRoot(p)) return left(p);                 // choice is irrelevant
  if (p == left(parent(p))) return left(p);      // return aligned child
  else return right(p);
}

/** Overrides the TreeMap rebalancing hook that is called after an insertion. */
protected void rebalanceInsert(Position<Entry<K,V>> p) {
  rebalance(p);
}
/** Overrides the TreeMap rebalancing hook that is called after a deletion. */
protected void rebalanceDelete(Position<Entry<K,V>> p) {
  if (!isRoot(p))
    rebalance(parent(p));
}
}
```

JAVA IMPLEMENTATION OF AVL TREE 3

```
/**
 * Utility used to rebalance after an insert or removal operation. This traverses the
 * path upward from p, performing a trinode restructuring when imbalance is found,
 * continuing until balance is restored.
 */
protected void rebalance(Position<Entry<K,V>> p) {
  int oldHeight, newHeight;
  do {
    oldHeight = height(p);                     // not yet recalculated if internal
    if (!isBalanced(p)) {                      // imbalance detected
      // perform trinode restructuring, setting p to resulting root,
      // and recompute new local heights after the restructuring
      p = restructure(tallerChild(tallerChild(p)));
      recomputeHeight(left(p));
      recomputeHeight(right(p));
    }
    recomputeHeight(p);
    newHeight = height(p);
    p = parent(p);
  } while (oldHeight != newHeight && p != null);
}
```